THE COGNITIVE DIFFICULTIES OF FIRST YEAR PHYSICS STUDENTS AT THE UNIVERSITY OF THE WESTERN CAPE AND VARIOUS COMPENSATORY PROGRAMMES

by

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ABSTRACT

This thesis reports the results of some five years of work with first year physics students at the University of the Western Cape in South Africa. South African education is presently characterized by large numbers of students who, judging from their poor results in their first year at university, are poorly equipped to meet the academic requirements of a university course especially in science-related disciplines. The purpose of this research was twofold:

(a) To determine the cognitive reasons for the poor performance of these students, characterized as disadvantaged in the study;
(b) To develop suitable instructional materials based on the determination in (a).

Feuerstein's theories of the cognitive development of disadvantaged persons were found to be extremely useful both in providing direction for the research as well as in interpreting the data generated by the study.

Thirty person-to-person interviews of about 45 minutes in length, were conducted with first year physics students as they attempted to solve two typical kinematics problems. Analysis of the interviews revealed that in addition to language difficulties experienced, students also displayed regularities in the types of errors made. Over and above this, students demonstrated significant cognitive difficulties with the analysis and elaboration of data in the problems. Many of these difficulties

can be related to the cognitive deficiencies documented in the work of Feuerstein. Additionally, a paper-and-pencil test was developed to examine whether students, after instruction, display any planned approach in their use of Newton's laws in solving typical physics problems. The results of the test with 86 first year physics students show that traditional physics instruction does not help disadvantaged students develop the ability to use Newton's laws in any structured manner.

Instruments were developed to assist students using the algorithmic approaches of Landa. Nine areas in the typical university first year mechanics course were analysed for the cognitive operations required to use specific laws or concepts in problem-solving. Booklets were designed which made these cognitive operations explicit in the application of the particular law or concept. A test group which used the instruments showed significant gains in quarterly examinations in the physics department, over a control group which received instruction in the traditional manner. Anecdotal evidence of the efficacy of the instruments is also presented.

The success of the research shows that it is possible to address the cognitive difficulties of disadvantaged students in physics within the framework of a first year university course. The approaches documented in this thesis give an indication of how disadvantaged students can be assisted and supported academically.

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CHAPTER 1

INTRODUCTION

1.1 General Background

Why do some students fare so much better academically than others? Why do certain academic subjects appear to be so much more difficult than others? Why are some people more intelligent than others? What is intelligence? How can it be measured? Does education make people more intelligent or does it simply separate those of different intelligence? These are questions which have long occupied the thinking of philosophers, psychologists and educators. They do not, however, seem to have been of much concern to natural scientists for it has always been tacitly assumed that anyone who can cope well with physics, mathematics, chemistry etc. must be 'intelligent'. There seems to have developed a kind of intellectual superiority - and even mystery - around the ability to handle the mathematical sciences, related to names such as Einstein, Newton and Maxwell.

In the last twenty years or so, however, the situation has changed dramatically with the growth of disciplines such as physics, mathematics and chemical education. The launching of the Russian Sputnik caused educators in the United States of America to re-evaluate their approach to the teaching of the sciences. Based on the psychological theories of Piaget, a movement developed which not only assesses curricula more carefully, but also looks at the intellectual quality of the students for whom such curricula are designed (McKinnon and Renner, 1971; Science Curriculum Improvement Study, 1970). Using mainly a Piagetian framework, attempts were also made to address the learning problems in science of persons who are variously classified as 'Third World', 'minorities', 'retarded performers','disadvantaged' etc.(as an example, see the SOAR project, 1978; Bauman R).

Within the South African educational scene, however, very little of such innovation has taken place. South Africa, with its colonial history, has inherited with some major modifications, the Scottish educational system. After twelve years of schooling, (Sub-standards A and B, followed by standards 1 - 10, spanning ages 6 - 17), three successful years at university will produce a bachelor's degree e.g. B.Sc. The sixteen universities in the country have very similar curricula in science subjects such as physics, chemistry and mathematics. It was into this environment that the University of the Western Cape (UWC), situated some fifteen kilometres from the centre of Cape Town, was legislated into existence in 1959 to serve exclusively persons who are classified in South Africa as 'Coloured', that is, neither White nor Black but with roots in both, although in lifestyle much closer to the White group. UWC is one of two parallel-medium universities in the country. Most first year courses in the Faculty of Science are given in both English and Afrikaans, with students thus able to choose the medium of instruction which they prefer. The University has grown from 180 students in 1960 to almost 5000 students in 1983, which underscores the increasing importance attached to tertiary education by the so-called Coloured group. Most of the students are first generation university students, that is, they have no close relatives who are university graduates. (In the past year or so, this has led to the UWC being dubbed 'The University of the Working Class'!)

From the outset it was apparent at the University that the majority of students in the Faculty of Science was experiencing great difficulties. This was demonstrated very graphically by the fact that the first year pass-rate in subjects such as physics, mathematics and chemistry has always been in the region of only 30%. Students recognition of the difficulties which they were experiencing with science subjects is also demonstrated by the fact that while the Department of Afrikaans-Nederlands, in the Faculty of Arts, alone has over 1000 students, the Faculties of Science and Dentistry together have little over 300 students (1982 figures). It is indeed remarkable that virtually nothing had been done to tackle the problems experienced in the Faculty of Science (where individual departments have had pass-rates as low as 17%) for almost twenty years.

One of the main reasons for the inactivity in this area can be attributed to the fact that it has always been possible to blame the badly-equipped high schools and the poorly qualified teachers for the product that entered the university's gates. Also it was felt that a tertiary education institution has a primary responsibility to uphold certain standards and so the staff did not deem it necessary to be particularly concerned with the learning problems of students. It was assumed that when the school situation improved one day, better products would reach the university and hence more would pass.

It was within this context, then, that the present research was conceived. A number of factors contributed toward the actual approach which was finally adopted. These include the following:

i) It was regarded as simplistic in the extreme to assume that changing the school environment was in itself all that was necessary to ensure better products from the school system. If this were true, why was it then that students from the best so-called Coloured schools, where there are well-qualified teachers and reasonable equipment, also experienced so much difficulty in first year science?

ii) The entire focus of attention, especially after the riots of 1976, seemed to be on the improvement of physical facilities such as school buildings, equipment, etc. Little attention was given to the recipient of the instruction. Is he/she able to cope mentally with a subject like physics, and if not, what is there in his/her thinking processes which inhibits him/her?

iii) The point of departure in conducting the research was that it should not simply be an academic, statistical study which analysed the situation in quantitative detail. Rather, it had to also attempt to "do something" to remedy the situation. The emphasis throughout was therefore on trying to identify factors which prevent students from understanding physics properly and then to develop approaches to overcome these factors. Thus the factors to be dealt with should be those amenable to remediation or compensation by the research. For example, it is well-known that many students, especially those from lower socio-economic areas such as Manenberg, Hanover Park, etc., have no place in which they can study privately or peacefully at home. Since factors such as this are essentially socio-economic they were not regarded as within the scope of the present research.

iv) That a meaningful impact can be made on the students' learning problems was supported by a well-known UWC statistic: the best students in first year courses in the Faculty of Science are those who are repeating the course, having failed at their first attempt. These students have a pass-rate of around 90% in first year and frequently go on to complete their degrees without further serious trouble. It might be argued that what is necessary is that the B.Sc. course should be spread over four years or that students simply be allowed to fail at their first attempt. The problem with this view is twofold:

a) There is the undoubted public stigma attached to openly offering the only four-year B.Sc. course in South Africa. A four year degree would be both socially and politically unacceptable and hence difficult to get approved. Also, since this "slow" degree would be mainly for Black and Coloured students it would undoubtedly exacerbate the already poor self-image of the groups involved.

b) There are also major financial consequences for the repeating or four-year students. Since they are regarded as first year failures, they are usually unable to obtain further bursaries or scholarships and hence most will never return to attempt first year again. Hence a potential source of successful students is lost.

However, from the point of view of this research, the fact that students are able to cope with first year, albeit at the second attempt, provided the motivation for the idea that there is nothing essentially wrong with their cognitive capacity as such. On the contrary, when account is taken of their socio-economic circumstances and background, it is apparent that there are - not unexpectedly - many talented and able individuals.

1.2 Defining the Problem

In attempting to examine the poor results of UWC physics students, the area chosen for examination was the cognitive functioning of the individuals in the Physics I class. While it was realized at the outset that affective and motivational factors obviously also play an important part in the overall results of students, the ability to cope mentally with subject matter seemed to be a pre-eminent factor. Additionally, since much of a typical Physics I examination, and indeed much of the normal class time, is taken up in solving problems based on the concepts developed, the study had to focus especially on the cognitive requirements of problem-solving.

The research literature, while considering in some detail the mental operations involved in problem-solving, has virtually nothing to say about the way in which disadvantaged students approach such solutions in various content-areas in general. In particular, very little work has been done with such disadvantaged students in physics. The traditional approach to dealing with underprepared students is to provide them with what is missing in their background in the area of content, especially mathematics. Thus special courses concentrate on covering large amounts of material in the usual lecture mode (see for example, Gordon, et al, 1980; Kotnick, 1975). Additionally, it is felt that tutorial classes as part of the first year Physics course, do much to assist weaker students. (As a case in point, the Physics Department at UWC has always been extremely keen on weekly tutorial classes in which students are allowed some time to solve specified problems before these are worked out on the board by a tutor. The consensus of opinion among students, however, is that these sessions are of only limited benefit since they do not know how to tackle the problems in the first place). However, of late it has been realized that certain mental operations which can be regarded as fundamental in dealing with any advanced physics curriculum may also be absent or of very low efficiency in these students. Two approaches, especially, deserve mention because of the manner in which they deal with this problem:

i) Using the cognitive developmental theories of Piaget, the SOAR project (Stress On Analytical Reasoning) has taken various unrelated content-areas and attempted to assist students in acquiring what they call "the ability to read critically and to reason analytically" (Lincoln C E, 1978). Some of the reasoning patterns focused upon are the control of variables, proportional reasoning, combinatorial reasoning etc. - mental schemas readily associated with Piaget's theories. Their approach is to present students with problems which range in content from concrete experiences to the more abstract and whose solution requires the application of one or more of the identified schemas.

ii) In a related but nevertheless distinct approach, McDermott and her co-workers (1980,1982) have developed a course which is preliminary to the usual first year calculus-based physics course, and has a heavy emphasis on the "hands-on" mode of presentation. The difficulties students have are grouped into three categories:

a) Difficulty with basic concepts such as mass, volume, density;

b) Difficulty with scientific (mathematical) representations such as graphs;

c) Difficulty with scientific reasoning as, for example, proportional and analogical reasoning.

In this programme, basic concepts are presented, mathematical representations are clarified and reasoning patterns are developed as students, working individually or in small groups, solve problems while working with wires and bulbs, tracks and iron balls etc. They utilize the content of physics exclusively to address the three areas above. They hold that

> "the ability of students to transfer reasoning skills to new contexts is limited. Furthermore, a course that stresses scientific reasoning but lacks a strong subject matter base can hardly be expected to help students overcome

conceptual difficulties". (Rosenquist, 1982)

In considering the reasoning patterns mentioned above, it needs to be appreciated that these patterns are themselves aggregates of more fundamental thinking skills. Thus, for example, in setting about the solution of a problem involving, say, proportional thinking, the data in the problem needs to be correctly organized, only what is relevant must be utilized, different relations may need to be established, more than one source of information may need to be considered, etc etc. Students in general, and disadvantaged students in particular, may display thinking difficulties in any of these operations.

The work of Feuerstein and his co-workers has been aimed particularly at disadvantaged persons (Feuerstein, 1980, 1979). More than twenty years of research on disadvantaged Israelis have led them to believe that the primary effect of disadvantagement can be identified at the cognitive level. They hold that in many instances the observed level of cognitive activity of a disadvantaged individual is no reflection of that person's potential. This can be appreciated from the following definition of a retarded performer:

> "one who shows a level of cognitive activity which is retarded as measured by his present performance but which does not necessarily reflect his potential cognitive capacity."
>
> (Feuerstein, et al, 1980)

They regard socio-economic circumstances and related deprivation as secondary causes of academic disadvantagement. The primary determinant of retarded performance is embodied within an interesting psychosocial theory called Mediated Learning Experience (MLE). It is postulated that a lack of MLE results in cognitive functions which are deficient in the three phases of the mental act viz.:

i) Input : this relates to the manner in which data in a given problem is interpreted by the person.

ii) Elaboration: this concerns the ability to use the data in searching for a solution in some orderly fashion.

iii) Output : this phase of the mental act enables the person to communicate the result of (ii).

By means of clinical interviews Feuerstein and his co-workers have developed a comprehensive cognitive profile of a retarded performer, characterizing his behaviour, inter alia, by means of deficient cognitive functions. These cognitive operations may be regarded as providing a much finer-grained analysis of mental functions than the schemas of Piaget. To use a chemical analogy, they represent a molecular analysis of thinking patterns as opposed to the more molar approaches adopted in the two programmes discussed above.

Using the work done by Feuerstein as a guide, the present study has endeavoured to determine whether a sample of first year physics students at UWC demonstrated cognitive deficiencies similar to those documented by the work done in Israel, when dealing with the content of physics. The person-to-person interview technique was employed to determine how students approached two typical problems taken from the kinematics section of the mechanics course. Thirty such interviews were conducted and the transcripts analyzed. It was found that the difficulties students experienced could be categorized as follows:

i) Semantic difficulties : it is clear that, in reading problems, students do not necessarily interpret words correctly within the context of the problem.

ii) Error factors : these have been defined as macroscopic thinking errors which manifest themselves as persons solve problems (Pickthorne, 1983). These can be related to the frequently documented misconceptions which students have of concepts in science (see for example, Osborne, 1980).

iii) Cognitive deficiencies : a review of the interview transcripts indicates clearly that students have great difficulties in analyzing data in a well-defined, ordered manner. This in its turn greatly influences their ability to approach the solution of the problems in a structured manner.

iv) A number experienced problems with elementary mathematical manipulation.

Additionally, all new first year physics students (some 200) were given a written test (before receiving any university instruction) to determine the extent to which they had developed any problem solving procedures in applying Newton's Laws. These concepts were chosen because all students would have studied these in depth at school level. This represented a test of cognitive abilities in the elaborational phase of the mental act. In analyzing this test, difficulties in categories (ii), (iii), and (iv) above are clearly evident.

1.3 Remediation Procedures

There are a number of programmes in operation which attempt to remedy cognitive malfunctioning in individuals (for examples of three programmes, see Arbitman-Smith, et al, 1984). All of these operate in a content-free environment. In the "Instrumental Enrichment" project of Feuerstein (Feuerstein, et al, 1980) instruments have been developed which embody specific cognitive functions. As students solve problems involving things like dots, geometrical figures, word problems etc., they are assisted to develop mental processes in which they may be deficient or of low efficiency.

It is immediately obvious that no such programme, regardless of how successful, could be attempted within the very rigid framework of a first year university physics curriculum. Indeed, whether transfer of cognitive operations is possible from a content-free environment to a specific content-area is still an open question. Also, whether in fact, transfer takes place within the same content-area still requires investigation.

What seems, apparent, however, is that if a section of physics, say, requires specific cognitive functions to be understood and applied especially in solving problems, then the absence or low efficiency of such functions in students will obviously mean that they will experience difficulties with such sections. Remediation is then possible on two levels:

i) Specific, identified cognitive deficiencies in students need to be addressed by a directed programme.

ii) The cognitive operations implicit in a section of content need to be identified and material developed and presented in a manner which takes cognizance of the fact that the necessary operations may not be very well developed in the students. By way of illustration : if it is established that students show an inability to plan as part of the elaborational phase of the mental act, then in presenting a concept which has a particular implicit sequence of steps in its application, this needs to be made explicit in the development of the concept.

Within the constraints imposed upon the present study by the traditional manner in which the UWC physics department operates, the general approach embodied in (i) could not be attempted.

In attempting remediation on level (ii) above, it needs to be appreciated that the cognitive functioning of students is not necessarily being improved. It may very well be, but no claims of such permanent remediation are being made. At best, what has been done can be described as compensation for the deficiencies in the cognitive functioning of students in the following ways:

a) On the strength of the list of cognitive deficiencies students demonstrate, the content-area chosen, viz. Mechanics, had to be carefully analyzed to determine where such functions are required in applying particular concepts.

b) On the basis of the analysis made in (a), instruments had to be developed, using the content in question, which made explicit the cognitive functions which are, of course, implicit. The instruments had to, in a sense, simplify the thinking for the students.

In developing the instruments, the work done by Landa (1976) was found to most useful. He holds that much of what can be described as automatic in the thinking of persons is nothing else than the application of well-internalized algorithms or heuristics which become automatic in the user by sufficient practice. This is an approach also lately adopted by Gagné (1982).

While, as stated above, no claims of permanent cognitive change in students are made the present study certainly points towards a possible means of producing such changes. While no evidence of a transfer of cognitive operations to other content-areas was sought, anecdotal evidence from students indicated that a degree of transfer was taking place. One student, for example, said that he was attempting to good effect in sections of zoology, the same approaches he had learnt in the physics lectures.

It seems apparent that if disadvantaged students can be confronted on all content-fronts with material prepared in a manner such that implicit cognitive functions required (which may be deficient in them) are made explicit as part of the presentation of the area under discussion, many more might not only find the subjects more intellectually within their grasp, but the remediation of deficient cognitive functions might well take place. An undertaking of this nature is obviously far more ambitious than that attempted here, but if it can be accomplished then our educational process would really be teaching many more people to think.

CHAPTER 2

A LITERATURE SURVEY OF CURRENT RESEARCH IN PHYSICS EDUCATION RELEVANT TO DISADVANTAGED STUDENTS

2.1 INTRODUCTION

The past fifteen years have seen a tremendous growth in published articles reflecting research in physics education. While this has resulted in the main from educators attempting to reconcile psychological theory and classroom practice, there is also a growing body of empirical evidence describing the manner in which students interpret various physics concepts. Additionally, there have been significant advances made in the development of instructional approaches to accommodate theory-based presentation of material. The use of theories of learning has led to a number of paradigm-based science programmes. These are:

i) The Reception Learning Paradigm : this is based on the educational theory of Ausubel as elaborated by Novak (Ausubel, et al, 1978; Novak, 1979).

ii) The Hierarchical Learning Paradigm based on the work of Gagné (Jones, 1979).

iii) The Developmental Learning Paradigm. This uses the theories of Piaget on the cognitive development of children and applies them to the learning and teaching of science.

iv) The Information Processing Approach of Cognitive Psychology, with its emphasis on how problem-solving actually occurs.

That paradigm-based research has indeed made a significant impact on classroom practice in the United States of America, has been well demonstrated by Peterson (1979). However, it needs to be remembered that

> "research in education, as well as the design of instruction, can be importantly influenced by the paradigm that guides the work of the science education researcher.... advances in a discipline arise from systematically employing a paradigm to guide the work and there are at any time competing paradigms. The crucial factor is that each must be guided by some paradigm, for ultimately all paradigms are modified or discarded. The issue is not whether the guiding paradigm is true, but whether it is useful".
>
> (Novak, 1979).

Since most of the work done with disadvantaged students has been based upon the Developmental Learning Paradigm, it is appropriate to consider the manner in which Piaget's theories have been applied to physics education. Additionally, Lochhead (1979) points out that

> "while Piagetian programmes are appropriate for students who are poorly prepared, courses in problem-solving are often oriented to the superior student."

But this is, of course, the crux of the difficulty - what is it in the cognitive makeup of disadvantaged students that makes problem-solving so difficult for many? It seems apparent that since much work in Cognitive Psychology is now being done in the area of problem-solving, an analysis of research being conducted may well prove illuminating with respect to disadvantaged students. Since little research (in physics education) has been reported on the application of paradigms (1) and (2) above to disadvantaged students, it does not seem necessary to include these in the present literature survey.

Any review of current research in physics education related to disadvantaged students would be incomplete if the large body of phenomenologically-based research on the misconceptions or alternative conceptions students hold of basic physics concepts were excluded. Interesting work has demonstrated that disadvantaged students also demonstrate these misconceptions. Important work in this regard has been carried out in South Africa. (Hewson, 1981).

2.2 THE DEVELOPMENTAL LEARNING PARADIGM

2.2.1 A Resumé of Piaget's Theory

The effect on science education of Piaget's theories of human cognition has been enormous. They have affected not only the view which instructors hold of their students but also the realistic expectations they can have of them. They have influenced the development of science curricula. They lie at the root of the "hands-on" approach to teaching science. They have been used as the underlying theory for the generation of many programmes to deal with the problems of underprepared and underperforming students.

2.2.1.1 How Intelligence Develops

It is fundamental to Piaget's theory of development that intelligence is not a static, immutable characteristic of a person. Rather intelligence is viewed as specified by some intellectual abilities and thus it is measured by determining to what extent these abilities have been developed. The human being therefore passes through stages of development in which certain intellectual functions are possible while others are not. How then does the process of development take place?

A central component in development is the idea of an operation. Note how Piaget (1964: 176, 177) defines this:

> "Knowledge is not a copy of reality. To know an object, to know an event, is not simply to look at it and make a mental copy or image of it. To know an object is to act on it. To know is to modify, to transform the object, and to understand the process of this transformation, and as a consequence, to understand the way the object is constructed. An operation is thus the essence of knowledge; it is an interiorized action which modifies the object of knowledge. For instance, an operation would consist of joining objects in a class to construct a classification."

and

> "These operational structures are what seem to me to constitute the basis of knowledge, the natural psychological reality, in terms of which we must

understand the formation, elaboration, organization, and functioning of these structures."

Piaget views development as a spontaneous process which occurs as the person interacts with a complex world and solves self-generated problems. In so doing, operational structures develop as he finds answers and thus he not only acquires knowledge but also learns how to learn. This therefore constitutes a Stimulus-Organism-Response (S-O-R) situation where the learner is never the passive recipient of knowledge but rather is the active participant in the learning process.

The process of development is seen as depending on four factors:

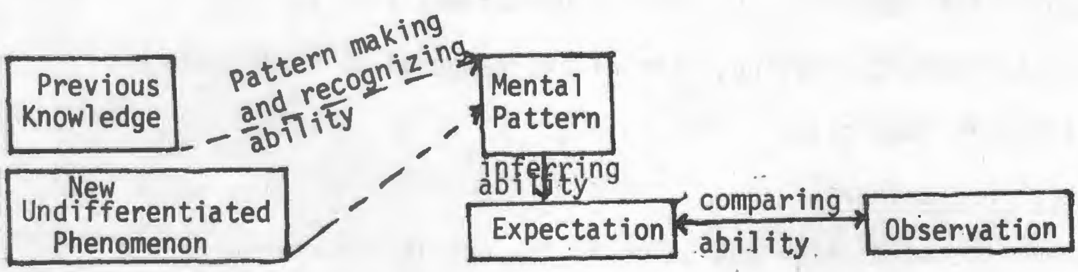
i) Maturation

ii) Experience

iii) Social transmission and

iv) Self-regulation

It is the fourth factor which Piaget regards as pre-eminent. A slight variation of Piaget's notion on this process has been proposed by Lawson (1979: 506):



If the expectation and observation agree, assimilation occurs easily. However, if the two differ, disequilibrium is present and no easy assimilation takes place. The operations available to the person at that stage of his development are brought to bear on the problem with the possible outcome being the development of further mental operations.

2.2.1.2 Stages of Development

Piaget postulates that intellectual development proceeds through four stages:

A Sensori-Motor

Practical knowledge develops along with the construction of a sensori-motor space.

B Pre-Operational

Here language develops but the ability to conserve (that is, the recognition that properties such as length, quantity, etc.remain invariant under certain transformations) is not yet formed.

C Concrete Operational

The first operations as defined above now appear. However, they are restricted because physical reality is always needed to give substance. Concepts which are developed can only arise as a result of first-hand experience. Operations such as classification, ordering, spatial and temporal abilities are now present in the child.

D Formal Operational

Hypothetico-deductive reasoning is now possible completely free of the presence of objects. Meanings are now given to concepts through reasoning. Models can be constructed and understood. Operations of propositional logic are now possible.

The presence of developmental levels across a large sample of British school children has been well demonstrated by Shayer and Adey (1981) using a series of Science Reasoning Tests (1979).

2.2.2 Application of Piaget's Theory to Physics Education

While there are many applications of Piagetian theory to science education (Kubli, 1979), there are two principal areas in which much of the research has been carried out. These are the determination of the cognitive ability of students and the matching of the curriculum to the various developmental levels of the learners.

A. View and Expectation of Students

Since the ability to employ hypothetico-deductive problem-solving procedures based on formal operational thinking patterns such as proportional reasoning, correlational reasoning and probabalistic thinking (Lawson, 1979), is intrinsic to any advanced physics curriculum, the presence in such courses of students who are concrete operational thinkers would be a cause for serious concern. Beginning in the early 1970's, a number of investigators examined the developmental levels of students in first year university classes. McKinnon and Renner (1971) demonstrated that in a class of 131 United States freshmen, 66 could be classified as concrete operational thinkers, while 32 could be placed in a transitional phase between concrete and formal operations. Many other investigators concur (Griffiths, 1976; Shayer, Küchemann, Wylam, 1976; Shayer, Wylam, 1978).

This in its turn, implies that many physics courses engage the bulk of students in a class on a cognitive level beyond their present ability. As Renner (1976) has pointed out:

> "each student must engage a subject in a manner appropriate to his or her present stage of development if he or she is to advance to the next stage of development."

He shows that concrete operational students who are required to interact with formal content neither achieve increased intellectual development nor understanding of the content itself. This realization has given momentum to efforts to:

i) produce suitable group tests of the developmental level of persons (Shayer, Adey, 1981; Lawson, 1978);

ii) correlate success in physics courses and Piagetian developmental levels in an effort to produce an instrument which can predict success (Barnes, 1977; Liberman, Hudson, 1979).

While (i) has been successful, (ii) has demonstrated that "students can somehow attain passing, if not high, grades in the more elementary of these courses even though they appear to lack the ability to think formally" (Barnes, 1977).

While at present researchers do not agree whether formal operational thinking as represented by the ten mental schemas is a unitary construct or whether each of the schemas should be regarded merely as an independent "reasoning pattern" (see Shayer and Adey, 1981, Chapter 7), there is agreement that students must be helped to acquire these abilities. As Karplus, et al (1979: 94,95) state

> "It would seem therefore, that teaching goals and teaching practice might well .... address themselves more consciously to the task of building formal operational structures."

B. Structure and Presentation of the Curriculum

The realization of the mismatch between curriculum requirements and the developmental level of the student for whom the curriculum is designed, has led to efforts to redesign the presentation of material based on Piagetian principles. The Piagetian "Learning Cycle" is given by Renner and Lawson (1973) as:

i) exploration : the student has concrete experience with materials.

ii) invention : establishing a new structure to explain the phenomena, perhaps with a new concept.

iii) discovery : further applications are discovered by students.

It is claimed that inquiry-oriented science courses such as the Science Curriculum Improvement Study (SCIS, 1970), promote more rapid intellectual growth (Renner, Stafford, 1970).

It has been demonstrated that properly designed homework problems can also assist the progress from concrete to some abstract modes of thinking through the process of self-regulation (Lawson, Wollman, 1975). Arons (1976) has demonstrated how eight instructional procedures and strategies can "impel students toward formal operational thought". Indeed, Schneider and Renner (1980) make a clear distinction between what they call "concrete and formal teaching".

In a slightly different approach Shayer and Adey (1981) attempt to remedy the mismatch between curriculum requirements and the developmental level of students for whom the curriculum is designed. Their "Curriculum Analysis Taxonomy" gives the developmental level required to cope with various topics in physics, chemistry and biology.

In essence then, the influence of Piagetian theory on science teaching has been

i) to make the laboratory rather than the chalkboard the focal point of the instructional exercise.

ii) to develop ways of helping students acquire the reasoning patterns associated with formal thought.

2.2.3 Application of Piaget's Theories to Disadvantaged Students

While there are a large number of physics programmes based on Piagetian principles which attempt to introduce acceleration mechanisms to assist students develop formal operational thinking, two programmes specifically directed at disadvantaged students will be discussed as being typical.

A. Project SOAR (Stress On Analytical Reasoning)

When reported upon (Lincoln C E, 1978; Carmichael J W, et al, 1980; Whimbey A, et al, 1980) the programme had four components:

i) Piagetian-based laboratory exercise in physics, mathematics, computer science, chemistry and biology

ii) Analytical reasoning/reading comprehension

iii) Vocabulary-building

iv) Quiz competition to reinforce (ii) and (iii).

The Piagetian-based programme consists of twenty five, three-hour laboratory exercises each organized to use the "learning cycle" of exploration, invention and application. The physics programme thus consists of five, three-hour sessions:

	Schema	Learning Cycle
Week 1 :	Control of Variables	Study of two systems which exhibit periodic motion
Week 2 :	Proportional Reasoning	Study of force required to move objects of varying masses
Week 3 :	Combinatorial Reasoning	Combination of switches which will ring a bell.
Week 4 :	Probability	Octahedral and cuboctahedral dice simulate radioactive decay.
Week 5 :	Recognizing Correlations	Relationship between variables in simple DC circuits

Carmichael, et al (1980) characterize the process as "a concrete approach because it introduces concepts at a concrete level before considering abstract relationships". It is claimed that students in the course show statistically significant gains toward formal operational thought.

B. Project PREP (Preparation for the Physical Sciences)

This programme has been concerned over a period of about six years with the development of a curriculum to meet the needs of "minority" students interested in pursuing science-related careers. The level of the course is equivalent to a university first year (McDermott, et al, 1980).

The curriculum is designed around two guiding principles:

i) concepts and reasoning are addressed together

ii) abstraction and generalization are preceded by direct experience.

Six modules have thus far been developed. These include Properties of Matter, Kinematics, Astronomy, Heat and Temperature, Electricity and Magnetism, Atomic-Molecular Model of Matter. The time taken to complete a single module varies from three weeks to one semester. Students attend for three sessions per week lasting two hours per session. There are at least five tutors to a class of around forty students.

What the programme attempts to accomplish can be deduced from the following student hand-out:

"Module 1 : Properties of Matter .

It is assumed that all courses using this curriculum will begin with the Properties of Matter module. Consisting of five parts, this module requires about one semester to complete. It begins with the construction of operational definitions of mass, volume and density. The materials have been carefully designed to compel the students to distinguish these concepts from one another. An investigation of properties of solutions leads to the introduction of the concepts of concentration and solubility. The subject matter background provided in the Properties of Matter module establishes a conceptual framework for the development of important scientific skills, such as the use of proportional reasoning, application of algebra to physical problems, interpretation and manipulation of straight line graphs, and construction of physical and mathematical analogies. In this module, as in all the others, a major emphasis is placed on helping students make connections among physical concepts, their scientific representations (words, graphs, formulas) and real world experience."

While it is nowhere explicitly stated, it is apparent that this programme also rests on broad Piagetian principles.

2.2.4 The Applicability of Piaget's Approach to University of the Western Cape (UWC) Physics Students

The South African educational scene is highly structured with great emphasis on what are vaguely called "standards", which are to be maintained at all cost. Within this framework formal curriculum development is rare. Despite the well-known difficulties of teaching science in "Black" and "Coloured" schools with laboratories either badly equipped or poorly utilized, little is done insofar as different teaching strategies are concerned.

The UWC is similarly constrained by tradition. As a so-called "ethnic" university it has an overriding concern with "standards" - usually those set by the "white" universities. Any attempts at innovation that do not square with approaches in "white" universities elicit responses of "inferior education" by students and the "Coloured" community alike. The professional physics community too, with its emphasis on "normal" research, is customarily slow in undertaking research into the needs of students and in incorporating the results of science education research into its usual lecture, tutorial and practical programmes. There is no appreciation for the fact that while the ultimate quality of courses must be maintained, the path followed to this end needs to be different for disadvantaged students.

That the UWC first year student needs help above the ordinary is clear. The first year pass-rate is of the order of 25%. In using Shayer's Science Reasoning Tests with 50 students (who are slightly older than UWC first years) at the Peninsula Technikon (a tertiary education institution also catering for "coloured" students, with emphasis on providing training for technicians in engineering and related disciplines), it was found that between 80% and 90% could be classified as at either the concrete operational or transitional level (that is between concrete and formal operational) (Lackay, 1982).

In a general survey administered to 300 incoming first year students at UWC in 1981, the following results were determined:

Question : Name two books on a scientific topic and if possible their authors that you have read recently.

Response : Those who had read	No books	One book	Two or more books
	63%	17%	20%

Question : For the following magazines, have you read them

a) often b) seldom c) never.

Response : Those who read	Never	Seldom	Often
i) Scientific American	75%	22%	3%
ii) Archimedes	37%	51%	12%

("Archimedes" is a South African science news digest written for senior schools)

Question : How often in your Standards 9 and 10 years at school have you yourself (or you plus a partner) performed an experiment in the school laboratory?

Response : Seldom to never : 78%

It would have required major administrative decisions to develop a laboratory-based Piagetian programme on the SOAR model (1978) or the hands-on approach of McDermott, et al (1980). Both the approaches, while effective, are very time-consuming. To incorporate this into the UWC Science Faculty, with its rigid year programme and inflexible timetable, would have been impossible. Financial considerations regarding students also make it difficult to introduce a pre-university year.

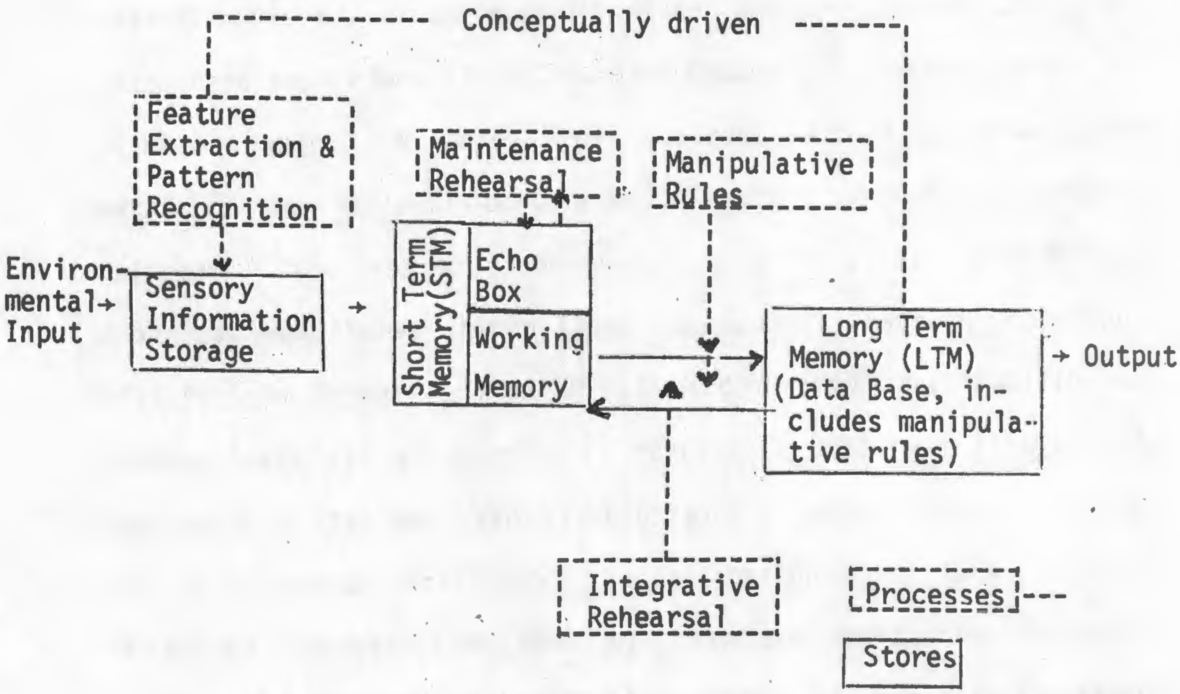
There is, however, another reason for seeking an approach different to the Piagetian. The Piagetian model as it would have been used, deals with operational levels. The taxonomy of Shayer and Adey, referred to earlier, gives a very good idea of the cognitive developmental level which a student needs to deal with particular concepts. However, it does not spell out in detail why student A has difficulty solving problem X. While the student may not have the general cognitive structures as evinced by his developmental level, is it possible to particularize his inability to a greater extent? What is required is a finer-grained analysis of the cognitive structures required to cope with various topics in physics. This will be discussed in detail in the next chapter.

2.3 The Information Processing Approach

The use in science education of the Information Processing Approach of Cognitive Psychology has grown markedly in the last ten years. Lately it has been proposed that Information Processing Psychology can form a very powerful "fourth paradigm" for science education (Stewart and Atkin, 1982).

2.3.1 The Information Processing Model of Memory

The essential features of the model of memory are given by Lindsay and Norman (1977) as follows:



According to this model human memory consists of at least three components with associated processes:

A. Sensory Information Store (SIS)

This is a transient store of sensory information (via, for example, the echoic and iconic memories) which gives a detailed replication of stimuli from the external environment as these impinge upon the human information processing system. The processes associated with this state are feature extraction and pattern recognition.

B. Short Term Memory (STM)

While this is also a transient memory state, information can be retained for up to some minutes by rehearsal. Information is processed serially up to maximum of 7 $\pm$ 2 chunks (Miller, 1956), where a chunk is a related segment of information not necessarily one discrete piece. It receives information from the SIS.

The STM appears to have two components :

i) Echo Box : Information can only be retained by rehearsal.

ii) Working Memory : It is from here that information can be integrated into the Long Term Memory by rehearsal. It also appears to serve as the "receptacle" for information retrieved from the Long Term Memory which is being processed and manipulated.

C. Long Term Memory (LTM)

The LTM is essentially infinite in capacity. Before knowledge can be used it must be transferred to STM, or the STM must contain a symbolic pointer to the location of the information in LTM. There needs to be not only a storage mechanism in LTM but also a means of access to the stored information, much like the indexing of books in a library. Thus (Simon, 1979 : 41):

"The LTM..... is itself a structure having two components. In analogy with an encyclopedia, we may say that the memory has both a text and an index. The information in the encyclopedia, contained in the text, is accessed rapidly by means of the pointers describing the index. The text of LTM is an associative structure - systems of nodes inter-connected by numerous links. Information can be retrieved from it not only via the index, but also by following paths of links from one node to another through intermediate nodes. Retrieval using the index we call recognition, retrieval using sequences of links we call association. The latter process is considerably slower than the former. Learning.... involves both (1) storing new nodes and links in the text of LTM and (2) elaborating the index to increase its powers of discrimination and recognition."

In addition to the search and retrieval procedure, the LTM also includes manipulative rules such as deduction, class inclusion and generalization. Simon, et al (1976) hold that:

"the behaviour humans will show in a task environment depends on the strategies they use. The qualitative and quantitative differences in performance between subjects are due in part to differences in their strategies for processing the information available to them; the changes in a particular subject's performance as he becomes experienced at a task are largely due to progressive changes in his strategies".

The concept of production systems gives insight into how strategies are developed. The production system (ps) is due to Waterman (1970) and has been refined by Klahr and Wallace (1976). In this model, the ps is a programme made up of a large number of productions, each consisting of two parts, a condition and an action. Thus if the condition for a particular production is satisfied, the action is executed e.g. condition: red traffic light; action: stop car. In terms of the LTM model stated above, the conditions are the index by means of which the actions retained in memory are activated. The operation of the production is described as follows by Klahr and Wallace (1976: 7):

> "The condition of each production is matched against the symbols in STM. If all the elements in a condition can be matched with elements (in any order) in STM the condition is satisfied. When a production "fires", the actions associated with it are taken. Actions can change the state of goals, replace elements, apply operators, or add elements to STM".

An overall strategy can be built up from individual productions in relevant order or sequence to enable a particular goal to be reached.

Wagner and Geeslin (1981) relate production systems to what they call, "the algorithmic component" of LTM i.e. "step-by-step procedures needed to reach some goal".

2.3.2 Application of the Information Processing Model to Physics Education

Much of the research carried out to date in using the information processing model (IPM) in physics education has been to analyse the manner in which persons, both novices and experts, solve problems in physics. Related to the manner in which problems are solved, is the way in which knowledge is organized in LTM i.e. how learning takes place. An outline of recent research in each of these related areas will be given below.

2.3.2.1 Learning in the IPM

Research has indicated that the knowledge a person possesses of a particular subject is highly interrelated. Thus, for example, physics students asked to tell what they thought of when given the words "river steamer", specified ideas about relative motion, or boats crossing streams at particular velocities, etc. (Larkin, 1978). Considerations such as these have given rise to the idea that information is stored in LTM in so-called "semantic networks". As Stewart and Atkin (1982: 325) express it :

> "One representational scheme, the semantic network, uses various symbols to represent "the net of interconnections among meaningful (semantic) components in memory" (Lindsay & Norman, 1977). There are two basic aspects of semantic networks : nodes, representing types of information (such as class concepts, names, or events); and lines representing specific ways in which the nodes are related.

> Semantic networks are models of how conceptual information might be stored in an individual's long term memory; as such they can be used to give meaning to terms such as "understanding" or "meaningful learning".

Larkin has demonstrated the presence of "large-scale functional units" of information in the memory of the physics "experts" as opposed to the rather discrete pieces of information held by the "novice" physics student. She shows that there is the need to help the student to relate bits of information into functional units as he or she learns (Larkin, 1978). As Fuller (1982: 46) expresses it:

> "physicists have organized their knowledge into large coherent "chunks" of information, more accessible than individual principles and equations."

Eylon and Reif (1979) studied the effects of "human internal knowledge organization". They demonstrate that the manner in which an experienced physicist retains information in memory demonstrates "hierarchical organization designed to facilitate selective information retrieval".

In related research, Clement (1978) proposes a "Model for Understanding" which consists of "the several types of knowledge needed for a person to understand a topic in physics". These are:

Internal Knowledge Domains

1. Practical Knowledge
2. Qualitative Physical Models
3. Concrete Mathematical Models
4. Written Symbol Manipulation
5. Objects, Events and Actions in the External World

The organization of knowledge in memory extends also to the means of retrieving it. As Larkin, et al (1980: 1336) express it:

> "Although a sizeable body of knowledge is prerequisite to expert skill, that knowledge must be indexed by large numbers of patterns that, on recognition, guide the expert in a fraction of a second to relevant parts of the knowledge store. The knowledge forms complex schematas that can guide a problem's interpretation and solution and that constitute a large part of what we call physical intuition."

It is postulated that this indexing is accomplished by organizing discrete pieces of knowledge into so-called "Condition-Action Units" (Larkin, 1978). The "condition" represents the cue held in STM for the "action" to be executed. It has been demonstrated that while physics textbooks generally teach useful actions, they seldom give the conditions under which the actions are to be implemented (Larkin, 1978; Larkin, et al, 1980).

2.3.2.2 Problem Solving in the IPM

Because the standard physics course requires the mastery of relatively few concepts and usually engages the student in the solution of well-structured problems, it has proved to be a fruitful area of research into the manner in which both experts and novices approach the solution of problems. Detailed observations can be made of individuals under controlled experimental conditions. The person-to-person interview technique, either video- or audiotaped is generally used as the individual works through problems.

Using this method, Reif (1977, 1981) has demonstrated that experts initially appear to be fairly imprecise in their verbal or pictorial analysis of a problem. They do not focus on the detail of the problem or consider relevant equations - as opposed to the novice who solves problems by accumulating discrete equations (Larkin and Reif, 1978). He gives the following prerequisites to problem-solving:

1. An efficient strategy to decompose the problem into sub-problems.
2. A knowledge base which contains a set of solvable problems that can act as building blocks for the solution of more complex problems.
3. A carefully organized knowledge base which allows easily retrievable information in different contexts.

As Fuller (1982) expresses it:

> "There appear to be two important characteristics of expert problemsolvers. First the experts approach solutions to physics problems through a process of successive refinements. They start with a gross description of the problem in words and drawings. Only later do they examine the details of the problems and introduce mathematics."

It has become apparent that the solution of problems requires a type of knowledge distinct from that usually associated with textbooks i.e. domain-specific knowledge. This has come to be called "procedural knowledge" and relates more to the manner of dealing with problems than the prerequisite facts required. In discussing the implementation of a model of thinking processes using a computer, Larkin (1982: 4) notes:

> "the proposed model explicitly separates knowledge which appears in the textbook from other knowledge which does not appear, but which is necessary to solve problems. There is a lot of this "other" knowledge and it is often not processed by learners. Making this other knowledge explicit allows it to be taught directly ....."

By utilizing computer-implemented models Larkin, et al (1980a) have demonstrated that for persons solving problems in physics "the main features accounting for different competences are differences in strategy for selecting physics principles, and differences in the degree of automation in the process of applying a single principle". Schoenfeld (1978) has shown that students' abilities to evaluate integrals can be greatly enhanced if they are given a coherent manner of approaching problems. His research methodology includes the observation of experts to determine

a) regularities, strategies or implicit rules experts may unconsciously use as they solve problems.

b) A decomposition of the problem solving process into smaller "chunks" which can be taught to students.

To assist in the development of the qualitative reasoning associated with problem-solving, attempts have been made to develop general heuristics applicable to any physics problem. These have grown in both detail and sophistication over the past ten years.

An early effort, acknowledged to be "primitive" by the authors, is given by Reif, et al (1976). The strategy consists of four major steps :

1. Description : Listing explicitly given and desired information and drawing a sketch.
2. Planning : Selecting relevant relations and outlining how these are applied.
3. Implementation : Executing the plan developed.
4. Checking : Checking the above steps and the answer obtained.

As the research has developed, the four general operations above have been subdivided into a large number of smaller, discrete steps (Reif, 1984). Lately the computer has been used to train students in the implementation of such a heuristic in physics. (Van Humbeeck, et al, 1982).

Clearly, cognitive psychology, with its elucidation of fundamental thought processes will continue to grow in its impact on both the teaching and learning situation. It has the potential for helping educators to also come to grips with the problems disadvantaged students bring to a study of physics.

2.3.3 Applications of Information Processing to Disadvantaged Students

While there is a growing interest in research into improving the thinking skills of individuals (see for example, Bransford, et al, 1984; Sternberg, 1984), little work has been reported upon in this field for disadvantaged students in science in general and physics in particular. An exception to this is the SOAR project, where one component of the programme is described as "The Cognitive Skills Approach to Teaching Analytical Reasoning" (Whimbey, et al, 1980). The program organizers describe this component as follows:

> "Cognitive Therapy : The second component in Project SOAR ...... was five hours per week of "cognitive therapy" designed to teach critical reading/analytical reasoning as measured on instruments such as SAT, the ACT, and the Nelson Denny Reading Exam."

The primary text for the classes was "Problem-Solving and Comprehension" (Whimbey and Lochhead, 1979). Students were given analytical reading and reasoning problems such as :

1. Cross out the letter after the letter in the word pardon which is in the same position in the word as it is in the alphabet.

The particular pedagogy used is to group students in pairs and let one of the pair solve a problem while thinking aloud. The role of the listener is then not only to observe, but also to check the accuracy of the problem-solver and to point out any errors immediately these are made. Impressive gains in student scores are reported (Whimbey, et al, 1980). The rationale behind the approach based on cognitive theory, is given by Lochhead (1980).

It is apparent then, that there is a great opportunity - and a pressing need - to apply the results of research into Information Processing to the learning problems of disadvantaged students in physics.

2.4 Research into the Misconceptions held by Students

A large body of empirical evidence has accumulated over the years which indicates clearly that young children, high school pupils and even university students hold "incorrect" scientific ideas of the world which are extremely resistant to change in spite of the formal instruction they receive in Newtonian mechanics etc. It has also been shown that similar ideas are held by large numbers of students. These can be classified as errors which are macroscopic in nature (as opposed to more microscopic reasoning errors) and can be identified with so-called "error factors" (Pickthorne, 1983). These pseudo-scientific ideas are variously called "misconceptions" or " alternative conceptions".

2.4.1 A Summary of Research in Physics into Misconceptions

Physics education is at present experiencing a burgeoning research effort in the field of misconceptions or alternative conceptions. It is clear that children bring with them to a study of science a vast complex of ideas and concepts from their experience in life. This "residual" science interacts with what they are taught and the results are not always what instructors assume they are. Examples of the scientific ideas of children in general are given by Rodriques (1980) while researchers like Trowbridge and McDermott (1980, 1981) and Osborne and Gilbert (1980), have studied specific topics such as kinematics and force respectively. A plethora of topics like these have been investigated either by clinical interviews of varying duration or by paper-and-pencil tests (for examples, see Viennot, 1979; Osborne, 1982).

In trying to determine children's ideas about force, Osborne and Gilbert (1980) cite the following as examples of what children believe in answer to the "Interview-about-instances cards" given to them. In answering the question "Is there a force on the car/bike?" (see examples of the cards given in the sketch on this page), they provided answers such as:

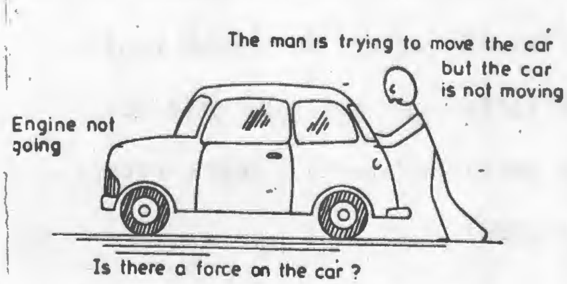
Car: "No because it can't feel anything, but there is a force on the man because he has to push it and that puts a force on him" (9 year old)

Bike: "No ... not really because he is not pedalling or anything" (9, 11, 13 year old)

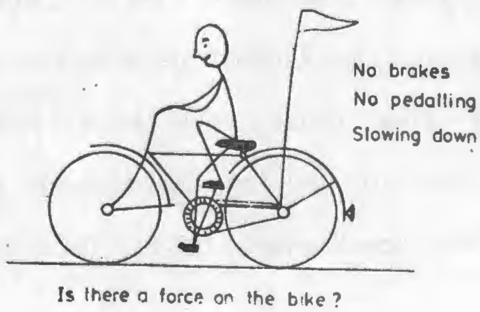
Bike: "Yes because it is forcing itself to stop" (9 year old)

Even after considerable instruction in science (to English "O" and "A" Level standards), students give the following responses to the "bike" question:

> "There's a force because of the bike's own mass ... the mass of the bike has come to such a speed that it won't just stop straight away ... the force is still in there ... in the bike ... the force was transferred to the person pedalling ... and it is now still adherent in the bike ... the bike still moves forward." (19 year old)



Is there a force on the car ?



Is there a force on the bike ?

There is obviously the question of language present, as is seen in the penultimate response above. Some idea of momentum as opposed to force seems to dominate this student's thinking (see Lawson, et al, 1981, p. 5).

Recently the emphasis is shifting from the documentation of misconceptions to

a) looking for the underlying cognitive reasons for misconceptions (Mehl & Volmink, 1983).
b) the development of instruction which recognizes the presence of alternative views of the world in those being taught (Osborne, 1984).

A comprehensive review of the state-of-art in misconceptions research as well as implications for instruction is given by McDermott (1984).

2.4.2 Misconceptions Among Disadvantaged Students

Few studies have been reported on the particular misconceptions held by disadvantaged students. The Physics Education group at the University of Washington in Seattle, USA, has included academically disadvantaged students as a separate group within a large study of first year physics students. They attempted to determine conceptions which students hold of velocity and acceleration by conducting individual demonstration interviews as students analyzed the manner in which metal balls moved along tracks (Trowbridge and McDermott, 1980 and 1981). The common misconception, that if two objects are at the same position they also have the same speed, was more widely held among disadvantaged students than the general student population. However, after instruction, disadvantaged students showed the greatest improvement. In the acceleration task it was found that disadvantaged students performed very much like the general student population, i.e. that both before and after instruction the concept of acceleration was not well understood.

In the single reported study carried out with black high school students in South Africa, Hewson (1981) examined the conceptions which students held of mass, volume and density. These were determined by means of clinical interviews on a floating-and-sinking task. Students confused mass/weight and heaviness, felt that only certain objects have mass, considered volume as equal to size or quantity and felt that density was related to "crowdedness", among a number of responses. An instructional strategy was developed using a model which produces conceptual change based on Ausebelian theory. A test group showed significant gains over a control group in the acquisition of the three scientific concepts.

Thus, while there are only a few studies which produce evidence specifically describing misconceptions among disadvantaged students, it is certainly safe to assume that such misconceptions do indeed exist among this group.

2.5 CONCLUSION

This review of current research in Physics Education indicates that relatively little attention has been specifically given to disadvantaged students. Most of the research has been conducted within a Piagetian paradigm with emphasis on the "hands-on" approach. In particular, little has been done to examine cognitive difficulties of disadvantaged students. Even within the SOAR programme with its module on pair problem-solving, it is not clear what cognitive mechanisms are at work (see Lochhead, 1980).
Thus, insofar as disadvantaged students in physics are concerned, there is a clear need for a more specific paradigm which can:

i) Identify the nature of cognitive difficulties which they may have,

ii) Suggest remediation mechanisms which can be attempted and which can thus be expected to influence instruction and instructional materials.

Even a cursory reading of any standard first year university physics text will demonstrate that there is very little which has been designed specifically with disadvantaged students in mind.

CHAPTER 3

DIAGNOSTIC AND REMEDIAL APPROACHES TO DISADVANTAGED STUDENTS IN PHYSICS

3.1 INTRODUCTION

The notion that an individual's ability to think is amenable to training has been propagated for a number of years. In 1976 De Bono published his book "Teaching Thinking" which says in its foreword that the book is based:

> "on what may well be the largest programme anywhere in the world for the direct teaching of thinking as a skill and, quite apart from this, on considerable experience in the teaching of thinking to somewhat demanding pupils".
>
> (De Bono, 1976: 8)

Bransford, et al (1984) analyze the nature of thinking and learning embodied in three programmes:

i) Feuerstein's "Instrumental Enrichment",

ii) Lipman, Sharp and Oscanyon's "Philosophy for Children", and

iii) Whimbey and Lochhead's short course in "Problem Solving and Comprehension".

To these, Sternberg (1984) adds the "Chigaco Mastery Learning Program" defining it and (i) and (ii) above as programmes which "train intelligence rather than merely measuring it". He suggests that (Sternberg, 1984: 39)

> "if intelligence can be broken down into a set of underlying processes and strategies for combining

the processes, then it is clear what we can do to improve it : we can intervene at the level of the mental process and teach individuals what processes to use when, how to use them, and how to combine them into workable strategies for task solution."

This view of intelligence is central to the programme developed by Feuerstein, et al (1980, 1979). It is the only programme of its kind in the world which was developed almost exclusively for disadvantaged persons. It documents the cognitive characteristics of disadvantaged persons in considerable detail. Using components of Feuerstein's Cognitive Map as a guide (see 3.2.3 below), it was possible to determine the extent to which physics students at the University of the Western Cape demonstrated similar characteristics to those found by Feuerstein with disadvantaged Israelis. Feuerstein's approach could thus be used as a diagnostic tool of cognitive characteristics in the UWC context.

Once an idea is obtained of the cognitive makeup of the physics student, intervention of the type suggested above by Sternberg becomes possible. It was found that the UWC physics students' grasp of concepts especially within a problem solving situation was greatly enhanced if cognitive deficiencies which were highlighted in the diagnostic procedure could be compensated by ordering necessary cognitive functions using the algorithmic and/or heuristic methods of Landa (1974, 1976).

In this chapter, an overview of the Feuerstein approach with some indication of its relevance to physics education, will be given.

The Landa approach, which represents a remediatory tool, will also be discussed. Later chapters will demonstrate the application of the methods to physics students at the University of the Western Cape.

3.2 The Feuerstein Approach - A Fruitful Paradigm for Understanding the Problems Experienced by Disadvantaged Students

As has been stated above, while it is possible to analyse cognitive processes, this does not necessarily elucidate the learning difficulties of disadvantaged students. While a definition of intelligence is given later, Piaget's theories have demonstrated very clearly that the notion of intelligence as something immutable and hence open to a purely psychometric approach is untenable. Intelligence, rather than being the static, measured result of some test, is an active dynamic variable subject to changes in each individual. This view provides the hope that human beings who because of history and circumstance are underperforming members of society, need not always have to operate in an environment suitably changed to meet their cognitive capabilities, but can have their cognitive processes improved to meet the challenges of a "standard" society. This approach lies at the root of the programmes developed by Reuven Feuerstein and his co-workers in Israel. This will now be discussed with an indication of the applications to physics education for disadvantaged students.

3.2.1 The Concept of Cognitive Modifiability

What causes retarded performance in an individual? Is it possible to change the cognitive abilities of human beings? The psychometric tradition of psychology with its measurement of the intelligence quotient (IQ) of each person, implies an overwhelming genetic role in the establishment of a human being's intelligence (Jensen 1969, 1973). Hence retarded performance becomes largely an accident of birth. The IQ test is, in this view, not only the predictor of what is presently the ability of the individual, but also largely what that ability will be in the future. Since tests have shown that certain groups do consistently worse on IQ tests than other, (Jensen 1977), the IQ becomes a useful tool as a moral justification for many types of categorization of peoples. The most telling argument against the notion of intelligence as something static has come from the work of Piaget, discussed above. The opposite position from the psychometric approach is that which implies that all cognitive differences are merely cultural, i.e. if the environment is changed, the retarded performance will be largely eliminated. That culture influences the manner of thought of individuals in that culture, is inescapable. Consider the following dialogue (Cole, et al, 1974: 187, 188):

"<u>Experimenter</u>: Flumo and Yakpalo always drink cain juice (rum) together. Flumo is drinking cain juice. Is Yakpalo drinking cain juice?

<u>Subject</u>: The day that Flumo was drinking the cain juice Yakpalo was not there on that day.

<u>Experimenter</u>: What is the reason?

2 verskillende
omgewings
Disadvantaged children

<u>Subject</u>: The reason is that Yakpalo went to his farm on that day and Flumo remained in town on that day."

This was from a study done on the Kpelle in Central Liberia. Note that the reply is plausible. It is not, of course, that dictated by logical necessity.

This indicates that the way in which information is processed is different among different people. However, to argue that a simple change of environment will also change the cognitive functioning of the individual, is, of course, to oversimplify the situation.

It (in common with the psychometric approach) fails to recognize a crucial component. Note how this is stated by Feuerstein, et al (1980: 7):

> "The neglect of cognitive processes has conspired to produce a widespread belief that intelligence is something that one either has or does not have and that attempts to change the structure and course of intellectual development are futile if not impossible."

Feuerstein's approach makes possible a different definition of intelligence:

> "Intelligence is usefully defined as a set of processes of logical thought, referred to henceforth as cognitive functions"
>
> (Ruth Arbitman-Smith, et al, 1982)

> "Intelligence is the capacity of the organism to use previously acquired principles, skills, and strategies for its adaptation to new situations."
>
> (Feuerstein 1979a)

Cognitive functions are then defined as process variables that are themselves compounds of native ability, work habits, attitudes, learning history, motives and strategies (Haywood 1977).
If this approach is accepted, then it follows that cognitive functions are themselves open to change. It is then not simply a question of altering certain specific abilities, but rather producing changes of a structural nature (Feuerstein, et al, 1980), i.e. this refers to the manner in which the person acts upon information which he receives. It thus becomes possible by means of a programme of intervention, to change the cognitive functioning of an individual, i.e. to produce "cognitive modifiability".
The success of one specific programme (Instrumental Enrichment) is documented in two books by Feuerstein, et al (1979, 1980) which cite various case studies.
Seen in this light, then, retarded performance is not some static definitive characteristic of the person, but rather a variable which is subject to modifiability by means of suitable intervention strategies. This can be seen in Feuerstein's definition of a retarded performer (Feuerstein, 1970) :

> "The cognitive structure of the culturally deprived, low-functioning individual is characterized by his low degree of modifiability i.e.learning by direct exposure to stimuli"

Note that the definition in no way implies that changes in cognitive structure are not possible. Rather it indicates that in his present state the individual is not able to react meaningfully to information reaching him - some type of intervention mechanism becomes necessary to acquire the necessary cognitive structures. The concept of cognitive modifiability implies not the question of whether change is possible but rather how and by how much. For disadvantaged students this means that instruction needs to be designed so that it not only provides the necessary content, but also the processes of thought required to acquire this content.

To enable this to be done it is important to examine how retarded performance arises. This will be done via a psychosocial theory called, by Feuerstein, "Mediated Learning Experience".

3.2.2 Mediated Learning Experience

How is cognitive structure developed in the small child? The model postulated by Piaget (1969) is that of Stimulus-Organism-Response (S-O-R). The organism in responding to and reacting with the stimulus impinging upon it, undergoes a learning experience. In this manner, cognitive structures are built up.

However, Feuerstein regards this as only one of the two modalities for the development of cognitive structures. Note how this is expressed (Feuerstein, 1979a : 365):

> "the sole use of the direct-exposure modality, even when it takes into account constitutional variations of the organism, will never explain differential cognitive development, horizontal decallage, and - what is even more important - the fact that so few people in our world attain the higher level of functioning described by Piaget as formal operations."

The second modality he calls Mediated Learning Experience (MLE) and it essentially interposes another factor into the S-O-R model of Piaget, namely S-H-O-R. "H" here, is the human mediator who interprets the stimulus for the organism both intentionally and with an interpretation which transcends the immediate needs of the situation. Thus, for example, the operation of a red traffic light on the movement of a motor car: the S-O-R would simply be that "red" implies "stop". However, the intentional human mediator can interpret this as more than a simple coloured light, but rather all that it represents in terms of an ordered life, the operation of government, etc. The experience becomes more than a simple command instruction, for here the organism is helped to become part of a decision-making process. Note in the following mediated operation how cognitive functions can be established by MLE:

Selection and Scheduling of Stimuli:

A caring parent does this naturally especially for reasons of protection from harm. Three things are accomplished:

i) The importance of some stimuli over others i.e.perception needs to be developed so that categorization can take place.

ii) Selection of certain stimuli over others for the needs of a particular situation.

iii) The mediator ensures that certain stimuli do not impinge upon the organism randomly; an awareness of both a temporal and spatial dimension is transmitted to the child.

These relate to how a person orders the world, how he interprets data, what he regards as more important for the needs of the moment etc. (It will be useful to compare the above to the list of cognitive deficiencies in the following section).

Feuerstein adds a number of other mediated interactions such as anticipation, imitation, the provision of specific stimuli, repetition and variation, etc.

What does mediated learning accomplish? Note how succintly this is expressed (Feuerstein, et al, 1980: 25):

> "In other words, MLE produces in the organism a propensity to learn how to learn, by equipping the organism with the tools necessary for this facility."

Seen in this light MLE becomes the proximal determinant (i.e. the factor with direct influence) in the etiology of under-performers. It transcends the distal determinants (i.e. secondary factors) such as genetic factors, socio-economic background, education of parents, etc. While any of the distal factors could spark the proximal one, (i.e. lack of MLE) this is not necessarily so. The distal factors on their own do not bring about retarded performance in an individual, as experience has often shown. Only if the distal factors also generate a lack of MLE, is retarded performance the result; i.e. a lack of MLE is the strong contributing factor toward retarded performance which is manifested by the non-development or underdevelopment of the cognitive functions which would enable the individual to deal with his environment. This situation is represented in the figure on the following page (Feuerstein and Rand, 1974).

In the following quote, the relationship between cognitive modifiability and MLE (Feuerstein, et al, 1980:19) is clearly described:

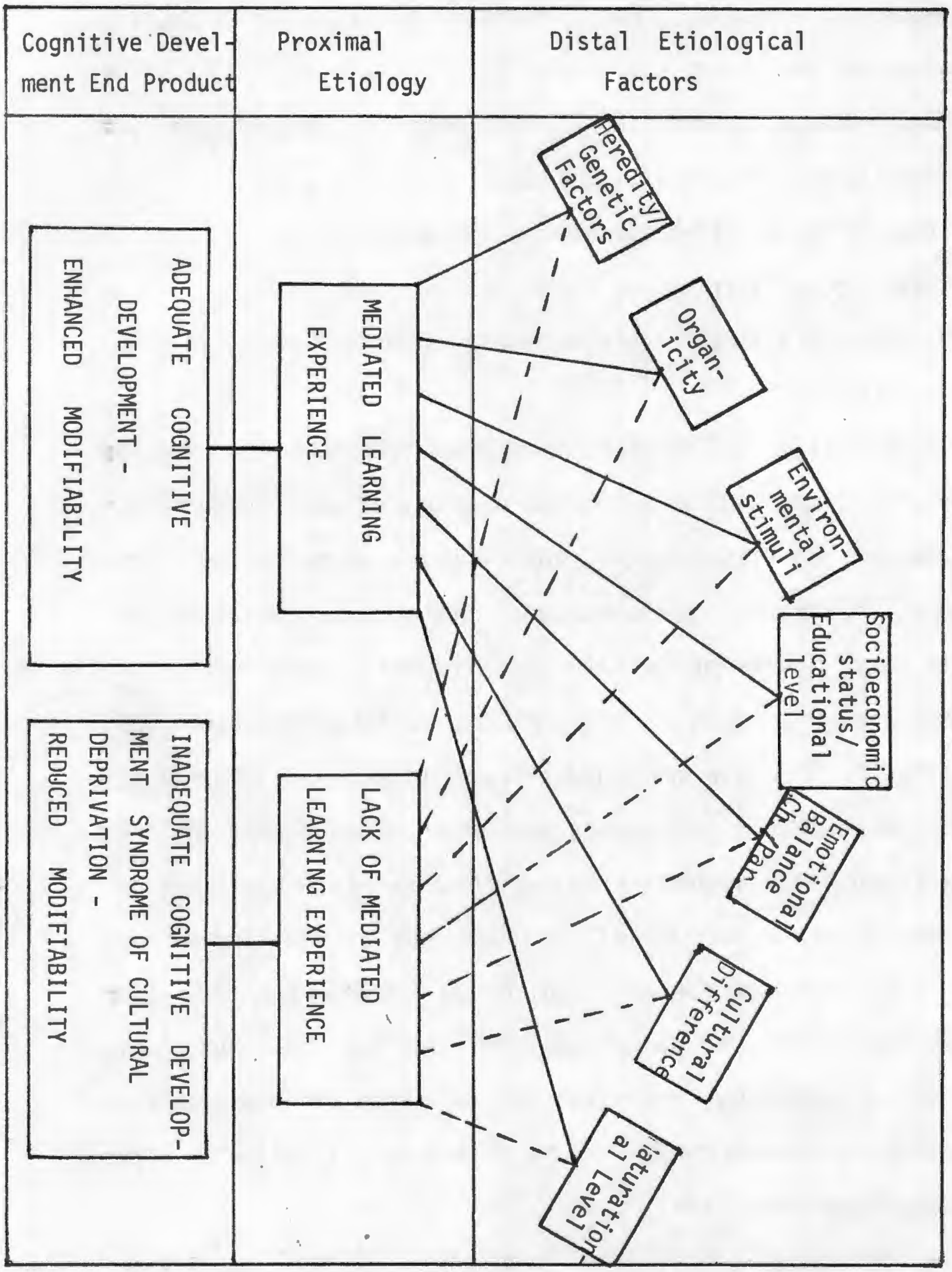


Figure showing distal and proximal etiological factors influencing cognitive development

> "What is at stake is not merely the transmission of specific skills or abilities but the development of the prerequisite cognitive schemata to enable an individual to derive maximum benefit from direct exposure to sources of stimulation. Our contention is that mediated learning experience is the foundation upon which cognitive structures are built and that, even as late as adolescence, major and significant cognitive modifications are possible."

If this is accepted, it follows that a directed attack at the results of a lack of MLE by suitable mediating programmes should be able to reverse the retarded performance. Feuerstein, et al, (1979, 1980) cite numerous case studies to underscore the value of this approach.

While Feuerstein gives many determinants for a lack of MLE, it is perhaps instructive to look at one possibility which may be particularly illuminating within the South African context and especially as regards "Coloured" students at the University of the Western Cape (Feuerstein, et al, 1980 : 39):

> "This attitude (that is, a very negative attitude) toward such an important and vital aspect of one's past cannot but result in a rejection of oneself and will affect in the first instance, the nature of the relationship between mother and child. By rejecting one's worth as a person, the mother limits her relationship with her child to the fulfillment

> of his biological needs. Inclination to provide the child with anything beyond his immediate requirements is diminished, if not eliminated. Many such parents openly declare that they are ignorant, illiterate and unskilled people and have nothing to give their children and, indeed, off-spring will grow up like them. In this manner, an entire community may relinquish its responsibilities in shaping the future generation. Clearly such an attitude will have a considerable impact on the self-image, identification, socialization, and moral and cognitive development of the child"

An examination of the history and the socio-economic circumstances of the "Coloured" group in South Africa shows a total rejection of the situation into which the group has been legislated and the second-class citizenship which it has inherited and is assigned in the land of its birth. It is no wonder that the transmission of culture is so difficult since it cannot identify with a purely "Black" culture and yet has for years found itself largely rejected by the "White" one.

This is certainly not given as the only reason for the poor functioning of "Coloured" students, but it does provide at least a very plausible explanation of why the prevalent group situation produces the results that it does. There is at present not sufficient evidence to postulate a rigorous, scientific, research-based social reason, but the above provides a possible insight arising from the theory of MLE.

The crucial factors for physics education are the cognitive deficiencies resulting from a lack of MLE. These will be considered next.

3.2.3 An Analysis of Cognitive Deficiencies

The lack of Mediated Learning Experience is postulated as the cause which has deficient cognitive functions as an effect. To give practical meaning to the effect, it is first necessary to analyse the constituents of human thought and then to place deficient cognitive functions within that context. This Feuerstein and Rand (1977) have done by means of a model which they call the Cognitive Map. The map has the following seven parameters:

i) Content. This is the subject matter being considered.

ii) Operations. These are defined as (Feuerstein, et al, 1980: 106)

> "an internalized, organized, coordinated set of actions in terms of which we elaborate upon information derived from internal and external sources".

It is important to note that operations rely on certain functions which are more basic; e.g. classifications are only possible if data can be properly identified and categorized. The operation is then a higher level thought process. We will return to this point in section (3.3).

iii) Modality, i.e. verbal, numerical, symbolic, pictorial etc.

iv) Phase. For purposes of convenience the mental act is divided into three phases, i.e. input, elaboration and output. This division is made for ease of description of the difficulties of underperformers. It must be realized however, that there is considerable overlap between phases.

This parameter is crucial for determining deficient cognitive functions.

v) Level of complexity.

vi) Level of abstraction.

vii) Efficiency. The extent to which cognitive functions are internalized and automatic will obviously determine the speed with which a mental act can be executed. The speed of operation may relate more to efficiency than to the absence of a cognitive function.

Deficient cognitive functions are categorized in relation to the three phases of the mental act. Of course, any of the other parameters may also be responsible for the difficulties encountered by a particular child.

It is important to note that the list of cognitive deficiencies enumerated by Feuerstein, et al, has been derived after extensive clinical observations of disadvantaged children over a period of twenty years using an instrument called the Learning Potential Assessment Device (LPAD). For details see Feuerstein, Rand and Hoffman, (1979), and particularly the large number of case studies cited. No claim is made that the list is either exhaustive and definitive or that all the functions are missing from the cognitive repertoire of all disadvantaged persons. Some may be present but not necessarily efficient.

Cognitive deficiencies which appear relevant to the teaching of physics are the following (Feuerstein, et al, 1980: 73, 74):

a) Input Phase

This involves the quantity and quality of data selected as a person attempts to solve a problem.

i) Blurred and sweeping perception

This relates to an imprecise delineation of data and incompleteness of data assimilated in defining a particular stimulus. This is not a total deficiency, i.e. there are situations in which perception can be sharp.

ii) Unplanned, impulsive and unsystematic exploratory behaviour

When presented with a situation rich in data, the person has no organized approach to examining the data or to determine those parts relevant to the solution of the problem at hand. Frequently a response is made incorporating only some data.

iii) Lack of, or impaired, receptive verbal tools and concepts which affect discrimination

Obviously if comprehension of the meaning of, or implications of, words is deficient, it will impair the quality and quantity of data collected from a given statement. It has been demonstrated (Haywood & Switzky, 1974) that poor verbal skills among disadvantaged students affect their data collection.

iv) Lack of, or impaired need for, precision and accuracy

Either all the data presented is not gathered or unrealistic approximations are made.

v) Lack of, or impaired use of, two sources of information

In this instance, the person does not look at the global situation but focuses either only on one source of data or looks at both separately without seeking any relationship between the various sources. There is an identification only of simple elements in each source.

b) <u>Elaborational Phase</u>

This involves the way in which the available data are used in reaching the solution to the problem.

i) Inadequacy in experiencing the existence of, and in subsequently defining, an actual problem

This relates firstly to the gathering of data, establishing relationships and appreciating that certain things are missing, incompatible etc. It is clear that if data are not carefully analysed, the existence of a situation of disequilibrium will not readily be perceived.

ii) Inability to select relevant as opposed to irrelevant cues in defining a problem

This is related to i) above since if there is no ability to focus on a problem situation then the relevance of cues cannot be appreciated.

iii) Lack of, or impaired, spontaneous comparative behaviour

The accent here lies on the fact that the comparative behaviour is spontaneous and is essential for taking separate units of information and integrating them into a larger coordinated thought pattern. As such it becomes vital for the establishment of relationships.

iv) Lack of, or impaired, interiorization

The ability to abstract and to use information in mental acts, is limited. Perceived experiences are usually concrete in nature. This obviously affects the ability to plan for a particular outcome as in problem-solving.

v) Lack of, or impaired, planning behaviour

This relates to the ability to see and construct the steps necessary to realize a particular outcome. These steps need to be arranged in space and time, must correspond to the cues established at input and must be related to the recognition of a certain problem situation.

c) Output Phase

This affects the ability to communicate the results of a particular elaborative process.

It is pertinent to note that other researchers give a list of cognitive deficiencies of learning-disabled children which are very similar to those of Feuerstein, et al. Those are listed by Arbitman-Smith and Haywood (1980) and include:

1. Defects in planning. There is no preliminary analysis of a problem.

2. "Defects in logic". This relates to the ability of children to understand verbal pictures.
3. Difficulties in establishing the relationships between two sources of information.

It is clear that there is wide agreement among researchers as regards the types of cognitive malfunctions among disadvantaged children. (See for example, Golick (1970); Wepman, et al (1975); Cruickshank (1977)).

3.3 The Relevance of the Feuerstein Approach to Physics Education For Disadvantaged Students

A recognition of the fact that disadvantaged students have cognitive deficiencies which affect the way in which they react to information presented to them, can meaningfully influence the type of instruction given to such students. Additionally, it can place in perspective the difficulties which students have with physics, especially with respect to problem-solving and concept application. How the Feuerstein approach affects both the acquisition of the Piagetian mental schemas by persons, as well as the research into the misconceptions held by students, will be discussed below.

3.3.1 The Influence of the Feuerstein Approach on the Piagetian Paradigm in Physics Education

As discussed in Chapter 2, there has been increasing recognition of the fact that the difficulties students have with physics do not relate simply to their understanding of the content of the subject.

For example, it has become clear that students lack many basic reasoning skills. Note the comment of McDermott, et al (1980: 137) as regards disadvantaged students:

> "We have found the reasoning difficulties faced by our academically disadvantaged students to be substantially the same as those reported by other investigators for the general student population, but more widespread and severe."

The same researchers give a list of some of the skills which need to be developed. These include: proportional reasoning, logical implication, combinatorial reasoning, control of variables, hypothetico-deductive reasoning. Further skills required for a mastery of physics content are furnished by Arons (1978).

While it is clear that these skills are widely used in teaching physics, it is perhaps not well appreciated that these reasoning skills are themselves aggregates of more fundamental cognitive functions. Clearly, if these cognitive functions are themselves deficient, then the higher level skills are very unlikely to be produced. By way of illustration let us examine some of the above list in terms of the cognitive functions implicitly required for their execution.

1. <u>Proportional Reasoning</u>:

To illustrate what is required in carrying out a task on proportional reasoning, consider the research done by Karplus, et al (1977). This involved tasks on proportional reasoning and control of variables. The proportional reasoning task is the following:

"Subjects were given answer pages and chains of about nine number one "gem" paper clips. They were then told that "Mr Short" on their papers has a friend "Mr Tall", also drawn on a piece of paper, and that the two figures when measured by a row of large round buttons placed side by side, were found to be four buttons and six buttons respectively, from floor to head. Then the items on the answer page were read aloud and the subjects were asked to be complete in their explanations about the height of Mr Tall ....."

Note the following "Response Categories" furnished by Karplus, et al:

"Category I (Intuitive): The explanation does not make use of all the data or makes use of the data in a haphazard or illogical way."

"Category A (Additive): The explanation focuses on a single difference (tall/short or paper clips/buttons) uncoordinated with other differences, and solves the problem by addition."

"Category Tr (Transitional): The explanation shows only partial proportional reasoning, or makes reference to concrete comparisons or iterations."

It is clear from the student responses to the above task that more is involved than the ability to deal with proportional reasoning. If a child has cognitive deficiencies on the input phase such as

i) Blurred and sweeping perception

ii) Unplanned, impulsive and unsystematic exploratory behaviour

iii) Deficient need for precision and accuracy in data gathering

iv) Dealing with data in a piecemeal fashion etc.,

then he would be incapable of coping with the problem _a priori_.

It must be stressed that no claim is being made that the children in the above study suffered from deficient cognitive functions. The point that is being made is that "lower" cognitive functions are implicit in higher order reasoning patterns.

2. Logical Implication

Physics is a logically coherent structure based on concepts, propositions etc. where a certain situation implies a particular outcome if principles, laws etc. are applied in a logical sequence. There is thus a logical necessity implied in the application of physical principles.

In its turn, all this would imply the ability to collect data describing a particular situation, it would mean that only relevant data would be used and that this could now be organized in some planned manner to produce a desired outcome, not necessarily very clearly seen at present. In addition to this, the person doing all the above must also deem the need for logical consistency to be essential (i.e. not the "So what?" response to a problem situation, as from the person in whom such a need does not exist). If the person does not have such a need, there would be no desire to search for a solution.

Thus apart from the obvious deficiencies at input, the following cognitive deficiencies on the elaborational phase will also impede logical implication:

i) Inability to select relevant data in defining a problem.

ii) Lack of spontaneous comparative behaviour.

iii) Lack of interiorization.

iv) Lack of planning behaviour.

v) Lack of a need for pursuing logical evidence.

3. <u>Performing Hypothetico-Deductive Reasoning</u>

Note how Arons expresses this skill (Arons, 1978: 213):

> "Students should be able to visualize in the abstract, outcomes that might stem from changes imposed on a given system or situation, whether it be in scientific, literary, historical, economic or political contexts, and affect such visualizations through reasonings within the basic principles or other rational constraints applicable to the system."

Hypothetico-deductive reasoning is an elaborational process (in the sense discussed in 3.2.3) and as such would be affected by many of the deficient cognitive operations outlined by Feuerstein. Examples of these would be:

i) Lack of, or impaired, interiorization

A person thus deficient, is characterized by a very limited ability to abstract. The thinking act is restricted to concrete experiences. This also leads to deficient planning for an outcome not readily visualized.

ii) Lack of, or impaired, spontaneous comparative behaviour.

Since comparisons between sets of data are not made spontaneously, difficulty is experienced in establishing relationships which are necessary for a progression of thought and indeed for coordinating thinking into hypothetico-deductive reasoning.

The above three examples are given to illustrate the existence of both "higher level" and "lower level" mental acts. The emphasis on Piagetian developmental theory in science education has resulted in operations such as proportional reasoning, control of variables etc., i.e. skills indicative of formal operations, being given much attention in the literature (See for example, Shayer and Adey, 1981). It seems clear, however, that "lower level" mental acts, need consideration as well. Or, to use a chemical analogy, it is necessary to examine mental processes on a molecular as well as a molar level.

3.3.2 Placing Alternative Conceptions in Perspective

In the light of the comprehensive list of deficient cognitive functions identified by Feuerstein, it is interesting to consider what exactly is being probed when attempts are made to identify misconceptions which students hold of concepts in physics.

It is, of course, hoped that a clear view is being obtained of their understanding of a particular concept, law etc., i.e. of the content of the subject. However, as has been argued in previous sections, the product of teaching (i.e. the content which teachers hope students will learn) is inextricably bound up with the process of learning it. Cognitive functions are brought to bear on the learning process which will inevitably influence not only what is learnt, but also how it is learnt. Thus it becomes a daunting task to unravel the two in any testing procedure.

This view is supported by Barowey and Lochhead (1981: 4):

> "It seems clear from our data that many students who have reasonable intuitions about qualitative physics are unable to apply these correctly because they cannot isolate critical features of the problem. They appear prevented from doing this by an inability to disregard irrelevant but perceptually obvious features of the initial presentation. We suspect that this failure to generate appropriate abstract representations may be linked to many of the common misconceptions that we and others have observed."

Consider the "simple" question associated with the picture in section 2.4.1 : "Is there a force on the car?" Experts would easily recognize that Newton's third and first laws are at issue here. Then to apply these laws some or other plan of action needs to have been developed, as will be discussed later. This involves the ability to recognize the data in the problem and the means to elaborate it. Finally there is the need to express cogently the results of that process of thought. Thus in saying that students give answers which do not square with Newtonian reality, is it possible to say at which point in this necessary sequence to the correct answer their thinking proved to be deficient?

This situation is exacerbated if examination-type questions are used to test misconceptions. Note the following question from Helm (1980):

•→ •

(1) (2)

Particles (1) and (2) are a long way from the rest of the universe but quite close to each other. The mass of particle (1) is half that of particle (2), and the two particles exert gravitational force on each other. If the arrow above represents the force exerted by (2) on (1) in magnitude and direction, which ONE of the following arrows represents the force exerted by (1) on (2)?

(a) ← (b) → (c) ⟶ (d) ⟵

Considering the amount of data that has to be sifted through, the decision that must be made of which concept to use, the planning procedure that needs to be applied to use the concept decided upon, it is clear that more is required to answer the question than simple rote-knowledge that "to every action there is an equal but opposite reaction".

The above is not to be understood as implying that misconceptions do not exist. They are obviously alive and well! However, the point being made is that the origin of the misconceptions as well as our view of how pervasive they are, need to be influenced by our analysis of the task structure established to test their existence. Clearly, the simple problem-solving type of procedure is fraught with cognitive difficulties. Perhaps a more fruitful type of approach is that represented by the laboratory-demonstration type of interview which seems to have less possibility for ambiguity as regards process rather than product with respect to concepts being tested.

An example of this is given by Trowbridge and McDermott (1980) in testing students' conceptions of velocity. After a "Criterion for understanding" the kinematical concept is established, "two speed comparison tests" are developed. The conclusion reached is that students equate "same position" with "same speed". This conclusion is reinforced with a paper-and-pencil test which produced similar results. Note the authors' interpretation of these conclusions:

> "Our interpretation of the responses ... is that these students lack an adequate procedure for deciding when two objects have the same instantaneous speed. Instead they focus attention on the perceptually obvious phenomenon of passing to make the required comparison. A successful comparison usually requires that an individual focus attention on the separation between the two balls and identify an instant when the separation is neither increasing or decreasing."
>
> (p. 1023)

It is clear from this well-defined and analysed study that care needs to be exercised in determining not only what is being tested but also how it is being done, so as to eliminate possible ambiguities in the interpretation of the results.

An interesting interpretation of the results of a misconception test is given by DiSessa (1982) in analysing "Jane's protocol" (p. 58) (a person-to-person interview with a student named Jane). He uses the term "distributed encoding" to emphasize the view that there are many factors contributing to the understanding of a particular concept. Note the following comments made in considering "a few commonsense classes of knowledge which contribute to understanding yet are sometimes regarded as Ancillary":

> "When an expert says a novice does not really understand an idea, it might well mean that the expert knows a context in which the student will not be able to apply the idea, i.e. the student lacks a way of interpreting the context so as to see the relevance of an idea",

and

> "This would be important in planning-like activities such as deciding whether and how to apply the theory."
>
> (p. 60)

An identification of weaknesses in students' cognitive processing could thus assist instructors in determining reasons for misconceptions which arise in students' understanding of a particular area of content. It also indicates that students' incorrect views of the world are not necessarily very deep-rooted. They may well be, but until some of the ambiguities of what exactly causes the misconceptions are eliminated, there can be no certainty in this regard. It implies that in giving instruction, great care needs to be exercised in supplying also the "ancillary knowledge". Instructors need to be aware, or need to be made aware, of how much apart from pure content is involved in internalizing a particular idea, i.e. the process needs as much attention as the product.

If this view is accepted, then we will be concerned not merely with how knowledge of misconceptions can influence instruction, but also how the recognition and implementation of necessary strategies for teaching the relevant processes in assisting the understanding of certain content, will affect misconceptions.

3.4 Remediation by Ordering Cognitive Processes Using Algorithms and Heuristics

A recognition of the fact that certain cognitive operations are difficult, if not impossible, for some students requires instruction to be shaped so as to compensate for what students may lack. This requires that a careful analysis be made of the cognitive functions necessary for dealing with a certain section of content. Once this has been established, it will then be necessary to shape the instruction given accordingly.

Cognitive operations as defined above in the work of Feuerstein would fall into the category of "other knowledge" discussed in the previous chapter (see 2.3.2.2). As such these operations may be difficult to identify since an experienced physicist would often apply the cognitive functions necessary for a certain task without being conscious of the mental operation he or she may be using. For example, any experienced and effective problem-solver begins the solution of a problem by careful analysis of data, both implicit and explicit (see 2.3.2.2). These almost obvious approaches of the expert have normally been developed through the solution of large numbers of problems.

Practice, however, often serves to conceal the many little steps which experts follow, by making these automatic (as anybody can discover by trying to strike a tennis ball with a racquet with any degree of precision and speed). To determine how the cognitive operations are used it is thus frequently necessary to break down an automatic process into its components. The role of algorithms and heuristics in effecting this automaticity will be examined below.

3.4.1 The Use of Algorithms and Heuristics by the Human Information Processing System

A general discussion of the human Information Processing System has been given in Chapter 2. There the Long Term Memory (LTM) was said to have a "Procedural Component". This is sometimes called the "Algorithmic Component" (Shavelson, 1981) where an algorithm is defined as "step-by-step procedures needed to reach some goal".

That humans automatically use algorithms in accomplishing some tasks can be seen from such diverse operations as baking a cake, to finding your way from point A to point B in a foreign city.

How then are algorithms developed as part of the thinking process? It has been very effectively demonstrated how this is possible by means of so-called "Production Systems". Thus, for example, Newell and Simon (1972: 32,33) discuss how a person crosses a street at a traffic light, as follows:

CONDITION	ACTION
Red light	stop
Green light	move
Move (left foot on pavement)	step with right foot
Move (right foot on pavement)	step with left foot

The entire procedure consists of productions (i.e. condition-action units), where, if a particular condition is satisfied, the corresponding action is carried out, while each action executed, triggers a related condition.

The efficacy of this approach has been demonstrated with the generation of so-called Adaptive Production Systems (APS) in studies in Artificial Intelligence. An APS grows by creating new productions. This it does by using strategies such as "learning by example" or "learning by doing" (see Neves, 1978). The Neves' APS is able to acquire the ability to solve algebraic problems by doing examples. One illustration from linear algebraic equations is given by Larkin, et al (1980: 1341):

"The following is a typical example in algebra:

$3x + 4 = x - 12$	
$2x + 4 = -12$	(subtract x from both sides)
$2x = -16$	(subtract 4 from both sides)
$x = -8$	(divide by 2)

......Neves' APS is capable of forming a production like, 'If there is a literal term on the right side of the equation, subtract that term from both sides of the equation'. Thus, it generates an appropriate condition to associate with the action.

It can be seen that three productions, one for each of the three steps in the example, will, when generalized to replace specific coefficients with variables, constitute a fairly general algorithm for solving linear algebraic equations in one unknown."

The idea of production systems, or, as he expresses it,

"theoretical propositions in the "if ..., then ... "form"

has been used by Landa (1976: 81) in developing algorithms for learning grammar.

It is perhaps relevant to differentiate between "algorithmic" and "heuristics processes" (See Landa, 1976, chapter 5). The basic properties of algorithms are:

i) Specificity : All actions of the person applying the algorithm are unambiguously determined and understood.

ii) Generality : The algorithm can be applied to a set of problems on a specific topic. The number of problems can be infinite.

iii) Resultivity : The algorithm will always produce the desired result which the user always achieves if given the necessary data.

Heuristic prescriptions on the other hand, tend to be more general in the requirements of each step; for example, "analyse the data in the problem" as opposed to "look at body A" as would be required by an algorithm. Thus heuristic prescriptions have as a result that:

i) not all people solve a problem.

ii) not all of them solve it correctly.

iii) not all of them solve it identically.

In our analysis of suitable procedures to use in examining topics in physics many of the prescriptions developed may possibly be placed on a scale somewhere between algorithmic and semi-heuristic. The term "algorithm" will however by used consistently throughout.

3.4.2 <u>Using Algorithms to Compensate for Cognitive Deficiencies</u>

> "A physics student has attended lectures, reviewed his lecture notes, and studies his textbook. Yet he cannot solve the problems in his homework. If enabling the student to solve these problems was one goal of this instruction, it has failed. The student could not use the lecture and textbook to achieve the goals of instructions." (Larkin, 1975)

Most physics instructors have experienced the apparent inability of students to apply knowledge especially in solving problems. It can be expected that this situation is especially severe with respect to disadvantaged students. Since necessary cognitive operations may be absent or of low efficiency in such students, the following remediatory steps are necessary:

i) A careful analysis must be made of the content-area in question to determine the cognitive operations required to understand the content and/or to apply it in solving problems.

ii) Students need to be tested as to the presence or efficiency of the cognitive operations required for content-mastery.

iii) The procedure whereby experts utilize the necessary cognitive operations with the specific content needs to be found and made explicit.

iv) Methods of teaching these procedures with the content-presentation, must be determined.

It is clear from the discussions above that algorithms are intrinsic to the application of knowledge in various fields. For example, this is readily acknowledged to be the case in doing simple arithmetic. After giving the well-known algorithm for the long division of numbers, Landa (1976: 78) makes the following comment:

> "There was a time when such algorithms were unknown, and only the most intelligent people could divide two moderately large numbers. After the algorithm was discovered they were eventually taught to children so that today students in primary schools are able to divide large numbers."

It is clear that concepts in physics are also open to algorithmization since they are usually applied in solving problems. A number of textbooks supply an algorithm for Newton's first and second law (for example, Tipler, 1982). However, few algorithms are normally supplied for any other concepts. Even in cases where the algorithm is supplied with Newton's laws, it is not regarded as something intrinsic to the law, but rather an addition to the text given only as a prescription for solving problems. For the most part, students, in dealing with various concepts in physics, are either shown how to solve a specific problem or given a number to solve on their own. In so doing, they of course develop some or other algorithmic procedure for coping with that type of problem based on the particular concept i.e. they are left to solve problems in order to develop the needed algorithm to solve further problems. While the best students are able to do this, experience has demonstrated that most students cannot. Indeed, if an algorithm is intrinsic to the application of many different concepts in physics and if students are to be frequently tested in their ability to apply the algorithm (by setting them problems to solve), then it is in fact doing them a disservice not to teach it explicitly.

The objection normally raised to algorithmic prescriptions is that it makes an automaton of a student rather than cultivating creative thinking. Is that a valid contention? It is important to realize that problem-solving is not a creative process per se. For example, ask any reasonably accomplished physics student how far a car will travel in 10 seconds at a constant speed of 5 m s^{-1}, and the answer of 50m will be immediately forthcoming. The problem is solved by a simple application of the equation:

$$\text{Distance} = \text{constant velocity} \times \text{time}$$

By extension of this idea, if an expert is asked to solve a problem which appears complex to a novice and the former immediately sets about solving it by virtue of having knowledge of the particular principle to apply as well as having a procedure for using the concept, then there is really not much that is creative in that process either. What is essential to recognize is the fact that regardless of the heuristic prescription that is followed in solving problems, ultimately the problem is solved by applying one of more principles, laws, concepts etc. Thus a particular problem may involve Newton's laws, principles of work and energy as well as kinematics all combined. However when the problem-solver realizes that one particular concept must be applied at a specific state in the solution, he needs to bring an algorithm to bear on the problem. This does not imply that a knowledge of algorithms as specified above will make the solution of complex problems immediately possible. But it should undoubtedly enhance the ability of students to use specific concepts in the solution of problems whether these are simple or complex.

Hence, what is being argued above is that while it is necessary to develop heuristic prescriptions for solving problems, it is also essential to algorithmize to the greatest extent possible, the concepts taught.

Indeed, since much problem-solving is given as an exercise after specific sections of the course have been covered, development of proper algorithms should not only enable students to solve more problems but also give them a clearer idea of what the concept is about. The need for an approach of this nature to complement the heuristic approaches being developed, is shown by noting that in the article by Larkin (1981) the greatest difficulty experienced by students in using Newton's second law was in determining the resultant force on the body.

It is suggested that the explicit teaching of algorithms to help compensate for cognitive deficiencies, will have the following benefits:

a) The proper algorithmization of concepts should enable greater understanding to be developed since the semantic and algorithmic components of memory interact and are not two separate entities. Note how this is expressed by Shavelson (1981: 133):

> "The distinction between the semantic and algorithmic components of memory is one of convenience. They systematically interact. At any point in time, a finite part of the semantic component is assumed to be working. Some of the nodes in the active part of semantic memory are linked to conditions of algorithms. If a condition or an algorithm is activated, the corresponding action is taken until the algorithm is executed."

As will be seen below, since no algorithm is developed without an interaction between specific content and the process to use that content, algorithms should elucidate detailed features of concepts. Salient parts of physical principles thus become clear.

b) It becomes possible to determine accurately at which point the thinking of a student is faulty if he is unable to solve a problem. Thus, rather than simply concluding that a student cannot cope with a concept, insight can be gained as to specific areas of weakness. This is especially important for disadvantaged students who are normally also underprepared in mathematical skills. It thus becomes possible to isolate details by simply seeing at which point application of the algorithm breaks down. Examples of this will be given below.

c) Greater confidence is built up by the student since a pattern can now be followed and established. As a feel develops for the way in which the algorithm is applied, some of the features become internalized and automatic and the student gets into the "learning-by-doing" cycle. As Landa (1976:92) expresses it:

> "As a rule, the transformation from one type of process (Algorithmic) to another (simultaneous post-algorithmic) proceeds by itself (spontaneously) in the course of repeatedly acting according to an algorithm"

In this way expertise is built up.

d) By making a planning procedure explicit in dealing with concepts, this transfers to students who become more conscious of the need to do this as well as being shown how to approach it. In so doing it is hoped that deficient cognitive functions can be remedied.

The benefits discussed above are not simply theoretical but have been demonstrated by the students who are the subjects of the study. This will be discussed later.

3.5 Conclusion

An attempt has been made above to show that the Feuerstein approach provides significant insight into the real learning problems of disadvantaged students. It makes specific why they will have difficulties with a structured subject like physics which requires a wide range of cognitive functions not only for understanding concepts, but also in the solution of problems.

Additionally, the Landa methods are proposed as a possible means of addressing cognitive deficiencies within the framework of physics. It needs to be stressed, however that this thesis does not necessarily espouse Feuerstein's theories in toto. Neither is it necessary to do so. The important features thereof for the purposes of this project, are the cognitive deficiencies which have been identified. As section 3.2.3 shows, there is general agreement among researchers about the existence of such deficiencies in disadvantaged persons.

It is now necessary to demonstrate that there are indeed cognitive deficiencies in the students who form the subjects of this study, and then to develop programmes to remedy these.

CHAPTER 4

DETERMINATION OF MOLECULAR AND MOLAR COGNITIVE DIFFICULTIES OF FIRST YEAR PHYSICS STUDENTS AT UWC

4.1 INTRODUCTION

There is general agreement that the field of testing human beings is fraught with difficulties. Major concerns of any investigator are, for example:

a) whether the tests administered, be they interview tasks, paper-and-pencil tests or any other type, in fact assess the ability being evaluated.

b) whether the responses obtained are unambiguous enough for conclusions to be drawn.

It is clear from the discussion in the previous two chapters that the application of various concepts in physics involves both process and product. It is therefore essential in testing to appreciate this dichotomy and to structure the tasks in such a way that this is explicitly recognized. Thus the particular cognitive function being examined must obviously be clearly required for the task and it must be possible to design the task to make clear its presence or absence in the students being tested.

This in its turn implies that any section of content used to test students must first be analyzed in terms of its cognitive requirements.

Thus, for example, in analysing the type of problem that students are required to solve in the kinematics section of the mechanics course, it was realized in this study that as soon as both the implicit and explicit data in the problem are clearly understood and the problem situation properly visualized, then the solution is obtained by the simple application of the appropriate equations. Thus before embarking on an indepth analysis of the cognitive characteristics displayed by the students during the person-to-person interviews documented in Appendix A, a brief discussion of the cognitive requirements of parts of the mechanics course is given. The strengths and weaknesses of the individual interview technique as opposed to a paper-and-pencil test, are also discussed. Finally, an analysis of the responses in the light of the Feuerstein list of cognitive deficiencies is discussed.

4.2 <u>The Mechanics Course Content</u>

The course in mechanics taught to first year students at UWC follows roughly the form of standard first year university texts, with that of Sears and Zemansky (1979) being very much the order of presentation. The course can be divided roughly into four sections:

1. <u>Equilibrium</u>:

Vectors : addition of vectors, subtraction, multiplication; components

Newton's First and Third Laws

Friction

Centre-of-gravity

Moments of forces; Second equilibrium condition

2. Kinematics

Definitions of displacement, velocity and acceleration

Graphical representation

Three equations for constant acceleration

Constant acceleration under gravity

Relative velocity

Projectile motion

Circular kinematics

3. Dynamics

Newton's second law and applications

Dynamics of circular motion

Newton's law of gravitation

4. Work, energy and momentum

Definition of work

Kinetic and potential energy

Conservation of mechanical energy

Power

Definition of impulse, Conservation of momentum

4.3 An Indication of the Cognitive Requirements of a Part of the Mechanics Course

In previous chapters it has been shown that the understanding and application of various concepts and laws in a particular content-area often require certain specific cognitive operations. Before testing for these in students it is necessary to demonstrate the nature of such cognitive functions involved in certain areas in mechanics. (A far more comprehensive discussion of this topic will be given in later chapters.)

A. Cognitive Functions on Input (see section 3.2.3a)

Observations of students over a number of years in the Physics Department at the University of the Western Cape have revealed that much difficulty is experienced with problems in kinematics. The content covered is usually the graphical representation of displacement; velocity and acceleration; motion under gravity (one-dimensional); projectile motion and relative velocity. While inordinate difficulty is usually experienced with the last two topics, it has often been astonishing how frequently students have difficulty with the application of the three standard equations for uniform acceleration. This is especially true if the problem deals with two objects moving simultaneously. A typical problem of this type, is the following:

> "At the instant that a lorry travelling with constant speed of 10 m s^{-1} passes a traffic light, a motor car starts from rest with an acceleration of 2 m s^{-2}.
>
> Determine :
>
> i) Time taken until they are level
>
> ii) Distance travelled."

An analysis of students' worksheets indicated that the misconception reported by Trowbridge and McDermott (1980), namely, that when the objects were level they had the same speed, was quite prevalent. In addition to this, however, a severe constraint on students' ability to solve the problem was their inability to extract meaningful data from the statement of the problem and especially to relate variables between the two objects.

In dealing with projectile motion, students have great difficulty in visualizing movement in two perpendicular directions simultaneously. They have little trouble in acknowledging that acceleration is taking place vertically while, since there is no force horizontally, there is constant speed in that direction. When it comes to solving problems, however, they are seldom able to sift through the maze of information, see what is relevant, where one begins with a solution and which formulae are applicable.

It seems clear that, especially in areas which lend themselves to both a semantically- and data-rich formulation of problems as a means of testing physical concepts, it is necessary to realize that more is required than the knowledge of the concept itself. If the student is not helped to analyse the data in a meaningful way he will have difficulty solving problems in spite of the fact that the physical principles involved are relatively straightforward, and well known to him.

B. Elaboration (see section 3.2.3b)

Consider the way in which Newton's laws are taught in most first year physics courses. There is normally a discussion of the law, perhaps with some demonstrations. Implications of the law are usually discussed as for example, the idea of inertial frames of reference in dealing with Newton I. The way in which understanding of the law is tested, however, is usually in a problem-solving environment. Frequently students at UWC say : "I understand what the textbook says about the law, but I do not know where to start in solving a problem". This is, of course, a not uncommon student complaint. Why is it? Simply because the solution of problems brings a number of other cognitive operations to bear apart from the knowledge of content. Other than cognitive functions related to the analysis of data, there are those which affect elaboration thereof, not least of all being some or other planned approach to the use of the particular concept or law. To illustrate, consider the use of Newton's Third Law: "To every action there is an equal but opposite reaction". There would have to be a realization that

a) Two bodies must be involved.

b) The action must be stated in terms of the effect the one body has on the other.

c) The reaction must then be computed in terms of the knowledge formulated in b).

All this implies the ability to recognize salient features of the physical situation as well as to formulate an approach by virtue of knowledge of the law itself. To argue that students will acquire this automatically is simply incorrect. Indeed the idea of tutorial sessions in physics, with the emphasis on the solving of problems, is to produce exactly this capability in students. However, our experience with students at UWC is that, in spite of many tutorials, many never do acquire it.

We will illustrate more concretely in subsequent chapters how the approaches set out above can be incorporated into a physics course. It seems clear that a recognition of the nature of the particular cognitive problems of disadvantaged students is vital for the development of suitable remediatory materials.

4.4 THE METHOD OF TESTING

It seems necessary at this stage to explain the method adopted in the present chapter to establish cognitive deficiencies (input phase) and to compare it to that used in Chapter 5 where the presence of a planning ability (elaborational phase) will be determined by means of a paper-and-pencil test.

The clinical interview, used widely in psychology, has, in recent years, been extensively used in research in science and mathematics education. It is a technique where a high level of interaction between subject and interviewer allows considerable probing into the responses given in answer to questions posed. An enormous amount of data can be accumulated in this way. We outline below the advantages and disadvantages of both types of evaluation.

A. Clinical Interviews

Advantages

i) More probing of student responses is possible.

ii) Greater detail is possible because of the interactive nature of this method.

iii) Immediate follow-up of responses made, becomes possible. Thus the interviewer is able to investigate different avenues raised.

Disadvantages

i) There is no necessary commitment to the ideas expressed since these can arise as a "top-of-the-head" response.

ii) The richness of data brings about its own problems of interpretation.

iii) The interviews themselves (usually approximately one hour in length) are very time-consuming and the data analysis is extremely laborious.

iv) At best, small numbers can be evaluated making statistical predictions hazardous.

B. Paper-And-Pencil-Tests

Advantages

i) Greater thought can be given both to formulating the questions asked and to answering them since there need not necessarily be any time constraints and there is not the presence of another person (the interviewer) which can have an unnerving effect on some persons.

ii) There seems to be a greater commitment to ideas recorded in writing than to those expressed verbally.

iii) Large numbers of subjects can be evaluated relatively easily.

Disadvantages

i) The method is limited in what it can test.

ii) The indepth probing resulting in the immediate pursuance of different avenues of thought is not possible.

In Chapter 5, a well-defined elaborational ability will be tested. The steps which are necessary to demonstrate this ability can be readily explicated. Thus a structured test for the presence of a planning ability can easily be constructed. A written test seemed ideal in this situation because of its potential for large numbers and group comparisons.

In the present chapter, however, no such previously determined structure, necessary to determine a cognitive function on input, suggested itself. Additionally, since it is not simply a matter of determining whether students are able to extract numerical data accurately, but rather whether the data implicit in the problem is meaningful to them, this aspect of data interpretation can more readily be probed using the clinical interview technique. Also, since cognitive functions on the input phase (related as they are to data analysis), involve visualizations of physical situations described only with words and no sketches, it is difficult to see how such abilities can be established in the paper-and-pencil mode. It should be mentioned that in conducting the interviews it was not deemed necessary to leave the students without prompts at the time when he/she appeared to be unable to proceed further. It was not a question of only determining what they could not do but rather in examining in what ways suggestions were incorporated by students into their thinking. In this way further insights into their ability to analyse data could be obtained.

In the present case, thirty interviews were conducted with first year students after they had completed instruction in the kinematics section of the course. The transcriptions and analyses of the interviews, each of which lasted between thirty and sixty minutes, are given in Appendix A.

4.5 PROBLEMS USED

The student is given a problem, asked to read it through carefully and then to write down both the explicit and implicit data. When these interviews were conducted, all the students had already covered the material in the lectures. The three problems given are as follows:

1. At the instant the traffic light turns green, a car starts from rest with a constant acceleration of 2 m s^{-2}. At the same instant a lorry, travelling with a constant speed of 10 m s^{-1} passes the car.
   a) How far beyond the starting point will the car overtake the lorry?
   b) How fast will the car be travelling at this instant?
2. A lift, height 3 m, accelerates upward at 2 m s^{-2}. At the instant that its velocity is 2 m s^{-1}, a screw falls from the roof of the lift.
   Calculate:
   a) how long the screw will take to reach the floor of the lift.
   b) what distance it has then fallen.
3. At the instant car A pulls away from a traffic light with acceleration 3 m s^{-2}, car B is 50 m from the traffic light travelling in the same direction as A but with speed 15 m s^{-1} and acceleration 1 m s^{-2}. Calculate how far from the traffic light B draws level with A and the velocity of each at this point.

4.6 ANALYSIS OF INTERVIEWS

In analyzing the thirty interviews conducted it was possible to establish four distinct categories of difficulties which students experienced as they attempted to solve the three problems posed. While these are listed separately, it will be apparent that the categories are not necessarily mutually exclusive. Obviously, the problems which students have with specific words or phrases (semantic difficulties) will influence the manner in which data may be analyzed or elaborated (cognitive difficulties). Similarly, cognitive difficulties on a molecular level will have an effect on the molar cognitive problems (error factors) of students (see section 3.3.2). The division between the four categories is made for ease of classification.

4.6.1 Semantic Difficulties

It is clear that students, in reading problems, do not necessarily interpret words correctly within the context of the problem. Frequently the meaning given to the word or phrase is the normal "everyday" one rather than the meaning required, and hence assumed, by the physicist who constructed the problem. The words or phrases misinterpreted are discussed below.

i) the screw "falls" in problem 2:

All thirty students interviewed indicated that the screw moves downward from the instant that it comes loose from the roof of the lift. When asked, all said that an object cannot fall upward. Yet of course, an object which is in freefall, i.e. moving under the influence of the force of gravity only, can move in any direction depending on its initial velocity. The word "falls" acted as a complete distractor here and even if all subsequent calculations were correct, students would still obtain an incorrect answer.

Indicative of the influence of the word on student interpretation of features of the problem is the fact that most assigned an initial velocity, u, of zero to the screw. One student correctly wrote that u = 2, but still indicated that it moves downward only. Others insisted that the motion of the screw is parabolic. One student realizing that motion is not downward only, assigned a parabolic motion to the screw but could not relate the initial velocity to an upward motion.

ii) <u>"the height" of the lift in problem 2:</u>

Two students interpreted the "height 3m" of the lift as meaning that the lift was 3m from the ground. They indicated this on the sketches they drew and related it to the variable "s" in the kinematic equations.

iii) <u>the "floor" in problem 2:</u>

Three students confused the word "floor" with the ground rather than relating it to the lift. One student even assigned a parabolic motion to the screw to enable it to reach the ground beyond the confines of the lift! It is clear that the context which the student ascribes to the problem will influence the manner in which words in the problem are interpreted.

iv) "at the same instant" in problem 1:

There seemed to be limited understanding of the expression "at the same instant". Although "t" is given as the same for both the car and the truck, there is apparently no clear perception of what the time period is for each object since there is no certainty of the initial or final velocities of each or of where the objects are when these need to be ascertained. The words do not seem to suggest any particular meanings or to carry special significance in students' attempts to understand the problem. It was seldom interpreted as implying a temporal scale for subsequent events.

v) "constant" in problem 1:

The word "constant" in "constant velocity" was interpreted in at least four cases as meaning that the acceleration of the truck is not zero. One student said of the truck that "the constant speed and the acceleration would give the same thing". In the majority of the interviews conducted the equation $s = v\,t$ for constant velocity, was not spontaneously associated with the expression "constant velocity" in the problem.

4.6.2 Mathematical Difficulties

While no systematic indepth probing of students' mathematical abilities was attempted during the interviews, the following obvious mathematical difficulties of students were repeatedly discerned.

i) the solution of simultaneous equations,

ii) assigning symbols to unknown quantities and then utilizing these in equations. Two students were also reticent about using a formula in which there was an unknown quantity.

4.6.3 Error Factors

Analyses of the transcripts of the interviews conducted also enable an assessment to be made of reasoning patterns which mislead students in attaining a solution to the problem or which prevent them from executing accurate steps toward solving the problem. These reasoning patterns can be classed as error factors, with the misconceptions which students may have of a particular principle as a subset of this broader designation (see section 2.4).

A list is presented below of error factors which have been discerned from the interviews.

i) The signs of the kinematic variables

Many of the students seldom assigned signs to the variables s, a, u and v in the three kinematic equations for one-dimensional motion with constant acceleration. In some cases signs were only inserted after prompting. It is apparent that the need for suitable signs never occurs to some students. Thus, for example, one student wrote the equation for the screw in problem 2 as: $3 = 2t + t^2$, after explaining that the 2 m s^{-1} is the upward velocity of the screw while 3m is the distance through which the screw falls. In another example, while the student perceives that the initial velocity of the screw is 2 m s^{-1} upward, the acceleration "g", acknowledged to be downward, is not given the appropriate sign. Some students showed a tendency to regard the acceleration due to gravity as always being positive.

In a number of cases, signs were given only to some of the variables but not to others. The following are possible reasons for students' neglect of appropriate signs:

a) Because one-dimensional motion is being described, students see no need to choose a suitable axis, in spite of the fact that the body may change its direction of motion.

b) The values, s, a, v and u in the kinematic equations are regarded as scalar rather than vector quantities. This probably arises because the equations are written without vector notation.

c) Students are used to examples where objects only move in one direction in the same problem and have thus not developed a strategy to apply when this is not the case.

ii) Concepts of acceleration and freefall

The interviews indicate that a number of students do not have a clear idea that in freefall an acceleration of g-downward is the only acceleration of the body. Some students attributed the acceleration of the lift, 2 m s^{-2}, to the screw in spite of acknowledging that the screw had broken free from the roof of the lift. This acceleration is viewed as somehow residual in the screw. Some used 2 m s^{-2} as the acceleration of the screw in an equation while one student first assigned an acceleration of (g-2) to the screw, later changing it to (g+2).

One student seemed to confuse acceleration and velocity. Thus he said that "the car will reach a constant acceleration there", in referring to problem 1.

Another student viewed acceleration due to gravity as if it were a force saying that "g is acting downwards on the lift".

iii) <u>Acceleration of a stationary object</u>

Two students had great difficulty appreciating that a stationary object can nevertheless have an acceleration. They felt that the car had to be moving before an acceleration could be ascribed to it. One student felt that "the car before reaching a constant acceleration of 2 m s^{-2} would first have an initial velocity".

This seems to be related to the difficulty which some students have in interpreting the exact point in time that the problem begins or ends, i.e. the instantaneous situation. This phenomenon is reflected by the student who gave the final velocity of the screw in problem 2 as zero because "it hits bottom here".

iv) <u>Nature of the problem</u>

For some students problem identification proves difficult. This can be regarded as a molar difficulty on one level as well as implying a cognitive deficiency, in that data analysis does not bring recognition of the parameters of the problem. The discussion here focuses only on the molar aspects.

It is apparent that the idea of lifts accelerating conjures up the well-known problem usually associated with Newton's second law. Some students thus introduced the idea of forces into their solution attempt. One student tried to incorporate the idea of a downward force on the lift by "m a". Two students attempted to calculate the tension in a cable. Another student likened problem 2 to that of the sandbag falling from a rising balloon and was unable to recognize that while in the latter case the sandbag falls onto a stationary surface, in the former the floor moves up to meet the screw.

Some students experienced difficulty in identifying one-dimensional motion. In both problems one student endeavoured to introduce a horizontal component. In problem 1 he drew a graph indicating an x-direction while in problem 2 he introduced a parabolic motion of the screw. A number of other students also did this with problem 2.

v) <u>The common misconception reported upon by Trowbridge and McDermott (1980)</u>, viz. that when two objects are at the same position they must also have the same speed, was held by a number of students.

One student was extremely persistent in this regard and could not be convinced of anything to the contrary. Another student postulated that the car and truck would travel together for some distance.

vi) <u>The related velocity of two bodies moving together</u>

In problem 2, few students could see that the screw and the lift have the same initial velocity. This appears to be a problem in addition to the semantic difficulties with the word "falls" discussed above. It was apparent from a number of the discussions that students did not immediately realize that at the instant that the screw comes loose from the lift it must have the lift's velocity. This also seems to be related to an understanding of the instantaneous situation, discussed in (ii) in this section.

vii) Intuitive feelings

In some cases students seemed to trust intuitive feelings regarding the physical situation rather than an analysis of the principle which applies. Two examples are the following:

a) One student insisted that the car will arrive at the meeting point in a shorter time than it will take the truck. This seems to rest on the intuitive feeling that the object which moves faster must take a shorter time regardless of the parameters of the problem.

b) The concept of force as it relates to motion is not always correctly understood. One student felt that the upward force on the lift must be greater than the downward simply because "the lift is travelling upward".

4.6.4 Cognitive Difficulties

A number of broad categories into which the cognitive difficulties experienced by students as they attempted to solve one or more of the problems may be classified, is given below. These categories are neither mutually exclusive nor unrelated. They reflect features which seem to be common to a number of the interviews and which relate to the manner in which the students analyze the data in the problem and then utilize the data to develop a solution to the problem.

i) Visualizing the physical situation

It is clear from the interviews that almost all the students make almost no effort initially to form a mental picture of the physical situation described in the interviews. Among these, however, there are two broad categories :

a) Those who after some prompting and help are able to form a clear idea of the physical situation. It appears that they do not necessarily experience a deficiency in their ability to visualize but rather that their data collection is extremely superficial and related more to numerical data and relevant (and irrelevant!) equations. This does not imply that after being requested to explain the physical situation they were able spontaneously to do so accurately. In many instances the degree of prompting and help was considerable.

In a number of instances students would engage in a visualization of the problem only as far as required to obtain data to insert into equations. In one case the student tried to visualize the problem only when no suitable equation occurred to him. Another student made only a cursory initial analysis of the situation at the traffic light in problem 1 and then looked for values to insert into an equation.

A few students found no difficulty in picturing the details in problem 1, but almost without exception needed considerable help in clearly identifying the essential features of problem 2.

This was reflected in the quality of the sketches drawn by some students. Thus while the sketches of problem 1 showed a clear perception of the situation, the sketches of problem 2 merely gave some numerical data without any clear indication of the movements of the bodies.

b) Those who appeared to be incapable of reading the problem and picturing clearly what was being described. In spite of prompting, help and direct requests to do so, some simply could not comprehend the physical situation. The following were observed:

- One student ignored all requests to explain the physical situation in problem 1. He simply continued to experiment with equations until the correct combination leading to a solution, was found. Since the answers to the problems were given, he continued inserting values into equations until he stumbled upon the correct answer by trial-and-error.
- In discussion separately with three students it became apparent that they did not understand that the truck passes the car at the traffic light. One student was extremely insistent that the truck only passed the car sometime after leaving the traffic light. In explaining what distance needs to be determined in problem 1, another said: "Between the starting point of the car and the point where the truck passes the car".

That accurate data analysis and the spontaneous visualization of the problem are closely related were well demonstrated by two of the interviewees. One of these students was the person who obtained the top mark in the matriculation (i.e. school-leaving) examination of the Department of Internal Affairs the previous year. Both were extremely careful about visualizing the problems before attempting any solution. They carefully fitted the data into the pictures which they had created. Even in the case where one of them did not see the manner in which the floor of the lift moves toward the screw, a short explanation sufficed to help him.

ii) Qualitative evaluation of the problem

It is noteworthy that almost all the students interviewed adopted the same strategy in attempting to solve the problems: they would read the problem, write down one or more of the kinematic equations and then try to fit the numerical data into the equations. Almost all the interview analyses contain the same observation : "Explicit, numerical data are noted and then there is an immediate attempt to relate it to known equations. There is no concern for qualitative aspects of the problem". It is apparent that the entire focus of attention is the equations and the numerical data. Once these are ascertained the problem assumes an entirely mathematical nature with the problem only being re-read when the student cannot proceed further.

That the initial choice of equation is central to the further search for data was observed with a number of students. Thus one student in approaching problem 1, made an analysis of what happens at the traffic light and then looked for values to insert into the equation $s = u\,t + \frac{1}{2}\,a\,t^2$ which she had first written down. When this led nowhere she chose the equation $v^2 = u^2 + 2\,a\,s$ and simply repeated the process. Another student, in searching for a suitable equation after reading the problem, wrote "a = distance x velocity" for problem 1 and then wrote "2 m s^{-2} = distance x 10 m s^{-1}" even though the acceleration applied to the car while the velocity of 10 m s^{-1} was that of the truck. Since the choice of equation is made even <u>before</u> numerical data are recorded, in many cases there is a definite arbitrariness about the equation chosen. Few students wrote down all three kinematic equations but chose either $s = u\,t + \frac{1}{2}\,a\,t^2$ or $v^2 = u^2 + 2as$, and moved from one to the other when one did not yield results.

It would be expected that the same two categories of students who experienced difficulty in visualizing the problem would also not make a good qualitative evaluation of the problem since these two abilities are closely related. While this is so, the ability to make a qualitative evaluation of the problem is more all-embracing than the ability to form a clear picture of the physical situation since in the former case features of the problem must be obtained which are necessary for its solution.

Thus it was found that the only students who were able spontaneously to extract features of problem 1 which led to a solution, were those who were acquainted with the type of problem (except the two good students mentioned above). One student was even able to relate an analogous problem which he had solved in the Applied Mathematics course. But even these students when attempting to solve problem 2 proceeded directly with numerical data and equations into which the data can be fitted. It seems that an evaluative strategy is not an essential part of their cognitive repertoire.

In most cases, then, students show a very limited ability to make a detailed problem analysis related to features other than numerical data. Some students clearly understand the relative motion of the bodies but do not extract features relevant to the problem solution. One student correctly described the motion of the screw but was misled by an initial drawing of a parabolic motion he had made. Another student when asked to describe the problem dealt vaguely with accelerations of the bodies.

It may be argued in the light of section 2.3.2.2 that the solution strategy adopted by students in this study is typical of the novice physics students. There are, however, two essential differences :

a) All the students interviewed had received considerable instruction in this section of work both at school and at university over a period of a year-and-a-half. They had also done a number of problems by the time they were interviewed.

b) The most important feature to consider, however, is the fact that the research discussed in section 2.3.2.2 related more to strategies which students do not adopt in solving problems. The present study deals with apparent cognitive inabilities which students have and for which remediation must be sought.

While the quality of sketches drawn, the way in which implicit data are analyzed and the identification by students of important points in the problem, are all clearly related to the qualitative evaluation which students make of problems, for purposes of clarity they are discussed separately below.

iii) The Quality of Sketches Drawn

While this is obviously related to points (i) and (ii) above, the following general observations can be made about the sketches drawn :

a) While some students saw the need to draw a sketch, the quality thereof was extremely poor and reflected the haphazard and often incorrect way in which data were collected and ordered. In one sketch of problem 2 it is not apparent whether the student is drawing the lift or the screw and how these are related.

b) Sketches which are drawn spontaneously by students are clearly only done for the recording of numerical data. The sketch shows no real visualization or interpretation of the problem.

c) A number of the sketches are completely meaningless.

d) Some students make no effort to draw a sketch and others do so only after prompting.

iv) Identifying Implicit Data

While implicit data are obviously related to the ability to visualize the physical situation and to make a qualitative evaluation of the problem, it does not automatically follow that if the student does these with some degree of accuracy that he/she will necessarily extract some or all the data implicit in the problem statement. Note the following examples from the interview analyses which clearly illustrate this :

- Implicit data are only extracted with great difficulty. A student sees, for example, that the distance travelled by both objects is the same (after some discussion), but cannot immediately see that the time taken by each vehicle to cover this distance is the same.
- While a second student makes good qualitative evaluations of both problems and, in problem 1 at least, relates time and distance for the two vehicles, he seems to ignore more obvious implicit data such as the nature of the motions of each body.
- A third student has a clear idea of the physical situation but does not sketch it out. As a consequence, implicit data are not easily apparent to her.

In a number of cases, however, the inability to visualize the problem clearly leads to implicit data being either ignored or wrongly interpreted. Most students had to be prompted not only to determine the implicit data but also to consider its relevance. Some students had to be encouraged to draw a sketch before they realized the meaning of the implicit data. With some students it was apparent that the ability to determine implicit data certainly exists but the need to do so in the problems given, was seldom perceived. With some students, in problem 1 they were easily able to relate distance, s, and time, t, for the two bodies and see that they were equal. They saw no need to explicate it. In the second problem where t is the same for the screw and the lift but where s for the lift means 3-s for the screw, most students seem unable to find the relationship. This seems to be related, in addition to their difficulties with implicit data, to the way in which they deal with two sources of information, as will be discussed below.

A further class of students is that which was simply unable to determine implicit data without considerable help. Even then they were not able to understand that in problem 1 the truck and the car travel the same distance from the traffic light to meeting point in the same time. Some would equate the distance but not the time, t. In spite of this they would often simply insert "t" in equations for the different bodies.

v) Goal and Problem Analysis

In analyzing the transcript of an interview with one student, the following comment was made:

"It is apparent throughout that he does not "see" the problem. His data analysis is haphazard and clearly affects his elaborational capacity to the extent that he appears unable to see how to proceed on his own at any stage without considerable prompting. His analysis does not appear to generate any meaningful related idea using the three kinematic equations."

Most of the students interviewed appeared to have no goals or sub-goals toward which they worked. The following features were observed

- Students do not appear to relate the data to what is required in the problem i.e. they make no goal analysis but simply insert data into formulae in the hope that the answer will simply "fall out".
- Students give no indication that they have a very clear idea of what needs to be determined or what is required to solve the problem. They read the problem, and then reason in a linear sequence from equations to the hoped-for solution via the numerical data. They seldom reason reflectively from the problem statement to establish sub-goals for themselves in an effort to find a starting point.
- The immediate result of their uni-directional thinking pattern in attempting a solution is that as soon as they are stuck they make no spontaneous re-evaluation of their approach. Rather, they tend to play around with the numerical data rather aimlessly. In most areas it is only

when the interviewer suggests a different approach that they will try it. They appear to make an impulsive initial judgment (as for example, which equation to use) and then to relate all they do to this without re-evaluating its accuracy.

A review of almost all the interviews conducted, shows that the majority of the students do not regard the problem statement as a piece of prose which must be carefully analyzed not only for all details but also word-for-word. The lack of both goal and problem analysis in the approach adopted by students can also be seen by the manner in which the two terminal points in both problems are treated in their data analysis. A significant number do not seem to appreciate that the events at the traffic light and again where the vehicles draw level in problem 1, define the problem. Note the following comments from the interview analyses:

- He does not appear to see spontaneously what the initial and final points are or, indeed, their relevance.
- His sketches show no clear definition of the important points in the problem. Both the initial and final points need to be made explicit to him.
- He does not determine the second important point in the problem but confines his attention to the traffic light and the numerical values there.
- She has difficulty relating the final point to what happens previously. She does not seem to see out of the problem formulation that the point where the vehicles

draw level has significance.

The majority of students were unable to identify clearly the two points in problem 2.

vi) Elaboration of information

In conducting the interviews it was very striking that when the data collection of students was defective in some way, they were seldom able to find cues which would prompt them into deciding what the next step was which needed to be taken. It was observed with many students that there was no orderly progression of thought in developing the problem - it was seldom clear what the student was attempting to do at any point. Some students tended to concentrate on single aspects of the problem (e.g. the velocity v in problem 1) and would then experiment with equations. Other students did not have a clear idea of what needed to be determined and the calculations made, tended to be rather aimless. Students seldom displayed any spontaneous comparative behaviour, i.e. they would seldom assess what they were presently doing in the light of what they had already found or what needed to be determined.

A number of the interviews indicates that students lack any planned approach to the problems. Two aspects of this problem were revealed:

a) The manner in which students accumulated the data in the problem did not provide them with the cues to decide on any particular approach. Some did not appear to perceive that a statement of constant acceleration of one of the bodies implies that the three kinematic equations must be used. For this reason some students attempted to use one of Newton's laws, parabolic motion and in one case even vector addition, in some vague effort at a solution. Some students gave the impression of a completely unstructured thought pattern with no particular approach. They did not appear to see the direction in which a line of argument was proceeding.

b) A large number of students was able to decide that the three kinematic equations needed to be used, but they were seldom able to decide which particular equation to use and why. It was apparent that while, in most cases, they could recall the equations, their use thereof had not developed in them any specific procedures. The following comments from the interview analyses were typical:

- Although he writes down the equations, he does not demonstrate any structured way of applying them and his numerical data analysis does not suggest any particular approach.
- It is clear that with problem 1 and more especially problem 2, he has no planned or structured approach. In problem 1 he is visibly surprised when he gets the correct answer.
- The equations evoke no structured thought pattern - as for example which equation is applicable for a specific situation.

The idea of the elaboration of a concept will be pursued further in the next chapter.

vii) Impulsivity; Unsystematic and Incorrect Data Analysis

As was noted in the discussion above, almost all students are very quick to decide on an equation and to relate numerical data to it. This impulsivity in dealing with the problem to be solved results in the student being unclear of what exactly needs to be determined. This in its turn makes his examination of the data very unsystematic. In addition to this, a number of interview transcripts indicates that students record data incorrectly, often confuse symbols and are incorrect in their analysis of data, i.e. they ascribe incorrect values to symbols. They also show no order in their manner of data analysis.

Note how some of the interview analyses relate the students' impulsivity to an incorrect use of the data recorded:

- Because of his initial haste to record numerical data and then to use equations he seems to lose sight of some of the data recorded. For example, after saying and writing down the initial velocity of the lorry, he later says that it is unknown.
- In the second problem he makes immediate (and incorrect) decisions as respects values (such as s) and immediately inserts these into equations.
- He shows a lack of accuracy in the manner in which he gathers the data. This can be attributed to the haste displayed in fitting values into equations.

In a number of cases, the interview analyses show that students are inaccurate in the data they collect. Note some examples :

- He demonstrates peculiar inaccuracies in data collection
  - after stating that the lift has an upward acceleration of $2\ m\ s^{-2}$ and recording it, he then says it has an acceleration of $g = 10\ m\ s^{-2}$.
- He does not easily see that the initial velocity of the car is zero. He speaks of the lorry as having constant acceleration. He is surprised to find the velocity of the lorry actually given in the problem after stating that it is unknown.
- He does not consistently use the data he has determined. For example, he writes $10\ m\ s^{-1}$ for the velocity of the truck on the sketch, but when it must be used in an equation he says that it is not given.
- His data gathering is very imprecise. For example, he gives the car a velocity of $2\ m\ s^{-1}$ which he indicates is perpendicular to that of the truck. Neither does he use symbols consistently (he confuses s and x).

In addition to this the large majority assign a value $s=3$ for the distance travelled in problem 2. Seldom is it specifically indicated which body covers this distance. Since it is the only value given in the problem which can be related to s, students appear to assume automatically that it must be the value required in the equation.

It was noticed in a few instances that students in recording the data do not always explicitly differentiate between the data of the two bodies. In one case at least this caused a student to equate the velocities of truck and car when these draw level.

viii) Relating Two Sources of Information

A difficulty which was common to almost all the students interviewed, was the manner with which they dealt with the two sources of information in the problem. An analysis of all three problems shows that no solution is possible by simply inserting values into equations. In problems 1 and 3 the data from the truck and car must be compared so that the simple relationship between the distances and times of each can be obtained. In problem 2 the distances covered by the screw and lift must be carefully related. Additionally in problem 2, the directions in which the two objects move are also different. No coherent picture can be obtained without seeing how the one body moves relative to the other.

The following degrees of difficulty were noted from the interview analyses:

a) In the most extreme case the students examined only one body to the total exclusion of the other. Note the following representative comments:

- He focuses entirely on the motion of the screw without relating it to the motion of the lift.
- She tends to focus entirely on one object in the hope that the answer will "fall out".
- She does not easily consider the two sources of information at once, focusing only on the car.

b) Students write down the data from both bodies but consider each in isolation. This seems to be the most common occurrence.

Typical comments from the interview analyses are:

- He examines the objects in problem 2 entirely on their own and makes no attempt to relate the movements of the bodies.
- Each object is dealt with separately and seldom is the motion of one related to the other.
- The two objects are not related. Each is examined separately and conclusions drawn from each independently.
- While he writes down the data of each object in problem 2, he makes no effort on his own to relate the two sources of information but deals with each entirely in isolation.

c) The student considers both sources but is not able to find a relationship. This aspect gave most students the greatest difficulty. While a number of them was able to establish the relationships in problem 1 after being prompted to visualize the problem carefully, almost none were able to do so in problem 2 without a very considerable degree of help.

4.7 Deficiencies Highlighted by the Interviews

It seems apparent from the above analyses that students seem to have general difficulties in analysing data in a well-defined, ordered manner. In an effort to categorize these difficulties more carefully, these will be related to the list of cognitive deficiencies enumerated by Feuerstein and his co-workers as discussed in Chapter 3.

1. Blurred and sweeping perception

Note the following characterization of this by Feuerstein, et al (1980: 76)

> "What characterizes the blurredness is a poverty of details or their lack of detail, a poor quality of sharpness, an imprecise definition of borders, and an incompleteness of the data necessary for proper distinction and description. The fact that an object or stimulus is perceived in a blurred way totally affects, as well as is affected by, the process of elaboration and output. Because we are not dealing with peripheral, sensory limitations, we are in the realm of a deficiency whose effect upon the elaborational process determines both the nature of the input and the subsequent output."

In almost every interview recorded, the inability of the students to give a clear visualization of the problems was noted. Even those who were able to do problem 1, had difficulty with problem 2.

This is also highlighted by the lack of clarity of detail of many of the sketches attempted. Some of the sketches were only attempted after prompting and even some of those which were spontaneously drawn, reflect a limited awareness of the parameters of the problem or indeed, of the actual physical situation. In many instances, the sketch was used to record only some of the numerical data.

It seems apparent that no real need was perceived to make a qualitative evaluation of the problem situation, with the focus of attention being the obvious perceptual clues embodied in the quantitative data. There is thus no clear delineation of the problem situation.

The blurred perception of the problem is reflected in the inaccuracy with which certain data are interpreted; for example, identification of s, the distance covered, with 3 m (the height of the lift), and also giving the screw an initial velocity of zero.

Another difficulty which can be related to a blurred perception, is that experienced by most in the lift problem in differentiating between one stationary observational system relative to which both lift and screw are moving, and one in which the observer is moving with the lift, i.e. one in which the screw falls directly from rest. The latter case is clearly the one more amenable to experience or a simple "gedanken experiment" and contributes to an undefined, vague perception of the nature of the problem. This can possibly be attributed to a misinterpretation of an experience or an easily imagined experience, placed within a context where it is not applicable.

That the deficiency of being unable to perceive carefully and accurately is indeed an inhibitor to a student's ability to elaborate a problem, is seen from the fact that in the interactive type of interview conducted with some of the individuals above, (as opposed to the strictly clinical, data collecting type) when they were helped to visualize the problems more carefully, they were frequently able to solve these without undue difficulty.

2. Unplanned, impulsive and unsystematic exploratory behaviour

Let us define this cognitive deficiency by taking some quotes from Feuerstein, et al (1980: 77, 78)

> "When presented with a number of cues that must be scanned, the individual's approach is so disorganized that he is unable to select those cues whose specific attributes make them relevant for a proper solution."
>
> "Instead it is the product of inadequate training in exploratory skills. This is reflected in a poor definition of the problem to be solved, a lack of goal orientation, and unsystematic exploration. Ultimately, the definition of a problem is itself a function of appropriate exploratory behavioural skills."
>
> "Here the major determinant of impulsivity is conceptual or epistemic with the acceleration of the response directly linked to the limited awareness of the need for additional data to produce the proper answer."

Some of the above features are readily observable in the responses of the students listed above. These include:

i) A number of the students read the problem, extracted only the numerical data and immediately proceeded to the equations in an attempt to solve the problem as it is perceived.

ii) Implicit data in the problem are not easily or automatically made explicit. Thus in problem 1, the distances, s, and the time, t, relevant for each body could be readily related by some (although not by any means by all), although it was not detailed in their answers. However, in problem 2, where relationships have to be established, i.e. x for the lift means $-(3 - x)$ for the screw, this could not be done readily by any of the students. Even the time, t, was not easily seen to be equal for both the lift and the screw. In two instances, the idea implicit in constant velocity, (acceleration = 0) was also confused.

iii) There was very little need felt by students for a careful data analysis of the problem statement in relation to a perceived physical situation even when the student was stuck and could not see which way to proceed. The normal strategy observed under those circumstances was to play around with the equations, trying those not already used.

3. <u>Lack of precision and accuracy</u>

This is defined as follows (Feuerstein, et al, 1980: 86, 87):

"Two categories of imprecision may be distinguished: missing data and distorted data. In the first category, the individual does not take the care to gather all the data he is offered and hence cannot use the information when he has to produce an answer or is required to report on a perceived object or event ...... In the second category, imprecision may be the result of a distortion of certain dimensions."

Relating the recorded interviews to the above, the following may be noted:

i) Students gave conflicting responses within the course of discussion as regards the same variable. Thus a number wrote explicitly that the initial velocity of the screw was 2 m s^{-1} but in sketching the situation, labelled the initial velocity as zero. They were also not able to resolve this conflict easily.

ii) A complete lack of axes or signs for variables was also noted in the responses of most students. It is very clear that signs are in no way regarded as a necessity by almost all the students interviewed. This does not mean that they are necessarily unaware of the need for a sign for certain quantities. It is especially conspicuous that in the case of "g", the acceleration due to gravity, a sign is usually given, either when writing down the value or when inserting it in an equation. The sign is usually negative.

iii) Students spent very little time reflecting on what they record in order to check it for accuracy. This resulted in their writing "t" for "t^2" and proceeding incorrectly. Or they would write down an incorrect numerical value after reading the problem and would then continue with the wrong value.

iv) A number of the students also did not distinguish between the symbols they used for the variables for the different bodies by using the necessary subscripts, etc. In one case at least, this resulted in the equating of velocities of lorry and car in problem 1.

4. <u>Impaired use of two more sources of information</u>

This is categorized as follows (Feuerstein, et al, 1980 ; 87, 88)

> "The function of lack of, or impaired use of, two or more sources of information is included as a deficiency at the input level, despite the fact that it is actually the outcome of an elaborative process. The use of two sources of information is a prerequisite to thinking because it is the basis of all relational thought processes .... If the individual constantly tends to relate to each source of information separately, either successively or alternately, but does not coordinate the two, the elaborative process will be impaired because the relationship between the two sources will not be available for further experiences."

A careful analysis of the interview transcripts and the observations of the students presented above, isolates this deficiency as perhaps being the root of the inability of some to deal with problems 1 and 3, and seems to be crucial in understanding the difficulties which all the students had with problem 2. The following observations seem germane to the situation:

i) Most students examined the numerical data of each object separately. Where there is overlap between the data of the two objects (as, for example, distance x travelled by the lift and distance 3 - x travelled by the screw) this was not done spontaneously.

ii) Even qualitatively, each motion appears to be analysed separately. The need to analyse the combined motion appears to be lacking and indeed the ability to do so was markedly deficient in most of those interviewed.

5. Lack of, or impaired interiorization

This deficient cognitive function is described by Feuerstein, et al (1980 : 97) as :

> "Planning cannot be conceived of without interiorized representation in the attempt to bridge between the present and a nonexistent future. It aims at a goal which, by definition, is not yet concretized. Planning is exclusively dependent on the representation of interiorized goals. The incapacity of the individual to represent to himself the future or the outcome of a transformation may severely limit his behaviour because it will reduce his judgement and, even more, confine his needs to the "here" and "now"."

It seems apparent from the cognitive difficulties discussed above that students do not make an "interiorized representation" of the problems. This can be deduced from the following:

- they made no qualitative evaluation of the problem and then seemed to lack the ability to extract essential features.

- their goal and problem analysis appeared to be superficial. Thus they were unable to establish goals towards which they worked in attempting to solve the problems. They were unable, in many cases, to appreciate the significance of the two terminal points in the problem.

6. Lack of, or impaired, planning behaviour

Feuerstein, et al (1980 : 98) relates planning behaviour and interiorized representation as above. There are, however, additional features in demonstrating planning behaviour:

> "Deficient planning behaviour is mentioned above in the description of interiorization with respect to the construction and representation of more remote goals. However, planning not only consists of the setting of goals that are located both temporally and spatially at a given distance from the "here" and "now" but also involves the dissociation of the aims set forth from the means to achieve them. This dissociation requires a further differentiation of the means in terms of the steps that are necessary to achieve the final goal."

The above considerations with respect to planning apply as much to organizing life's activities as it does to solving a problem in physics. In the latter case the ultimate goal is established by the problem statement whereas the steps to that goal need to be carefully constructed from accurate data analysis leading to the application of some law, principle or equation.

From the considerations under "Elaboration of Data" above, it is easily seen that the students in this study

- simply fitted values into equations in an effort to obtain an answer. This is often the extent of the planning done by most of the students. When they could proceed no further they seldom re-evaluated what they had done in an effort to adopt some alternative strategy.
- showed very little structure in their approach to the problems, often concentrating on discrete features to the exclusion of all else.
- did not analyze data sufficiently accurately for a plan to crystallize therefrom.
- did not show a planned approach in their use of the three kinematic equations. The interviews showed plainly that they did not even follow the elementary step of writing down the possible variables s, a, v, u, t and then determining which of these are given for each object.

4.8 Conclusions and discussion

The insights gained from the interviews were extremely illuminating in clarifying some of the very real learning difficulties of the students in this study. Even a cursory reading of the interview transcripts should make it obvious that a knowledge of the particular concept, law or equation which must be applied to solve a problem, is only a part of what is necessary to find a solution. Physics, like any other science, generates its own vocabularly which not only includes specialized terms but which might also give normal words a particular meaning within a certain context. The word "falls" is an example of this. Since the interviews demonstrate the extent to which semantic difficulties serve as distractors, the determination of such words and their influence on the learning process is an interesting field for further research. While much research in Physics Education has been devoted to the determination of the misconceptions which students may have, the present study also indicates that "Error Factors" in general - molar reasoning difficulties - play an important part in preventing students from dealing with certain sections of content effectively. Some of the error factors determined, are the following:

i) Students seldom assign signs consistently to objects moving in a straight line. This difficulty seems to be caused by the fact that vector notation is seldom, if ever, used with the three kinematic equations.

ii) The concept of freefall is not readily understood by some. A few students seemed to feel that the acceleration of the body before it is in freefall is somehow residual in the body.

iii) Some students did not realize that a body with a velocity of zero can have an acceleration. This difficulty is related to the confusion which some displayed as to the instantaneous situation of a system.

iv) Some students gave evidence of trusting intuitive feelings which they may have had of a particular situation rather than employing the physical principle which applied.

Further examples of error factors are given in the text. It seems apparent that fruitful research can be done of sections of content in physics to determine such error factors. Appropriate material can then be developed to remedy such difficulties.

Error factors can also be related to the molecular cognitive problems of students. An example of the influence of the latter on the former has been given (see Mehl and Volmink, 1983). For this to be done effectively, an analysis of concepts in terms of cognitive requirements needs to be done. This, too, is a field of research which needs to be explored.

A recognition of the cognitive difficulties which UWC physics students display has proved extremely illuminating in understanding why these students do so badly in the normal physics course. The cognitive operations which many either do not or cannot employ, include the following:

i) Visualization of the essential features of the problem,

ii) Making a qualitative evaluation of the problem,

iii) Making explicit the implicit data in the problem,

iv) Establishing the relationships between two sources of information,

v) Planning an approach to the solution of the problem.

It needs to be emphasized that this research has not necessarily shown that these cognitive difficulties are actually deficiencies in all students. Some students have never been taught to analyze data carefully or to plan an approach to the solution of a problem (a rote-learning approach to the teaching of science by unqualified teachers will certainly not include such refinements). In other cases, however, the interviews indicate clearly that some students simply cannot execute such cognitive operations. Relating the perceived cognitive difficulties of UWC students to the Feuerstein list demonstrates that many students in this study show varying degrees of the cognitive deficiencies documented in the research carried out in Israel. It remains to be demonstrated how these difficulties can be addressed using the content of physics. This will be done in Chapters 6 and 7. Indeed, how recognition of cognitive difficulties can affect university teaching has already been reported (Mehl, 1984).

CHAPTER 5

ANALYSING THE PRESENCE AMONG STUDENTS OF PLANNING IN USING CONCEPTS

5.1 Introduction

The cognitive function being examined in this chapter is "the ability to plan in using a particular concept". Since planning ability is an elaborative process, it is necessarily specifically linked to the concept under discussion since different concepts would require different approaches. In testing for the presence of some planned approach, therefore, the concept must be broken down into the steps required for its application. Students can be tested on:

i) their realization that the steps are necessary when applying the concept; and

ii) their ability to execute the steps.

It is important to understand exactly what is being examined; on the strength of Feuerstein's list of cognitive deficiencies detailed in the previous chapter, it is claimed here that it is likely that the students will display little planning in their approach to various concepts. The purpose of the test administered was to validate this premise (section 5.2). It is imperative to realize that it is not the ability to plan which is *directly* examined in the test. This can be seen by recognizing the distinction that needs to be drawn between the manner of testing for cognitive deficiencies on input (see chapter 4) as opposed to elaborational deficiencies. The ability to analyse data, while obviously influenced to some extent by a knowledge of content, can be examined without recourse to much concept understanding. However, an elaborational ability is very clearly linked to an understanding of the concept being elaborated. To test for the existence of an elaborational ability thus implies that some sort of instruction must have been given. After this instruction the student can be tested as to the presence of a planned approach in the use of the concept taught. It is a premise of this thesis that without instruction specifically designed to address particular cognitive deficiencies (in this case the ability to plan) students will not develop these cognitive abilities through the normal, "classical" manner of teaching physics. Without the necessary planning done for them, disadvantaged students will continue to experience difficulties in coping with specific concepts in physics. To test an elaborational function, therefore, a section of the course needs to be tested in which students have received a fair amount of instruction.

To reiterate therefore : what is being tested in this chapter is not the ability to plan (directly in the test), but rather whether a planning capacity in the use of particular concepts already exists in the students after school instruction (thus the ability to plan is indirectly tested). The test attempts to assess not only the ability to execute the steps of an established procedure in the use of a particular concept, but also whether these steps are deemed to be necessary by the students. The latter can be done by determining whether the steps are recognized as important or indeed, if they are even recognized at all.

The test designed was limited to the application of Newton's three laws of motion. Since all the students tested had done Physical

Science up to and including their final year at school, it was felt that they would be reasonably well acquainted with the mechanics section of the work which is dealt with in some depth in their penultimate school year and then revised in their final year in preparation for the school-leaving examination. They would also have been confronted with problems on these concepts. Consequently a planned approach would be more likely to show up here than anywhere else. The test was administered to all students in their first week at university, before they had received any formal university instruction.

To design the test, the planning procedures intrinsic to the application of Newton's three laws had to be made explicit. This was done in the manner outline in 5.3.

5.2 The Test Administered

The following is the test given to all first year physics students at UWC in their first week of the first academic term (the kind of answers anticipated are indicated in brackets in italic type where possible).

FIRST YEAR ASSESSMENT TEST - PHYSICS NAME: ....

1. State Newton's First Law:

   a) In words *(A body will remain in a condition of rest or of uniform motion in a straight line unless acted on by an unbalanced force)*

   b) In symbols *($\vec{A} + \vec{B} + \vec{C} + - \ldots = \vec{0}$*

      Define the symbols *($\vec{A}$, $\vec{B}$, $\vec{C}$ - different forces on body*

2. State Newton's Second Law:

   a) In words: *(The acceleration of a body is proportional to and in the same direction as the resultant force acting on it, and inversely proportional to the mass of the body)*

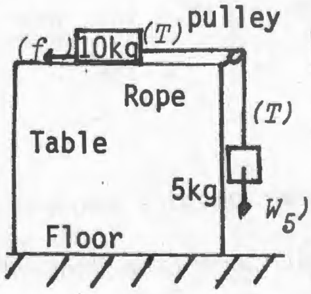
b) In symbols: $(\vec{F} = m\,\vec{a})$....................

Define the symbols: (F = Resultant force. a = acceleration of body
m = mass of body....)

3. State Newton's Third Law: (To every action there is an equal but opposite reaction).............

4. Which of Newton's laws would you use if you had to solve an equilibrium problem?

Law	Reason
(1st	*It deals with a body at rest*
3rd	*Establishes the forces acting on a body)*

5. 

Masses of 10 kg and 5 kg are connected by an inextensible rope as shown in the sketch and the system is in equilibrium. Calculate the force of friction on the 10 kg mass. Please read this carefully and then answer the questions below:

a) How do you understand the expression "the system is in equilibrium"?

(the 10 kg and 5 kg connected by the rope, are stationary)...

b) Which of Newton's laws would you use to solve the problem?

(1st and 3rd).......

c) Draw in all the forces which you think are important to solve the problem on the sketch above, give each a symbol and name them below:

Symbol	Name of Force	Symbol	Name of Force
(T	*Tension*	W_5	*Weight of 5 kg)*
(f	*frictional force)*		

5. d) Solve the problem:

(5 kg: sum of forces on 5 kg $= W_5 - T = 0$

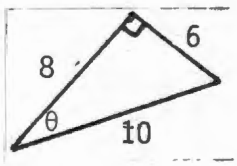
10 kg: sum of forces on 10 kg $= T - f = 0$, $\therefore f = W_5 = 50N$)

6. A person walks due North for 3 km and then due East for 4 km. How far will he be from his starting point? ....*(5 km)*....

7. In a right-angled triangle, two sides have length 5 cm and 12 cm respectively. Calculate the third side. ....*(13 cm)*....
Is more than one answer possible? ....*(Yes)*....

If so, why? *(12 cm can be the hypotenuse)*

8. Please write down:

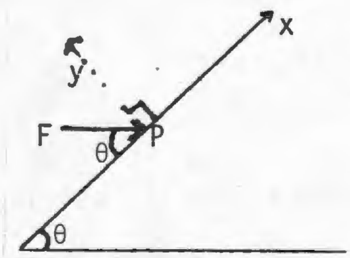


a) sin θ = ...*(0,6)*....

b) tan θ = ...*(0,75)*....

c) cos (90 - θ) = .*(0,6)*..

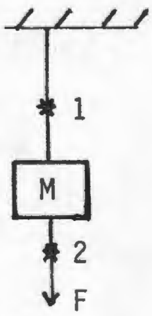
9.



A force F acts at a point P on an inclined plane. Determine: component of F in x-direction (parallel to plane): .*(F.cos.θ)*......

component of F in y-direction (perpendicular to the plane): ......*(-F sin θ)*......

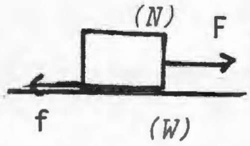
10.



i) If the rope in the diagram is given a sudden downward jerk at F, does the rope break at point 1 or 2? .*(2)*.....
Why? *(Inertia of M resists change)*..

ii) If the rope is gradually pulled downward at F, does the rope break at point 1 or 2? ....*(1)*.......
Why? *(Weight of M adds to downward force)*

11. a)



(force of friction)

The mass M is at rest on the floor with the forces acting as shown. Write down the equations describing the situation:

(F - f = 0) ....

(N - W = 0) ....

11. b) In the sketch in 11 a), are there any other forces <u>ON</u> M?

Tabulate:

FORCE	HOW CAUSED
(N	*Reaction of ground on M)*
(W	*Force exerted by earth on M)*

c) If instead of being at rest, the body is accelerating along the floor:

i) Which of Newton's laws is applicable? *(2nd)* ....

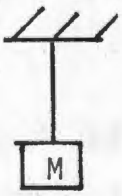
Why? *(It deals with accelerated bodies)* ....

ii) Write down the equation(s) describing the situation.

(F - f = M a) ....

(N - W = 0) ....

12.



A body of mass M, is suspended from the ceiling by a rope. Given the following bodies, state whether a force is exerted on mass M by each, the reason for the force and its direction (up or down).

	BODY	FORCE(Yes/No)	REASON FOR FORCE	DIRECTION
1.	Rope	*(Yes*	*Tension in rope*	...*Up)*.....
2.	Ceiling	*(Yes*	*very slight gravitational*	*Up)* ....
3.	Earth	*(Yes*	*weight due to gravity*..	*Down)* ....

State whether mass M causes any forces to be exerted ON the following bodies, give the reason for the force and the direction (up or down)

	BODY	FORCE(Yes/No)	REASON FOR FORCE	DIRECTION
1.	Rope	*(Yes*	*Reaction to tension*	*Down)* ....
2.	Ceiling	*(Yes*	*gravitational*	*Down)* ....
3.	Earth	*(Yes*	*gravitational attraction*	*Up)* ....

13. Two masses M_1 and M_2 are completely isolated in the universe (sufficiently distant so that no outside masses influence them), but they are close enough together to exert a gravitational force on each other. If $M_1 = 2M_2$, and the force exerted by M_2 on M_1 is 20N as in the sketch, show the size and direction of the force exerted by M_1 on M_2.

5.3 Analysis of the Test Presented

To enable a meaningful analysis to be made of the test presented, it is necessary first to establish the nature of the planning procedure which is in fact being assessed. While this is done in detail in Chapter 7 for Newton's laws, for the purposes of the test in this chapter, it is only necessary to establish the broad steps involved in using the particular law and to test whether these are present. This will therefore be discussed before the test analysis is given.

5.3.1 Establishing Steps in a Planning Procedure

A. Newton's First and Second Laws

The laws are stated above as part of the test. Since the First Law can be regarded as a special case of the Second (acceleration = 0), the steps required in applying the laws are fundamentally the same for both, except that those for the Second Law would be more detailed.

The following steps are regarded as essential if the laws are to be applied, especially in solving problems:

STEP 1: Determine which law is applicable to which class of problems.

STEP 2: The body (or bodies; subsystems) in question needs to be isolated. If there are connected bodies, the law needs to be applied to each separately.

STEP 3: The forces acting on the body need to be identified.

STEP 4: Directions need to be decided upon. The direction of acceleration (or possible direction of motion) normally establishes one direction. The other is then chosen perpendicular to this direction.

STEP 5: The relevant equations must be applied.

Newton I: $\Sigma \vec{F} = 0$. There needs to be a recognition of the fact that the vector addition of forces is necessary. This implies the use of components of forces along directions established in Step 4. The ability to use trigonometric relations and Pythagoras's theorem is obviously necessary.

Newton II: $\vec{F} = m \vec{a}$. The same considerations apply as above. The realization that $\vec{F}$ is the resultant force is obviously crucial.

B. Newton's Third Law

To apply Newton's Third Law the following steps need to be recognized and applied:

STEP 1: Two bodies (or subsystems) need to be isolated (as dictated by the specific problem, naturally).

STEP 2: The action must be defined in magnitude and direction as caused by body (1) on body (2).

What is being averred is that if the above steps are absent as the students try to apply Newton's laws, then they have not developed a proper planned procedure for dealing with the laws, but that these are being applied in a haphazard manner which is likely to lead to difficulties.

5.3.2 Analysing the Test

Questions 1 - 3:

1. *State Newton's First Law : a) In words; b) In symbols. Define the symbols.*
2. *State Newton's Second Law: a) In words; b) In symbols. Define the symbols.*
3. *State Newton's Third Law.*

The ability to formulate the laws correctly is obviously necessary if they are to be used effectively. No proper use can be made of equations if the symbols used are not properly described in the statement of the law.

The ability to rewrite the laws in terms of symbols, where this is possible, is necessary for the laws to be used in the solution of the normal type of physics problems. This is, of course, especially true for Newton's First and Second Laws, although not for the Third. It is equally important to be able to give each symbol its proper meaning.

While students may not always remember the statement of the law, the planned use of the law in solving problems can reasonably be associated with equations for its use. A lack of equations (or an incorrect formulation thereof)with properly defined symbols, would thus indicate a lack of any plan to use the law in solving problems.

The above three facets are, of course, not absolutes. Thus, if a student cannot state a law or write down equations to use the law or define the symbols in a law satisfactorily, it does not necessarily follow that he cannot use the law in some planned manner. These are merely indicators that it is unlikely that any procedure has developed. The questions which follow can also be regarded simply as indicators. However, in their totality the questions should give a fairly strong idea as to the presence of a planned approach or not.

Question 4: *Which of Newton's laws would you use if you had to solve an equilibrium problem?*

The choice of the correct law in dealing with a particular type of problem is crucial if one is to plan for a specified outcome. It is not reasonable to assume that a planned approach for use of a concept has developed if there is no recognition where it is to be used.

The students are being tested to determine whether their application of the laws has any structure, that is, whether any realization has developed that there are key features which have to be recognized in using the law. Thus the reasons given for using the law in solving an equilibrium problem can demonstrate how the student thinks in relation to the manner of application of the law. It could also indicate how laws can be misapplied.

Of course, any of the three laws would constitute a correct answer. The reason given would, however, show whether the answer given is valid.

Question 5 a)

a) How do you understand the expression "the system is in equilibrium"?

b) Which of Newton's laws would you use to solve the problem?

c) Draw in all the forces which you think are important to solve the problem on the sketch above, give each a symbol and name them below.

What is being examined here is not only the meaning of the term "equilibrium" but more importantly whether the student realizes the importance of correctly identifying "the system" as the first step when applying Newton's first and second laws. Thus the answers to questions 5 a) and 5 d) will both indicate the importance in the mind of the student of isolating the particular bodies involved in the solution of the problem.

Question 5 b):

This is a test of consistency between questions 4 and 5 b).

Question 5 c):

This serves to identify the following:

i) whether students have indeed identified the "system" relevant to the problem. This is illustrated by their indicating just those forces which act on the 10 kg and 5 kg masses.

ii) whether they are able to execute the second step of the procedure, namely, to draw in all the forces acting on the bodies, not only by naming the forces but also by giving the correct direction and point of application of each.

iii) whether they can interpret the influence of other bodies on the system, that is, have some idea of how Newton's Third Law is applied.

Question 5 d): *(Solve the problem)*

This serves to show:

i) whether students see the need to deal with each body separately and apply the law to each.

ii) whether they are able to see the importance of finding the resultant force on each body

iii) whether they are able to calculate the resultant force either on each body separately or on the system as a whole.

iv) whether they appreciate the need to use certain equations and have the ability to do so.

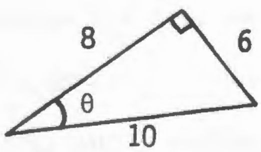
Questions 6 and 7

A person walks due North for 3 km and then due East for 4 km. How far will he be from his starting point?

Pythagoras's theorem is used extensively in determining resultant forces. These questions simply test for this mathematical ability.

Question 8

Please write down:



a) $\sin\theta =$

b) $\tan\theta =$

c) $\cos(90 - \theta) =$

Trigonometry is a fundamental skill required in dealing with the components of forces in the normal applications of Newton's laws and would be utilized in any planned procedure. (See 5.3.1, Section A, step 5)

Question 9

A force F acts at a point P on an inclined plane. Determine: component of F in x-direction (parallel to plane): component of F in y-direction (perpendicular to the plane):

The inclined plane is very frequently used in the first year course and students will already have dealt with it in their penultimate school year. This question tests:

i) their ability to apply trigonometric relations in a practical situation often encountered in problems.

ii) whether they see the need to assign the correct sign to the components.

These must often be executed as a part of any procedure in solving problems on equilibrium or accelerated motion.

Question 10

i) If the rope in the diagram is given a sudden downward jerk at F, does the rope break at point 1 or 2? Why?

ii) If the rope is gradually pulled downward at F, does the rope break at point 1 or 2? Why?

This is essentially a question which tests the students' understanding of the concept of inertia, also defined as part of Newton I. It is taken directly from one of the textbooks used by students in their penultimate school year. The idea of inertia is intrinsic to Newton's First Law. As is argued in the following chapter, an implicit algorithm is another intrinsic feature of this law. The first part of the question tests to what extent the concept of inertia has been made explicit for students. The second part of the question tests to what extent the steps in section 5.3.1 A are used by them.

Question 11

a) The mass M is at rest on the floor with the forces acting as shown. Write down the equations describing the situation.

b) In the sketch in 11 a) are there any other forces ON M?

c) If instead of being at rest, the body is accelerating along the *floor:*

i) Which of Newton's laws is applicable? Why?

ii) Write down the equation(s) describing the situation.

a) Both the object and the relevant forces acting are given. This question then serves as a test of the students' ability to write down the necessary equations. It will also serve to indicate whether students have any appreciation of the fact that a particular body has been isolated and that the forces (necessary for solving the problem) acting on that body, have been indicated. It also serves as a cross-check of questions 1b) and 2b).

Since equations are requested (and not just one) it tests whether the need is felt to draw in all the forces acting on the body (such as the normal force and the weight) and whether the resulting equation can be set up.

b) This tests the ability to apply Newton III to the stated problem.

c) The following points are being examined in this question:

i) whether the student is able to relate Newton II to the idea of an accelerating body.

ii) the crucial idea here is to see whether the student realizes that in applying Newton II the resultant force is necessary rather than the simple F = m a application, where F happens to be a force acting. This tests whether there is any correlation with question 2 b).

In asking for "equation", the idea of orthogonal directions, one parallel to the motion, the other perpendicular to it, is implied. This therefore examines whether any idea of axes occurs to the students.

<u>Question 12:</u>

A body of mass M, is suspended from the ceiling by a rope. Given the following bodies, state whether a force is exerted on mass M by each, the reason for the force and its direction (up or down):

i) Rope ii) Ceiling iii) Earth

State whether mass M causes any forces to be exerted ON the following bodies, give the reason for the force and the direction (up or down) : i) Rope ii) Ceiling iii) Earth

The first section of the question determines if the student can formulate the "action" in Newton III in terms of the effect of the different bodies on the mass, M.

The second tests whether the "reaction" can be formulated in terms of the "action" established in the above.

Implicit in the two sections is also the recognition that two bodies must be identified in the application of Newton III. It is possible from the responses to determine whether or not there is a recognition of these three features in students' application of Newton III.

<u>Question 13</u>

Two masses M_1 and M_2 are completely isolated in the universe (sufficiently distant so that no outside masses influence them), but they are close enough together to exert a gravitational force on each other. If $M_1 = 2M_2$, and the force exerted by M_2 on M_1 is 20N as in the sketch, show the size and direction of the force *exerted by* M_1 *on* M_2.

This is a slight variation of one problem of a test set to determine misconceptions among students and teachers with regard to understanding basic concepts in physics (Helm, 1980). This question is discussed in Chapter 3, section 3.3.2. The point made there is that it is unlikely that among disadvantaged students many would correctly answer the problem because of the number of cognitive processes required to arrive at the correct solution. The question has been included to test this premise.

In analysing student responses to all these questions, the emphasis is not on a pure statistical analysis of numbers of correct answers, with the possible exception of the last question. Rather, the nature of student responses as these relate to the overall claim of the test, is of paramount importance. However, since the test also serves to give some idea of the relative abilities of test and control groups (see below), it is useful to show how these compare on the various questions.

5.3.3 Analysis of the Results

The number of answer sheets completed in English was 42 while the number in Afrikaans was 44. These form the test and control group respectively. For convenience, where necessary, the Afrikaans statistic will be given in brackets after the English with an "A" to designate it as such. Where comments have been written down from the Afrikaans answer sheets these are translated into English. The data are given in this form: the number in the group; % of the group which that number represents.

Question 1 a):

22; 52,4% (31; 70,5%A) gave the correct formulation.

Question 1 b):

Nobody in either section gave the correct or nearly correct equations, although many wrote "F = m a" without specifying anything.

Question 2 a):

11; 26,2% (10; 22,7%A) gave the correct formulation.

Question 2 b):

32; 76,2% (36; 81,8%A) had the equation written down under one of the three laws.

Question 3:

26; 61,9% (31; 70,5%A) wrote the law down correctly.

The results of 1 a), 2 a) and 3 are to be expected, that is, that greater numbers would remember the 3rd law. Since the 2nd law is the most difficult to formulate, it would have the least number who would remember it correctly.

The fact that none could formulate Newton's first law in symbols, is revealing. It seems that students are simply not used to the fact that an equation is applicable. This does not of course, mean that they cannot set up an equation which has the resultant force equal to zero in solving an actual problem. But it is meaningful to contrast this number with the 76% (82% A) who knew the "formula" for Newton's 2nd law. Thus while the 2nd law is learnt with the related equation as a necessity, there is no such situation with Newton I. It is doubtful whether students would be able to solve an equilibrium problem with any sort of structured approach if they do not realize that there is an applicable equation. The fact that there is only a statement of the law could be indicative of a rote-learned approach. It should be possible to check the above conclusion with question 4 d).

While a high percentage of the students are acquainted with the equation "$\vec{F} = m\vec{a}$", (the vectors were not taken into consideration in arriving at the number of correct responses), it needs to be determined whether their application of the equation would be very meaningful. An indication of this would be their ability to define the symbols, m, a and especially F, correctly.

28 of 32 (32 of 36 A) simply define "F" as "Force". 3 English-speaking students specified that it is the "resultant force" while one said that both F and a are vectors. In the Afrikaans section, 2 wrote that it is the "force acting on the body", 1 called it an "unbalanced force," while 1 called it an "applied force". Whether students are simply being vague in their definitions and non-specific can be tested by their responses to question 11 c).

Question 4:

The following were the choices exercised and the numbers who selected these:

	No response	1st law	2nd law	3rd law	2nd & 3rd
English	5	10	10	16	1
Afrikaans	9	11	10	14	-

Reasons Given:

1st Law: 4 of 10 (1 of 11, A) gave no reason for their choice.

The following are typical of the responses:

i) "This law discusses balanced and unbalanced forces"

ii) "It concerns maintenance of equilibrial forces to continue balanced states of motion"

iii) "Equilibrium is constant with many forces involved"

iv) "Opposite forces should be equal for equilibrium"

v) "For something to be in equilibrium there should be no force acting on it, or all the forces acting on it should be in equilibrium"

vi) "Because inertia is a state of equilibrium"

2nd Law: 5 of 10 (5 of 10, A) give no reason for their choice.

The following responses are typical:

i) "F = m a, W = m g, These quantities are used"

ii) "Because you must know which forces are involved"

iii) "Dealing with M and Force i.e. you can find acceleration"

iv) "The masses will have acceleration acting on them"

v) "The equilibrium of an object depends on the different forces acting on it"

vi) "because an object at rest is in equilibrium".

3rd Law: 5 of 16 English (4 of 14 A) gave no reason while some merely restate the law. The following are other responses:

i) "The force on one side of the fulcrum can be equated to the force (in the respective direction) on the side"

ii) "As there is an equal but opposite force of reaction for every action, thus giving equilibrium"

iii) "The forces in equilibrium are the same"

iv) "Because a state of equilibrium should be reached having an action and a reaction"

v) "A body can only maintain equilibrium if two equal and opposite forces act on it"

vi) "Because the force with and against gravity plays an important role in solving equilibrium problems"

vii) "Exertion of forces"

viii) " $\because$ the resultant force is equal to the forces of the three different points"

19 of 42 (45%) students in the English section while 19 of 44 (43%) students in the Afrikaans section either gave no answer or no reason with their answer.

Of the students who chose the 1st law, it appears that those like i), ii) and iv) have some idea of equilibrium, iii) is vague, the reasoning of v) is circular, although perhaps just badly expressed, while vi) is not easily understood.

The respondents who chose the 2nd law either concentrate their comments on forces (i, ii and v) or do not see that an accelerated object is not in equilibrium(iii, iv).

Three of the Newton III respondents (ii, iv, v) seem to feel that action and reaction forces act on the same body. It should be noted that all the above responses are typical of both the English and Afrikaans sections.

It seems clear from analysing the responses that the students are very haphazard in dealing with "equilibrium". Apart from the fact that there seems to be little indication that there is any structure in their approach, they show no clear idea of what the laws are or mean, they have only a vague idea of various concepts such as "uniform motion", "acceleration" and indeed "force". It is difficult to see how many students are in a position to deduce suitable algorithms for applying the various laws when they show not only confusion of concepts, but also a very vague realization of where the laws are applicable.

Question 5 b):

This is considered before question 5 a) since it is simply a test of consistency between it and question 4.

	No Response	1st law	2nd law	3rd law	2nd & 3rd
Question 4:	5 (9A)	10 (11A)	10(10A)	16(14A)	1
No who chose same in 5 b):	2 (2A)	5 (5A)	7(9A)	5(3A)	1

Thus 20; 47,6% (19; 43,2% A) were consistent in their choice of law to apply in dealing with an equilibrium problem. 14; 33% (10; 22,7% A) chose Newton I as applicable.

Question 5 a):

In analysing the responses to this question, the following seven categories are distinguishable:

a) No response: 1 student both English and Afrikaans

b) Only the forces are mentioned without any reference to masses or "system": 6 English, 12 Afrikaans. Examples of the responses are:

   i) "When the opposing forces are balanced i.e.equal"

   ii) "The resultant forces are equal"

   iii) "means that it is equal or balanced"

   iv) "The magnitude of the two forces has "cancelled" out movement in any direction (closed system)"

   v) "The forces which act on each other are equal"

c) The answer simply says: "no movement" . : 1 student English.

d) The answer alludes to the two masses and the fact that forces influence them: 10 (9 A). Examples are:

   i) "There is no resultant external unbalanced force acting on these bodies"

   ii) "The different forces on each mass piece are equal and opposite and are balanced"

   iii) "The two forces cancel each other and there is no physical reaction between the objects"

   iv) "The forces acting upon the bodies are the same causing no movement"

e) Only the movement of the bodies is mentioned: 12 (44 A) students. Examples are:

   i) "The two masses are at rest"

ii) "Both the 10 kg and 5 kg masses are balanced - neither of them is moving"

iii) "The 5 kg mass is not pulling the 10 kg mass down any further since it has reached equilibrium"

iv) "Object is stationary"

f) Reference is made to forces which affect "the system": 10 (14 A). Examples are:

i) "When all the forces that exert a pulling effect are balanced, the system is said to be in equilibrium"

ii) "There is no movement, thus the forces being completely balanced giving equilibrium. The system is constant."

iii) "There are no external forces exerted on the stable system"

iv) "The forces acting on and in the system are balanced"

v) "The resultant of the system is zero"

g) Only the unchanged position of "the system" is mentioned: 2 (4 A)

22; 52,4% (13; 29,5% A) of the respondents isolate the bodies in their answers. It is noteworthy that no response includes the rope as part of the system. In determining how meaningful the isolation of the bodies is in the minds of students, it is worthwhile to analyse more carefully the quality of the responses which refer to forces. 26; 61,9% (35; 79,5% A) explicitly mentioned forces in answering the question. Of these, 11: 26,2% (17; 38,6% A) make reference to the forces acting on the bodies or "the system". 15; 35,7% (18; 40,9% A) simply talk of "resultant force", "two forces", "opposing forces" etc. An analysis of question 5 c) should reveal whether "the system" is clearly identified as a necessary feature, however.

It seems clear from the analysis that students appreciate the role which forces play in establishing an equilibrium condition. However, only 5; 11,9% (5; 11,4% A) relate the forces to specific bodies that is, clearly identify the system involved.

The overall impressions gained in reading the responses to this question are:

1. Students display no clear understanding that forces are applied to specific bodies in an equilibrium situation.
2. Their approach to applying the laws is relatively undefined and vague.

Question 5 c):

Number who indicate:

	No forces	1 force	2 forces	3 forces	4 forces	5 forces	6 forces
Eng	2	0	13	13	9	5	0
Afr	2	2	24	5	5	5	1

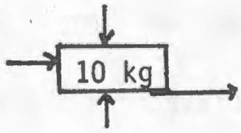
1 force: The forces specified are "gravitational" and downward".

2 forces: "force of gravity" : 9 (13 A) No. who specified the direction: 4 (6 A)

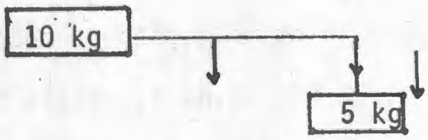
"friction": 8(16 A) No. who specified the direction: 4 (1 A)

3 (1 A) correctly identified the weight of the 5 kg mass and the friction on the 10 kg mass as the forces necessary to solve the problem. Generally the direction of the forces are very haphazardly drawn, in-

dicating very little certainty of the system. Thus, for example: the frictional forces are indicated by:



gravitational forces:



No single response in either section gave the tension correctly.

<u>3 forces</u>:

Frictional forces: No. who stated: 11 (4 A) Correct direction: 6 (2 A). Those who did not give the correct direction, either did not indicate or gave directions in the rope, to the right on the 10 kg mass or up or down on the 10 kg mass.

Weight: No. who stated: 12 (4 A) Correct direction: 6 (2 A) 5 in the two sections combined indicated the weight as acting downward in the rope or outside the body. 2 in each section gave no direction. Tension: No. who stated: 6 (3 A) Correct direction: 1 (0 A).

Most gave no direction for the tension.

Of the 13 (5 A) responses which indicate three forces, no single response correctly identified the three forces above correctly and specified the directions.

<u>4 forces</u>:

Frictional forces stated: 6 (4 A) Correct direction: 3 (2 A)

Weight: Stated: 9 (5 A) Correct direction on the 5 kg : 4 (4 A)

Tension: Stated: 4 (3 A) Correct direction: 2 (1 A)

Only one person in each section drew in the three forces with correct directions. It is noteworthy that here some draw in the forces with no apparent appreciation of what the system is, that is, forces were drawn as acting on the floor, table etc.

5 forces:

Friction:	Stated:	3 (5 A)	Correct direction: 2 (3 A)
Weight:	Stated:	5 (5 A)	Correct direction on 5 kg: 4 (4 A)
Tension:	Stated:	4 (3 A)	Correct direction: 2 (1 A)

Here also additional forces make their appearance. Thus, for example:

"F . force exerted on floor by table"

3 respondents indicated forces exerted on the floor or on the table. No single response in English and 2 in Afrikaans correctly indicated all the forces necessary.

In summary:

i) Only 4 respondents in each section were able to indicate the necessary forces and draw the correct directions. Clearly the ability to execute the step: "Draw in all the forces acting on the system", is absent.

ii) Responses indicate that many do not clearly identify "the system". This is also seen by the manner in which forces are drawn in on the various bodies in the problem. Not only directions, but also points of application are confused.

iii) Responses show that Newton I and Newton III are frequently confused.

The responses to questions 5 a, b, c clearly indicate that the students generally have no understood-structure for dealing with Newton's first law as required in an equilibrium problem. This should become even clearer from an analysis of question 5 d).

Question 5 d):

The following categories of answers were discernable:

CATEGORY	AFR	ENG
No attempt	12	10
Correct answer	7	7
Acceleration is calculated	8	3
Weights of two bodies added or subtracted to furnish an answer	3	11
The weight of 10 kg mass given as answer	7	5
Miscellaneous; undefined method	7	6
N =	44	42

It is clear from the small numbers in each section who manage to solve the problem correctly, that the students have no clear idea of how to apply the relevant concepts or indeed that they have any procedure to do so. In spite of the fact that the sequence of steps in question 5 leads to the answer in an ordered, planned manner, less than 17% of students in each section gave the correct answer. It seems clear that not only do they lack any planned approach, but they are also not able to execute the necessary steps even when these are made explicit. Of the 14 correct responses, 6 applied the limited strategy of simply looking at forces which balance at opposite ends of the system. Only one person in each section wrote out the equations for both bodies. This limited strategy will, of course, give difficulty in any problem which is slightly more complex.

From the above responses it is also obvious (confirming what teachers know, of course!) that students confuse concepts. The idea of calculating acceleration seems to arise from a straight application of F = m a. Students seem also to use intuitive ideas like subtracting weights of bodies to determine how movement occurs. Or, they focus only on one section of the system, as in calculating the weight of the 10 kg mass. If the answers are related to the directions of the forces in question 5 c), then it is easy to see how inability with one step in a procedure, affects the other.

QUESTION	No. Correct; Eng	No. Correct; Afr
6	41 (97,6%)	39 (88,6%)
7 a)	38 (90,5%)	44 (100%)
7 b)	21 (50%)	25 (56,8%)
8 a)	37 (88,1%)	37 (84,1%)
8 b)	37 (88,1%)	35 (79,5%)
8 c)	26 (61,9%)	32 (72,7%)
9 x-component	5 (11,9%)	7 (15,9%)
y-component	5 (11,9%)	5 (11,4%)
correct sign y-component	0	0
10 (i)	7 (16,7%)	5 (11,4%)
(ii)	14 (33,3%)	17 (38,6%)
11 a)(F - f = 0)	9 (21,4%)	5 (11,4%)
(N - m g = 0)	2 (4,8%)	0

In question 11 a), the number who simply wrote "F = m a" or "f = m a" are:

English: 19 (45,2%) Afrikaans: 23 (52,3%)

It is noteworthy from the above how similar the responses are between the two groups not only on the understanding of concepts, but also in mathematical abilities as seen in questions 7, 8 and 9.

Question 11 b):

Normal force: English: 9(21,4%) correct. Afrikaans: 8 (18,2%)

Weight : English: 21(50%) correct. Afrikaans: 24 (54,6%)

It is once more significant that almost half the students in each section do not correctly identify the weight as acting, while about 80% apparently have little idea of the nature or source of the reaction.

Question 11 c) i)

Choices:	No Response	1st law	2nd law	3rd law
English	6	5	28	3
Afrikaans	3	9	31	1

Question 11 c) ii)

English: 4 Afrikaans: 0, correct responses of equation F - f = m a. One person also gave the equation N = m g. Virtually all who responded, wrote as the equation F = m a.

Question 12:

The answers in this section were assessed to be correct if both the action and reaction forces were given accurately.

1. Action-reaction pairs between rope and mass, M:

Correct responses: English 12 (28,6%) Afrikaans 12 (27,3%)

It must be stated that a number of the answers were only marginally correct.

Typical of incorrect pairs of responses are the following:

	REASON FOR FORCE	DIRECTION
i)	"hanging weight	up
	The hanging body	down"
ii)	"The weight of the mass	up
	The weight of the body	up"
iii)	"friction between the two prevents it from dropping/	upwards
	it tends to pull it	downwards"
iv)	"The rope tends to pull m upward	upward
	Upward force of tension in string	upward"

Many of the responses are simply nebulous, and taken together, meaningless. Examples are i) and ii). Others are ridiculous, as is the first reason in iii). (the idea of friction appeared in a number of responses). Others, such as iv), do not clearly distinguish that the action and reaction forces act on different bodies.

2. Action-reaction pairs between ceiling and mass, M

The reason for giving question 12 (2) was to determine if students could distinguish between forces exerted between bodies themselves (in this case very weak gravitational forces) as opposed to those transmitted through another agency (in this case, the rope). This would determine whether there is any discernment of the two bodies involved in an action-reaction pair. It was not expected that students would identify the gravitational force. Hence the accepted answer would be "No".

No. who answered no to both parts: English 7 (16,7%); Afrikaans 5 (11,4%). As a check on whether they give the same answer for both sections that is, see that an action and reaction must exist simultaneously:

Number who changed answer to the question "FORCE (Yes/no)" that is, "yes" once and then "no" (or vice-versa) for a given action-reaction pair:

English: 18 (42,9%) Afrikaans: 17 (38,6%)

Typical responses are the following:

	REASON FOR FORCE	DIRECTION
i)	"the rope exerts a force on the ceiling	up
	the mass through the rope pulls at the ceiling	down"
ii)	"no force	/
	the pull of the rope on the ceiling	down"

3. Action-reaction pairs between mass, M and earth

Number of correct responses: English 5 (11,9%) Afrikaans 5 (11,4%). The number who felt that the earth exerted a force on mass, M, but that there was no reaction force (or vice-versa) was significant:

English 22 (52,4%) Afrikaans 25 (56,8%)

Some sample responses are the following:

REASON FOR FORCE	DIRECTION
i) "gravity on M	down
Only gravity is acting on M and not vice-versa	non"
ii) "Two bodies attract one another ($G = \frac{m_1 m_2}{r^2}$)	down
The same reason as above, but the force here is very much greater because the earth has a very large mass	up"
iii) "Force of gravity	downward
Gravitational force acting on M	up"
iv) "Gravity pulls the force down	downwards
M pulls back on gravity more than 10 m s^{-2}	down"

In analysing the responses, it is apparent that students have no clear idea:

a) that two bodies are involved in applying Newton III,

b) of what constitutes action-reaction forces.

Thus clearly, no procedure exists for applying Newton III with most students. Since this is so, it is apparent that very few would be able to answer question 13.

<u>Question 13</u>:

Correct responses: English 5 (11,9%) Afrikaans 3 (6,8%)

5.4 Discussion and Conclusions

On the basis of the data and discussion presented above, it is reasonable to make the following conclusions:

1. The students have a very confused grasp of Newton's laws. Additionally, no clear idea seems to exist of which law to apply to which situation.

   While the relatively low numbers who could even formulate Newton's laws correctly are interesting (and surprising, given the rote-learning approach to science of many high school pupils), it is significant that the only law which evoked an equation was the second as seen from the responses to question 3. It is clear that the first law is not seen in any problem-solving context. This is supported by the responses to question 4, where only about 25% in each group indicated that they regarded the first law as applying to an equilibrium problem. Even among this group, however, few could offer a sound reason for their choice and only approximately a half were consistent in choosing Newton's first law for use in an equilibrium situation in question 5b.

2. There appears to be no procedure either implicit or explicit which is used by students when applying the laws nor could individual steps in a procedure be executed.

   It was apparent from the response to question 5, where less than 20% of the students in each section solved the problem correctly, that school-instruction had not developed in students either the ability to isolate essential features of Newton's first law, or, indeed, to sequence these features in any meaningful way for solving problems. It is clear that they are also incapable of executing individual steps in a procedure, such as, for example, drawing in all the forces on a body. This is evidenced by the fact that only four students in each section were able to indicate the necessary forces and draw the correct directions. This is corroborated by question 11b where only some 50% of students in each section were even able to identify the weight as a force which acts on the body.

The same situation applies with respect to the use by students of suitable equations. In question 11a only 21,4% English and 11,4% Afrikaans students wrote the correct equation, $F - f = 0$ for Newton's first law, while only four English and no Afrikaans students managed the correct equation for Newton's second law in question 11c.

The answers to question 11 indicate that with Newton's third law as well, no procedure for its use exists with most students. Because of the large amount of data in problem 13, very few students - five English and three Afrikaans - managed the correct answer (as expected).

3. It is noteworthy that the students in the two language groups were strikingly similar in their approach.

It seems apparent that during their secondary school science instruction no planning procedure for using Newton's laws has developed in the students in spite of their formal acquaintance with and knowledge of the laws. Additionally, it appears that the language difference is not a particularly significant one.

CHAPTER 6

COMPENSATORY PROGRAMMES TO ADDRESS DEFICIENCIES AT INPUT

6.1 Introduction

It is now necessary to demonstrate how algorithms can be utilized to develop, or compensate for, deficient cognitive functions as students deal with the first year Mechanics course at the UWC. It must be clearly realized that no claim is being made that the algorithms developed are unique, for some textbooks and especially books on problem-solving, do contain similar algorithms. However, what has been attempted here is to make the development of the relevant algorithms an integral part of the presentation of the concepts. This approach has flowed from the realization that an algorithm is intrinsic to a particular concept in a problem-solving situation as much as are other ideas regarded as implicit in a more theoretical way. Thus, for example, the ideas of inertia and frames of reference are essential elements of Newton's first law which are usually highlighted in a discussion of the law. However, these ideas are of little direct value in solving the normal textbook type problems such as a ladder against a wall or a body on an inclined plane. And yet it is in the latter situation that students are usually tested as to their understanding of the law. Thus an appreciation of the algorithm would obviously prove of more value in this instance than would ideas like inertia.

It must however be stressed that in this research algorithms were not given to students as recipes to churn out answers to problems.

Nine booklets were developed in an effort to compensate for deficient cognitive functions. These are given in Appendix B. The form of the booklets was followed in the formal lectures. The development of the algorithms formed an essential part of the teaching process. The point of view adopted in this thesis with regard to algorithms needs to be placed in perspective. No claims are being made that the explicit development of algorithms necessarily makes concept understanding complete, nor indeed that perfect problem-solving ability is thereby attained. Other workers (see for example, McDermott, et al, 1980) have demonstrated clearly that much hands-on experience is also necessary to enable disadvantaged students to come to grips with fundamental concepts such as density, velocity etc. The relevance and limitations of algorithms to problem-solving have already been discussed in 3.4. It is, however, claimed that while algorithms are not sufficient, they are certainly necessary in understanding concepts properly, especially if the concepts are required for the solution of problems.

In Feuerstein's analysis of deficient cognitive functions (see for example, Feuerstein, et al, 1980) those on the input phase are regarded as more difficult to address than the elaborational deficiencies. It seems apparent from a consideration of the deficient functions (in Chapter 3) that while it is relatively simple to deal with a single elaborational difficulty like a lack of planning as an entity, this is not possible with deficiencies on input. Thus while the test described in Chapter 5 could be used to isolate planning as being something which students lack, the interviews discussed in Chapter 4 generally present a multiplicity of cognitive functions as being lacking. As has been discussed previously, the kinematics section of the course normally contains problems rich in data. This is especially so in the case of relative velocity and projectile motion. Booklets were developed for each of these. While the topic of circular motion is usually treated for instructional purposes as an extension of Newton's second law, and as such falls in the elaborational phase (see Chapter 7), because of the need to visualize a three-dimensional situation and relate the data relevant to this usually on a two-dimensional drawing, it too has unique data-collecting features and for this reason was placed under the input phase. A detailed booklet is given for this section. The booklet on Work and Energy makes use of the fact that the problems in this area, while containing much detail, can usually be solved by applying one equation if the expressions for work, potential energy and kinetic energy are known and can be carefully evaluated in the light of the given problem.

An attempt has been made in the booklets to address the identified deficiencies on input highlighted in Chapter 4. Some of the ways in which this was done are as follows:

a) Unplanned, impulsive and unsystematic exploratory behaviour:

   To systematize the manner in which the available data are scanned as well as to make it more goal-oriented, that is, to enable an approach to a solution of the problem to be found, the data collection is algorithmized in all four booklets. As will be seen in more detail in the next chapter, this automatically produces a planned approach and prevents the impulsivity associated with incoherent attempts at the solution of problems.

b) <u>Blurred and sweeping perception:</u>

In an attempt to prevent this as students read through problems, the initial discussion in each booklet to establish the equation used or the principles involved, has focused on implications which might easily be missed. For example,in dealing with relative velocity, it is not necessarily apparent which velocities are at issue if the problem specifies a particular direction of motion or involves an interception or point of closest approach. This potential problem is made explicit. Additionally, in the booklet on circular motion, the student is encouraged to visualize the problem and to this end help is given by sketches etc.

c) <u>Lack of precision and accuracy:</u>

To prevent data from being overlooked, the algorithms play a very useful role. In addition to this, the booklets contain specific mechanisms to oblige students to use the data correctly. For example, in the booklet on projectile motion, the 'point in question' in the problem is highlighted as being of fundamental importance when relating the data accurately to the problem situation. Also, the choice of direction for the axes is made a particular element of decision.

d) <u>Impaired use of two or more sources of information:</u>

This has proved to be a particularly difficult function to address. The following examples of attempts to do so are given:

i) In discussing projectile motion, students frequently find it difficult to relate motion in x- and y-directions simultaneously, to the composite motion of the body. The theoretical discussion in the booklet attempts to do this and to develop understanding of the issue.

ii) In dealing with relative velocity, difficulties in visualizing the nature of the relative motion are experienced by students because they do not relate the movement of the two bodies but rather they focus on the actual movement of each body. This is discussed in the first section of the booklet.

The above discussion does not represent an exhaustive list and other factors will become clear as the individual booklets are discussed.

It is necessary to note that the format employed in booklets 6 - 9 differs substantially from that in booklets 1 - 5. The design employed in nos. 6-9 is the following:

i) Theoretical discussion: this is done to establish principles, derive equations or to address the type of problems enumerated in a - d above. It is done in a manner usually not encountered in textbooks, since the accent rests on the ability to use the concepts discussed to solve problems.

ii) The algorithm is established by solving actual problems. Here too, two methods are used:

a) Each step is demonstrated across a spectrum of problems.

b) Each problem is completely solved by stepwise use of the stated algorithm.

While method (a) is favoured for establishing the intrinsic nature of the algorithm, the coherence of the presentation can sometimes be lost.

Each of the four booklets will now be discussed and then analysed. The complete booklets are given in Appendix B. The results with the test and control groups will then be discussed.

6.2 Relative Velocity

This is a part of the course with which students generally, and disadvantaged students in particular, have great difficulty. It is not unusual at the UWC to have an entire class fail to solve a problem of this type. The booklet attempts to remedy the situation. The approach outlined in the booklet was also that adopted in the lectures. The same is true of the other three booklets as well. (Appendix B67 - B79)

6.2.1 Discussion of Mech Bl 6 : Relative Velocity

The relevant vector equation for dealing with the velocity of a body A relative to a body B, is:

$$\vec{V}_{AE} = \vec{V}_{AB} + \vec{V}_{BE} \quad \text{..........} \quad (6.1)$$

where E represents the reference body, usually the earth, and

$\vec{V}_{AE}$: velocity of body A relative to the earth,

$\vec{V}_{AB}$: velocity of body A relative to body B,

$\vec{V}_{BE}$: velocity of body B relative to the earth.

The equation is established by discussing the intuitive ideas which students have of the relative velocities of two objects moving in the same or opposite directions, where both move relative to the stationary earth. While students will easily give answers of 0 and 120 km h^{-1} for the relative velocity of two cars each with speeds of 60 km h^{-1} (moving in the same or opposite directions respectively), they are usually unable to say what the velocity of the one relative to the other is when the cars move at right angles to each other. By showing that in the case where the cars move parallel to each other, one vehicle is "stopped" by a suitable back velocity which must then be added vectorially to the velocity of the other object as well, the vector equation with subscripts suitably ordered, is established for the case of velocities in any direction.

The following algorithm is developed by a stepwise solution of four problems:

1. Determine the three bodies involved, A,B,E and establish which is the reference body, E,
2. Establish which of the magnitudes $|\vec{V}_{AE}|$, $|\vec{V}_{BE}|$ and/or $|\vec{V}_{AB}|$, are known.
3. Establish which of the directions $\vec{V}_{AB}$, $\vec{V}_{AE}$, and/or $\vec{V}_{BE}$, are known.

Remember

a) The actual direction in which a body travels is given by the direction of its velocity relative to the earth.
b) For an interception of two bodies A & B, the relative velocity $\vec{V}_{AB}$ must be along the line joining them.

4. A vector diagram of the velocities in accordance with,

   $\vec{V}_{AE} = \vec{V}_{AB} + \vec{V}_{BE}$ is required.

5. Determine the unknowns.

Before considering the application of each of the above steps to the four problems, an analysis is given of the fact that the actual direction in which a body travels is given by its velocity relative to the reference body as well as demonstrating that the direction of the relative velocity will be along a line joining the two bodies in the case of an interception. These are incorporated as (a) and (b) of point 3 of the algorithm.

Steps 2 and 3 of the algorithm are executed together and students are encouraged to construct the following matrix:

	MAGNITUDE	DIRECTION
$\vec{V}_{AE}$:		
$\vec{V}_{BE}$:		
$\vec{V}_{AB}$:		

6.2.2 Analysis of the Approach

The following points are noteworthy:

i) The necessary equation (6.1) is derived so that important features of the intrinsic algorithms are made explicit. Thus, for example, the fact that three bodies, one of them a reference body, are involved, is made clear. Additionally, the vector nature of the equation is emphasized, and the ordering of the subscripts is also shown.
In this way the need for proper planning (discussed in 4.7 No. 6, p. 125) is addressed.

ii) For data to be meaningful more information is frequently necessary than is given in the usual textbook approach. This is well illustrated by the significance of the directions of both actual velocity (relative to the earth or some suitable reference body) and relative velocity. It is both unnecessary and unrealistic to expect students to come to these insights first in the course of solving an actual problem. Indeed, many of them never do! By making this explicit the "blurred and sweeping perception" students may have is countered.

iii) The approach adopted with the problems is to make the intrinsic input algorithm plausible as well as to supply the necessary practice in applying it.

iv) The reason for placing the data in a matrix-arrangement, is for neatness and clarity of detail. In this way students are taught the importance of ordering data carefully. This also enables the data in each problem to be examined more systematically and works against the impulsivity of data analysis associated particularly with many disadvantaged students.

v) The point is emphasized that since we are dealing with a vector equation there can only be two unknowns among the six variables. Students are thus helped to search for the four knowns, be these implicit or explicit. This also addresses the problem of "unplanned, impulsive and unsystematic exploratory behaviour".

vi) The importance of not taking a given quantity simply at face-value, is emphasized in Steps 2 and 3, Problem 4b (p. B75) in dealing with the direction of the raindrops. Students are taught to be careful in their evaluation of data thus counteracting a "blurred and sweeping perception".

6.3 Projectile Motion

The aim of the booklet is to assist the student in applying the cognitive processes involved in solving problems to the case of uniform motion under gravitational acceleration. There are a number of factors involved in a proper understanding of the motion of an object in two-dimensions under the influence of gravity;

a) the dynamics of the situation, namely that only the gravitational force is acting, needs to be understood as implying that acceleration takes place only in a vertical direction with constant velocity horizontally;

b) the manner in which any motion is split into its components and vice-versa, requires a particular insight.

The booklet does not supply all this and no claim of sufficiency of the approach is made, rather that it is necessary. Since the focus of the booklet is the proper analysis of data in problems on projectile motion, sufficiency should not be expected. The booklet is found in Appendix B, pages B81 - B95.

6.3.1 Discussion Of Mech B1 7 : Projectile Motion

The booklet is introduced with a discussion of the manner in which a velocity can be viewed in terms of its components. This concept is then applied to parabolic motion with special emphasis on the following points:

i) The same time, t, applies to corresponding components of the motion.

ii) The difference in nature of the motion in the x- and y-directions.

iii) The importance of choosing a positive direction for the vertical axis.

A stepwise development of the following algorithm is then made using four typical examples.

1. Choose suitable axes x (horizontal) and y (vertical) and the origin at the point of projection of the object.
2. Determine the x- and y-components of the initial velocity, V_o.
3. Return to the problem and find and write down the coordinates of the point or points in question.
4. Analyse the vertical motion taking place under constant acceleration, $\pm$ g:
   a) Fill in from the problem the values of the 5 variables:

      a = $\pm$ g (sign depends on direction of y-axis

      + y upward : -g

      + y downward : +g)

      s = y (y - coordinate of point in question, STEP 3)

      t = ? (same as for horizontal coordinate, x)

      $V_o = V_{oy}$ (Step 2)

      $V = V_y$ (at highest point, $V_y = 0$)

   b) Use the appropriate kinematic equation to determine the unknown(s).
5. Use the horizontal motion

   $x = V_{ox}\ t$, where necessary.

Each problem is considered carefully in terms of the requirement of each step of the algorithm.

6.3.2 Analysis Of the Approach

The interviews in chapter 4 highlighted two features which were also experienced in using this booklet:

a) The difficulties which many students have with the choice of axes and the signs of the kinematic variables. This is addressed in this booklet by making the choice of axis, with origin, a definite point of decision as step 1 of the algorithm. Point 4(a) also indicates the necessity - and importance - of the correct choice of signs for the acceleration.

b) The fact that students do not always interpret words correctly within the context of the problems. An example of this is the word "lands" in problem 2. This is discussed with students on p.B85.

The following are the cognitive functions required for dealing with problems on projectile motion :

i) The use of two sources of information : The initial part of the booklet is devoted to addressing the fundamental difficulty with this type of problem : seeing a composite motion in terms of its components. It is apparent that students have difficulty in going from the components of the motion to the composite motion and vice-versa. Related to this is the fact that the common time, t, for the x- and y-motion is only grasped with difficulty.

ii) The need for precision and accuracy in data analysis: While this is obviously necessary with any problem, the "blurred and sweeping perception" of disadvantaged students and their impulsivity in data analysis often prevent them from proceeding carefully with an indepth analysis of the problem statement. Some ways in which the booklet is designed to facilitate accuracy in data analysis are the following:

a) A careful analysis of discrete features of the problem is forced on the student by the algorithm.

b) A decision with regard to each kinematic variable is necessitated by step 4.

c) The discussion of each step with each problem enables the student to see the importance of implicit data. For example, the initial velocity of the object in problem 3 is discussed (see pp. B86 and B87).

d) The emphasis on the "point in question" in Step 3 is important for the purpose of correctly identifying the data in the problem. This also facilitates greater goal orientation in analysing the problem.

iii) The most obvious attribute of the booklet is that it presents a coherent manner in which the data in the problem can be organized.

6.4 Circular Motion

The discussion embarked upon in the booklet is more ambitious than that normally attempted in a South African first year physics course. However, experience has shown that limiting the discussion of circular motion to the case where the speed around the circle is constant inhibits the discussion because it does not allow the interesting details of the acceleration due to a change in speed (tangential acceleration) to be contrasted with that due to a change in direction (radial acceleration). Indeed, the fact of a third direction perpendicular to the plane of the circle of rotation and hence constituting a direction in which there is equilibrium, normally remains completely hidden from the students. The student approaching the problem for the first time fails to appreciate the textbook idealization and explanations unless these facts are faced from the outset. While the tangential acceleration is not stressed in the booklet, its discussion allows the radial acceleration to stand out in greater relief. The booklet is found in Appendix B, pp. B97 - B111.

6.4.1 Discussion of Mech Bl 8 : Circular Motion

The initial discussion in the booklet is designed to highlight the essential features of circular motion and to contrast clearly the origins of the tangential acceleration as opposed to the radial acceleration. At the same time the question of constant angular velocity associated with different linear velocities (particles at different radii from the centre of rotation) is discussed. That acceleration can arise not only from a change in the magnitude of the velocity but also from a change in its direction, is carefully illustrated. The radial and tangential directions are then easily seen to define the plane of the circle, with a third direction perpendicular to the plane of the circle being one in which the object is in equilibrium. It is shown that while at each point on the circle, a resultant force is not necessary in the tangential direction, for circular motion a resultant centripetal force <u>must</u> be present. The resultant force perpendicular to the plane of the circle is shown to be zero.

The following algorithm is then established and applied to the solution of four problems:

1. Visualize the geometry of the problem in three-dimensions.
2. Identify the plane of the circle of rotation and the centre of the circle.
3. Draw in all the forces acting on the body.
4. Draw in the three axes Ra, Ta and $\perp$.
5. Set up the equations:

$$F_{RAD} = m\,\omega^2 R = \frac{m\,v^2}{R}$$

$$F_{\perp} = 0$$

6. Set up the tangential equation if necessary.
7. Solve the problem.

A comparison of this algorithm with those developed in the following chapter for Newton's first and second laws, will show a large measure of similarity. However, since problems on circular

motion depend so crucially on a visualization of a three-dimensional situation to analyse the data in the problem effectively, it was placed under the input phase.
The distinction between the kinetic friction opposite to the direction of motion and the radial static friction which causes the circular motion, is very carefully drawn (see problem 3a, pp. B104 and B105). This is a matter on which textbooks are frequently silent, merely speaking of "friction" as being the force supplying the radial component.

6.4.2 Analysis of the Approach

The following points may be noted:

i) The first step in the algorithm requires students to visualize the problem. As has been demonstrated by the interviews in Chapter 4, the ability to visualize is a recurring difficulty among the majority of students. It is apparent that some sort of physical representation would be very useful at this stage. The discussion in the booklet on pp. B101 - B104 attempts to help by means of the sketches which are drawn as well as suggestions to simulate the exact physical situation. Step 2, the identification of the circle of motion and its centre, suffers from the same difficulties as those just mentioned. Indeed, Step 3, drawing in all the forces acting, can, if not done carefully enough, be a source of confusion if students have difficulty in associating correct angles with forces relative to the three perpendicular directions. It is apparent that to make the sketch with proper appreciation of directions, means that the problem visualization must be accurate. These considerations are associated with two deficient cognitive functions on the input phase of the mental act:

a) Blurred and sweeping perception. This is discussed in detail in section 4.7, p.118.

b) Impaired spatial orientation:

   Note the following from Feuerstein (1980:83,84) "Clinical observations of culturally deprived individuals have demonstrated again and again an impairment in their level of functioning in tasks requiring spatial orientation."

   While, in the severe cases, the retarded performer cannot distinguish right from left, in general terms disadvantaged persons may suffer from "impairments in representing, projecting, conceptualizing, structuring, and organizing space."

   Seen in this light then, it is not surprising that the students continue to have difficulty with circular motion with its inherent necessity to conceptualize three-dimensional events. The suggestion of a practical setting to address the problems of spatial visualization is supported by Feuerstein (1980:85):

" .... the spatial orientation may be dealt with by the culturally deprived in a more concrete operational manner ....."

It is apparent, then, that the booklet on its own will not address all the cognitive difficulties on input of the students in this particular section. It is interesting to note that similar problems have been experienced in studying students' difficulties with electrical circuits (Feher, 1983).

ii) The nature of the algorithm as well as the detailed discussion in the booklet of the various features associated with circular motion, train the student to be aware of certain data and to see the entire problem in terms of the three component features of the movement of the body. In this way the student is assisted in being accurate in his/her data analysis as well as focusing attention on the important features separately.

iii) The student is assisted in searching the problem for implicit data by the insistence on the need for the presence of a radial force even if it does not appear to be given (see p.B105).

6.5 Work and Energy

It may perhaps be strange that this is also classified under the input phase, but it should be clear below why this is so. Again there is a single equation which is applied and the problems can usually be solved by application of this single equation. The concept of conservation of mechanical energy is also apparent as a special case. The booklet is found on pp. B113 - B126.

6.5.1 Discussion of Mech B1 9 : Work and Energy

An effort is made in this booklet to relate carefully the concepts of work done by a force on a body and the energy gained or lost by the body. It is shown that an example of these two concepts arises when a body accelerates from an initial velocity, V_0, to a final velocity, v, while moving through a vertical height, h. The equation

$$(F\cos\theta - f)\, s = (E_k + E_p)_{FINAL} - (E_k + E_p)_{INITIAL}$$

where F = External resultant force acting at an angle to the displacement, s.

f = force of friction

E_k = kinetic energy

E_p = potential energy,

is derived.

The various terms are carefully explained (see p. B115). In cases which involve a weight, Mg, the reasons for placing the weight on the right hand side of the equation as contained in the expression for potential energy, and not with the external forces acting on the body, are discussed in detail (p. B114).

The following algorithm is developed by solving four problems:

1. Determine whether any external forces, other than the weight and the normal reaction, act on the body.

   Determine the total work done by these forces (p.B113).

2. Identify the first situation of the system and choose an appropriate reference level.

   a) Determine E_k (Use symbols if values are not given)

   b) Determine E_p

   c) $(E_k + E_p)_{\text{INITIAL}}$

3. Determine the final situation of the system.

   a) Determine E_k (Use symbols if values are not given)

   b) Determine E_p

   c) $(E_k + E_p)_{\text{FINAL}}$

4. $$(F\cos\theta - f)s = (E_k + E_p)_{\text{FINAL}} - (E_k + E_p)_{\text{INITIAL}}$$

5. Solve for unknowns.

This is the only booklet where each problem is solved completely before the following one is tackled.

6.5.2 Analysis of the Approach

The booklet attempts to overcome the cognitive difficulties of students in their analysis of data, as documented in this and previous chapters, in the following ways:

i) The algorithm provides a structure within which the data in the problem can be organized to meet the requirements of the work/energy equation.

ii) The two important points in the equation viz.the initial and final situation of the system, is made a particular point of decision in reading the problem formulation. This has the same effect as "the point in question" when dealing with projectile motion (see p.175 point d).

iii) In problems 2 to 4 an "analysis of the problem" is provided. This is the first attempt to assist students to make even a cursory qualitative evaluation of the problem statement before actually applying the algorithm. The introduction of this important procedure was delayed until it could be "seen" to be useful and naturally incorporated into the problem-solving routine being developed.

6.6 Procedures Adopted with Test and Control Groups

The research has been conducted throughout on the 1B students of the first year UWC physics group. These are students who need physics for dentistry, pharmacy etc. but who do not plan to continue further studies in physics. Each year they are divided into two groups on the basis of language preference and are thus either an English-speaking or an Afrikaans-speaking group. These formed the Test and Control groups respectively. Each group received the course notes in their own language. Virtually no student in either group purchased the course textbook, "University Physics" by Sears and Zemansky, which is available only in English. While it may be argued that different results can be expected *a priori* by virtue of the language difference, our experience at UWC over the years indicates the opposite. Thus while no claim is made that there are no differences, it has always been striking that the English and Afrikaans sections seldom differ by more than a percentage point or two on average in their results in class tests or, indeed, in the final examination. As will be shown below, the students' course averages in areas other than that subjected to the research remained very similar.

In addition, as seen in Chapter 5, where the results are given of the pretest to establish the presence of a planning ability in students when applying Newton's laws to problem-solving, there is remarkable similarity between the two groups. In all the results analyzed only those students who were doing the course for the first time were considered - i.e. those repeating the course were excluded. No special class tests were given to the groups. They were subjected to the usual periodic tests which had the same form as has been the practice over a number of years in the Physics Department at UWC. The performances of the test and the control groups on those questions which required the application of a particular concept, were compared.

While the statistics extracted provide general indicators, it is not felt that they need be the only barometers of the success or otherwise of the approaches developed. While the statistical results presented below demonstrate very clearly that the approaches adopted to compensate for cognitive deficiencies were successful, students responses to the approaches adopted are also important. These will be discussed.

Finally, a word needs to be said about the method of instruction and the instructors used. The test group was taught by the author using the approaches developed in this thesis. The booklets which are outlined and discussed in this chapter and the next were given to the class as ancillary material to the lecture presentation. Throughout the lectures the emphasis was laid on the development of algorithms wherever possible. No section of the course which required the solution of problems was taught without the algorithm implicit in the concept being developed and illustrated. Each group had 5 x 40 minute lectures and a 1 x 40 minute tutorial per week. The tutorial was the usual problem-solving session where the students attempted to do the problems themselves. Help could be called upon from the demonstrators available. Because the teaching strategy with the test group took longer than usual, some of their tutorial periods were used for instruction. This meant that the test group had fewer opportunities to solve problems than did the control group.

The control group was taught by a person who has had forty years experience in teaching first year physics. He taught only the first year course and had indeed been asked to help at UWC because of his ability as an instructor, especially with first year students. He is Afrikaans-speaking. It is thus fair to say that he would give good "traditional" physics instruction. His enthusiasm for the task was favourably commented on by both staff and students. The purpose of the above is to indicate that there was no attempt to"load" one group at the expense of the other. A genuine effort was made within the framework of a normal degree course to structure things so that the merit of the different approaches could be assessed. As explained earlier the structure of the South African university degree course and the pressures on UWC to conform to the standard model would not permit a more radical approach.

6.7 Results and Discussion

While it may be felt that recourse to statistics is the way to validate the approach outlined in this thesis, our view is that it is only one of the avenues that must be used. Since we are dealing with people and not things, statistics can at best only be an indicator of the effectiveness of the method. The best intentioned researcher must acknowledge that there are many variables over which no control can be exercised. For example, in spite of the fact that test and control groups may be carefully matched, individuals in one group may, for example, work more diligently than those in another. It could, of course, be argued that the test group would work harder because of success achieved with the approach .... etc. etc. etc. The point is that many imponderables are involved when we deal with humans.

For this reason, two other less quantitative, but nonetheless important, indicators will also be considered:

a) Since we are dealing with efforts to produce greater structure in students' thought processes, this should be reflected in the test papers and examination scripts of the students. This will be examined below.

b) It has been our experience that students are not slow in expressing their feelings about the manner in which the lectures are conducted. UWC has a somewhat obvious record of student protest and comment on the university itself as well as on events outside. Thus unsolicited comments may also be accepted as valid indicators - especially since students who were repeating the course (having failed the previous year) were subjected to a method different to that of the previous year (in the first year that the new approach was attempted in the lectures).

In what follows in this section, then, both the formal statistical results as well as the less quantitative considerations outlined above will be presented. These two factors apply both to this chapter and the next where cognitive deficiencies on the elaborational phase are discussed. No very sophisticated statistics were attempted as the data did not justify such procedures. Straight averages to test questions were calculated with 95% confidence intervals being established. These were obtained by:

$$\text{Confidence Interval} = \text{Standard Error} \times 1{,}96$$

$$\text{where Standard Error} = \frac{\text{Standard Deviation}}{(\text{Number in the group})^{\frac{1}{2}}}$$

If the differences between test and control groups are large enough, then the differences in the results would be statistically significant, given the 95% confidence intervals.

It is perhaps important at this point to reiterate the main differences in strategy in teaching the test and control groups: with the test group each concept was developed with the implicit algorithm made explicit in a stepwise manner using relevant examples where these served an instructive purpose. An attempt was made in this way to develop a planned approach to the students' later use of the concept in solving typical problems. Very few problems were actually solved in class during the five lectures and one tutorial period per week, as explained above.

With the control group, on the other hand, the usual broad discussions of the concepts were conducted with the application of the concepts to problem-solving being illustrated by the solution of actual problems both during the lectures and the tutorial sessions.

The test and control groups were both assessed as part of the normal quarterly tests of the Department of Physics. The questions used are of the types usually associated with first year physics class tests at UWC. Verbatim examples of these are the following:

Relative Velocity

A pilot wishes to fly due North. A strong wind is blowing at 100 km h^{-1} towards the East. The speed of the plane with respect to still air is 200 km h^{-1}.

a) In which direction should he head his plane?

b) Calculate the speed of the plane with respect to the ground.

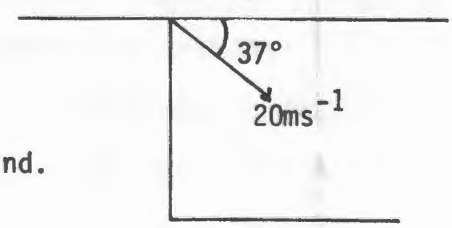
Projectile Motion

A person throws a stone from the top of a cliff 100m high with a speed of 20 m s^{-1} at an angle of 37° below the horizontal. Calculate:

i) Time taken to reach the ground;

ii) How far from the foot of the cliff the stone lands;

iii) Velocity with which it strikes the ground.



Circular Motion

A book lies horizontally on the seat of a car moving with speed 15 m s^{-1} around a bend of radius 40 m. Calculate μ if the book is on the point of sliding.

6.7.1 Statistical Results

The results given below were obtained during the usual quarterly class tests. While only one set of results is given, similar figures were obtained over the two years that the approaches outlined above, were implemented.

Relative Velocity

	Test	Control
N	52	48
$\bar{x}$	3,72 $\pm$ 0,56	1,59 $\pm$ 0,64
%	53,1 $\pm$ 8,0	22,7 $\pm$ 9,2
σ	2,06	2,27
S E	0,29	0,33

(N = number of students; $\bar{x}$ = average score;
σ = standard deviation; S E = standard error)

The confidence intervals (3,72 $\pm$ 0,56) and (1,59 $\pm$ 0,64) for the means in the test and control groups do not overlap. So it may be claimed that there is a significant difference in the performances of the two groups.

Projectile Motion

	Test	Control
N	52	48
$\bar{x}$	3,76 $\pm$ 0,58	2,36 $\pm$ 0,62
%	47,0 $\pm$ 7,2	29,5 $\pm$ 7,7
σ	2,12	2,19
S E	0,29	0,32

As above, the confidence intervals for the means in the two groups do not overlap and it can therefore be concluded that the performances of the two groups show significant differences.

Circular Motion

	Test	Control
N	52	48
$\bar{x}$	2,01 $\pm$ 0,50	1,79 $\pm$ 0,51
%	50,2 $\pm$ 12,4	44,6 $\pm$ 12,9
σ	1,83	1,82
S E	0,25	0,26

No significance can be attached to the apparent better performance by the test group.

No results are available for work and energy. Administrative problems prevented this (students action in the university in general, prevented normal class tests of this and other sections). It is interesting to note that no significant differences are noticeable with circular motion (see discussion in Section 6.4.2 above). The differences for relative velocity and projectile motion are maintained in subsequent tests and indeed in the final examinations. Over the two years that these approaches were tried, differences of similar magnitude were observed between test and control groups.

It is instructive to compare the averages for the Mechanics section to those of other sections where the approach was not used.

<u>March</u> :

	Test	Control
Mechanics:	52,4%	28,5%
Optics:	26,7%	30,9%

<u>June</u>:

	Test	Control
Mechanics:	49%	36,6%
Thermodynamics:	30%	35%

It is interesting and significant that it is only in the Mechanics section that there are appreciable differences between the two groups. The Optics and Thermodynamics courses were each taught to the two groups by the same lecturer.

A comparison with the results in years prior to the experiment are also instructive. It was not the first time that the two lecturers had presented the course to the Physics IB English and Afrikaans groups. Indeed, each had taught the course in the previous three years. In the year preceding the experiment the averages were:

<u>March</u> :

	English 1B	Afrikaans 1B
Mechanics:	36,8%	34,6%

<u>June</u> :

	English 1B	Afrikaans 1B
Mechanics:	50,5%	49,8%

The class had been divided on the basis of language for four years when the new approach was tried and for the first time an appreciable difference in the Mechanics section, but in no other, was recorded between the two groups. While the Hawthorne effect obviously has to be carefully considered, the students were given no reason to suggest that they were being observed and because of the language difference there is reason to believe that communication between the classes regarding the lecture content and methods was minimal. The differences in test scores enumerated above were maintained throughout the year.

The improvement was not limited to one year; for both years that the approach was attempted with the Physics IB group, results similar to those given above, were obtained.

6.7.2 Analysis of Worksheets

As has been discussed above, since an attempt has been made to develop some structure in the students' cognitive processing abilities, it is not simply a matter of whether a particular problem is solved or not, but rather whether any structured approach could be discerned in the manner in which students set about the solution. While no statistical data were accumulated in this area, physics staff members who viewed both sets of worksheets, noted the following:

a) The test group, far more so than the control, gave every indication that they knew how to proceed in attempting a solution.

b) The test group's solutions were more neatly set out than those of the control, indicating less hesitancy in approach.

c) Relevant equations were more readily arrived at by the test group. As one observer remarked: "It is clear that this group (the test) knows what to do."

6.7.3 Unsolicited Comments

While this may rightly be regarded as only a general indicator of the success of a particular approach, the volume and consistency of comments received, while not only being very gratifying, seem to indicate otherwise. The following are noteworthy:

a) Class attendance remained high in the test group for the duration of the course - unusually so not only in comparison with the control group but also as compared to attendance for other sections of the first year course (Optics and Thermodynamics) for the same group.

b) Those repeating the course were unanimous that the approach in the first "Test year" as compared to that of the previous year (given by the same lecturer) showed an enormous "improvement".

c) Many students approached the author to comment on what they called the efficient teaching method which they claimed really helped them understand the concepts being discussed. All said that the development of the "strategies" (as these were initially called in class) was particularly helpful.

One student, after the first class test, told the author that while at high school he had always failed physics, at UWC he had actually passed a physics test for the first time. He attributed it to the way he was taught.

d) One student even claimed that he had used the method of planning which he learnt in physics to good effect in studying a section in zoology.

6.8 Conclusion

On the strength of the results, both statistical and non-quantitative cited in this chapter, it can reasonably be concluded that the original premise has been substantiated: recognition of cognitive functions which may be deficient and attempts to make these explicit in the concept being considered, will improve not only students' understanding of concepts, but also their ability to use them in solving problems. It seems clear that students can be assisted to make not only a more careful, but also a more accurate, analysis of the data in a problem. This, however, requires that the concept must be analyzed very carefully by the teacher for the types of data that it generates. Especially data which may be regarded as implicit, need to be explained to students. The use of appropriate algorithms can also be effective in focusing the attention of students on the important data associated with a particular concept.

This does not necessarily mean, of course, that the efficiency of the skill is very high. However, the approach outlined supplies a guideline to enable students to accomplish better data analysis.

The very structured environment within which the research had to be conducted, prevented a different utilization of time to work for a greater efficiency in the exercise of the cognitive function. Thus, laboratory sessions can be used to reinforce some of the ideas discussed in the lectures. In dealing with circular motion, for example, it is relatively simple to design an experiment involving the analysis of data in three-dimensions. The examination of the manner in which suitably designed practical experiences could remedy or compensate for deficient cognitive functions would, in fact, be an interesting field of research.

CHAPTER 7

COMPENSATORY PROGRAMMES TO ADDRESS DEFICIENCIES IN THE ELABORATIONAL PHASE

7.1 INTRODUCTION

The four booklets discussed in the previous chapter were designed to address particular cognitive deficiencies related to the manner in which the many details in the problem statements were analyzed by students. The algorithms developed enabled the students not only to organize the data in the problem carefully, but also to approach the solution of the problems in a more structured manner. In each of the booklets, however, a theoretical preamble was given with the algorithms being used only in solving problems.

In the case considered in this chapter, however, in an attempt to develop in the students a planned approach to the use of Newton's laws, friction and the second equilibrium condition, the booklets are used to demonstrate that the algorithm is intrinsic to the concepts. The booklets show that understanding of the concepts and hence their use in a structured manner imply that certain essential features of the concepts must be recognized. In an effort to highlight the salient details of the concepts the booklets (and especially 1, 2 and 5) are designed to answer specific questions raised at the beginning of each. In so doing, not only are details clarified, but the intrinsic algorithm is made more easily assimilable. It is clear from the results of the test described in Chapter 5 that disadvantaged students do not extract these features on their own in spite of instruction they may receive about the concept. The booklets thus go much further than the normal textbook, not only in the analysis of concept details, but also in sequencing these features in an attempt to produce a measure of planning in the use of the concepts in solving problems. It is noteworthy that except for booklet 4, no specific problems are solved. Rather, situations typical of those included in a first year university mechanics course (e.g.inclined planes and pulleys) are analyzed in detail. It was found that because the approaches developed in the booklets usually took longer to teach in the formal lectures than the traditional method of teaching these sections, some of the weekly tutorial period of 40 minutes had to be used to keep the test group abreast of the control group. Effectively, therefore, the test group worked through fewer actual problems in the classes than did the control group. The fact that the former did so much better in the examinations (which consisted exclusively of problem-solving) than did the latter in both years of testing then becomes particularly significant.

It is worth mentioning that student enthusiasm for the methods used was extremely high in both years. Not only was this evidenced by the large class attendance in comparison with other sections of the physics course, but also by the innumerable positive comments received from students.

In what follows in this chapter then, the five booklets are discussed and analyzed, after which test questions and results are presented.

7.2 Newton's Third Law

It is customary in most textbooks to consider Newton's first law before the third law. However, the second step in the algorithm for Newtons's first law requires "drawing in all the forces acting on the body" (see section 7.3). Since this cannot really be done properly without an understanding of Newton's third law, it has been found useful to reverse the order of presentation of the two laws.

7.2.1 Discussion of Mech Bl 1 : Newton's Third Law

This booklet, like all the others, is written for use in an interactive manner since it was first devised to help students with self-study. Early in the research, however, it was appreciated that the approach in the booklets had to be adopted in the lectures as well if meaningful results were to be obtained.

After a statement of the law the following five questions are posed to guide the student through the essentials of the law as well as to make clear the features of the intrinsic algorithm used in applying the law:

i) What do the terms "action" and "reaction" mean?

ii) On what do the "action" and "reaction" operate?

iii) How many bodies are involved in the law?

iv) If all forces have an opposite force, how is motion ever possible?
Or, a related idea : what is the difference between Newton I and Newton III?

v) How must the term "unbalanced force" be understood?

Each of the questions is discussed using the following four examples:

1. A fist hitting a table.
2. A hand pulling on a rope attached to a box on a rough surface.
3. A person standing on the earth.
4. The earth revolving about the sun.

The terms "action" and "reaction" are clearly defined and illustrated using the four examples. In discussing the bodies on which each force acts, the following three frequently-used forces are explained:

a) the normal reaction, R_N
b) the tension, T
c) the frictional force, f.

Since these forces are used frequently in both statics and dynamics problems, their introduction at this stage makes their later utilization much easier.

After the above analysis of the action-reaction pair it becomes obvious, in answer to question (iii) on the previous page, that two bodies must be identified in the application of the third law.

Additionally, a discussion of why two forces acting on the same body cannot be an action-reaction pair (a common mistake among students, as can be seen from the test in Chapter 5 above), is given.

7.2.2 Analysis of the Approach

The algorithm is explicitly stated on the last page of the booklet after an indepth discussion of the law. It has the following steps:

1. Establish whether it is indeed a problem involving N III, i.e. whether action or reaction forces must be determined. Note that it may be part of a larger problem where you must determine the forces exerted on a body by other bodies.
2. Isolate the TWO bodies involved.
3. Determine what is the action:

   i) Size ii) Direction iii) Body on which it acts.
4. Remember that the reaction must have:

   i) Same size ii) Opposite direction iii) Must act on the body producing the action.
5. Test whether your answer can fit into the following:

   ACTION : Force exerted by body A on body B.

   REACTION: Force exerted by body B on body A.

The main cognitive deficiency being dealt with in this chapter is the inability of students to develop some planned procedure in using the law. The demonstration of the algorithm implicit in the law serves a twofold purpose :

a) Students are given a structure within which to apply the law.
b) Students are assisted in developing the ability to plan by a careful analysis of all the details of a law or concept.

The following points are noteworthy:

i) By stressing underlying meanings, the student is led to an appreciation of the implicit algorithm. Thus, for example, by examining the nature of the action and the reaction, the students come to an understanding of the fact that two bodies must be involved in the application of the law. The various steps of the algorithm thus follow naturally from the insights gained. Additionally, this emphasis on detail also assists in a careful analysis of the data in a problem statement and thus works against the input deficiencies discussed in the previous chapter.

ii) An attempt is made to help the student see both the implication and the meaning of the words used. In this way it is hoped that the rote-learning approach which characterizes many students' approach to physics can be overcome. Thus the "I know what the law says but I cannot use it to solve problems" syndrome can be addressed. It is hoped that the ability to plan in using the law will be cultivated through such insight. The semantic difficulties highlighted in Chapter 4 above are also addressed by such an approach.

7.3 Newton's First Law

The algorithm for Newton's first law is frequently given in textbooks. The similarity of the algorithms given by various textbooks (see for example, Sears and Zemansky, 1979; Tipler, 1982) with the one developed here serves to underscore its intrinsic nature. In applying Newton's first law to the solution of problems, there is indeed an approach which is dictated by the very nature of the law.

7.3.1 Discussion of Mech Bl 2 : Newton's First Law

The booklet is introduced by the following statement of the law and related questions:

Newton 1 : A body will remain at rest or continue with uniform motion in a straight line, unless acted upon by an unbalanced force.

Would you please look at the above formulation of the law and answer the questions below? (without turning the page please!)

1. How many bodies are mentioned?
2. a) What is an "unbalanced force"?
   b) If a single force acts on a body, can the body remain at rest?
   c) Which forces are mentioned in the law?
3. The law is sometimes written as:

$$\Sigma\vec{F} = \vec{0} \text{ i.e. } \Sigma F_x = 0 \ \& \ \Sigma F_y = 0$$

   a) Can you prove this?

b) What is implied in saying $\Sigma F_x = 0$, $\Sigma F_y = 0$?

4. How can you use the law to solve problems on equilibrium?
5. Can you use the law to give a definition of force?
6. What property of the body makes it act as described in the law?
7. What is implied by "at rest or continue with uniform motion in a straight line?

Sufficient time is taken to develop each point 1 to 3 adequately since these recur frequently not only in the statics but also in the dynamics part of the course. No effort is made to solve specific problems, however. Rather, seven equilibrium situations involving bodies or systems on rough surfaces and inclined planes held by ropes passing over pulleys are presented. Each of the first four questions posed above is discussed in detail using these seven situations as illustrations.
After establishing the need for the law to be applied to bodies individually, considerable time is spent on drawing in the forces acting on the body. Here again the role of the reaction force, tension and friction is discussed. It is apparent that an understanding of Newton's third law is essential to execute this step. The fact that the resultant force has to be zero is shown to mean that the sums of x- and y-components must individually be zero. In discussing the three questions in this way, the nature of the intrinsic algorithm is then established.

Finally questions 5,6 and 7 are discussed very briefly.

7.3.2 Analysis of the Approach

In answer to question 4 : "How can you use the law to solve problems on equilibrium?" the following algorithm is presented:

To solve problems on equilibrium:

a) Isolate the body or bodies whose equilibrium is to be obtained.

b) Draw in all the forces acting ON the body or bodies.

c) Draw a convenient set of axes so that the forces acting on the body can easily be split up into components along these axes. If two or more bodies are involved in the problem, different sets of axes may be necessary. Remember that once you have a system of axes you have positive and negative directions. Therefore you must decide if any force or component along an axis is + or - .

d) Now set up the equations $\Sigma F_x = 0$ and $\Sigma F_y = 0$ remembering:

   i) Some forces need to be split into components in the direction of the axes.

   ii) Each force or component must be assigned either a + or - sign.

The approach adopted in the booklet is to demonstrate the intrinsic nature of the algorithm. It is made apparent to the student that the application of Newton's first law to solve problems requires that certain specific details in the problem statement needs to be recognized in a definite order. The idea of planning in the use of concepts with the algorithm as the vehicle to achieve this, is continually emphasized. The following may be noted:

i) As each of the first three points in the booklet is discussed, the procedure to follow in using Newton's first law to solve problems is written down as a series of STEPS. The idea is to demonstrate to the students that an intrinsic algorithm needs to be made explicit to facilitate proper planning.

ii) As in the case of Newton's third law, a careful analysis is made of words relevant to the development of the algorithm as well as implications of the law. Under the former, the idea of "a body" can be classified while the choice of suitable axes falls under the latter. As demonstrated by the test in Chapter 5, students do not automatically see the idea of "one body" or "forces on". It is interesting to note the reaction of students in the lecture presentation to the question: "How many bodies are mentioned?" By this time in the course, a good rapport usually existed between lecturer and students and they would usually comment freely on questions posed. Yet in the case of this particular question they were reticent to commit themselves. This underscores again the point made in Chapter 5 above that the essential details of the law leading to a planned procedure in its use are not apparent to the students.

7.4 Friction

While an algorithm with Newton's first law is common, one dealing with friction appears to be absent from the usual first year university textbooks. Indeed it is not immediately apparent how one can be developed. Since frictional forces play a role in many problems both in statics and dynamics, it was therefore decided to develop the booklet, and the necessary algorithmic approach.

7.4.1 Discussion on Mech Bl 3 : Friction

This booklet differs from the previous two in that no questions are posed initially to guide the thinking of the student. The reason, of course, is that no law is being analyzed. Rather a discussion is given of the way in which the frictional force arises. Some of the properties of the force of friction are given. The following three aspects related to friction are then discussed:

1. The direction of the force of friction

   The way in which the direction of the force of friction can be established is demonstrated using seven different examples typical of a first year university physics course.

2. The considerations which apply if the system is stationary. Here two possibilities are presented:

   a) The force of friction is sufficient to prevent movement taking place but is not a maximum.

   b) The object is on the point of moving so that the following expression applies

$$f = \mu_s R_N ,$$

where f = force of friction

R_N = Normal reaction

μ_s = Coefficient of static friction

It is stressed that this relation only applies if the body is on the point of moving.

3. The system is moving so that the same expression applies as in 2(b) with the coefficient of kinetic friction replacing the coefficient of static friction.

   The fourth point in the booklet now repeats steps 2 and 3 of the algorithm on Newton's first law, viz. drawing in the forces on the body and choosing a suitable system of axes.

7.4.2 Analysis of the Approach

The algorithm developed to deal with problems on friction is the following:

1. Determine the direction of the force of friction i. e. find the direction in which the system moves, or tends to move.
2. If stationary, two possibilities exist:
   a) The friction is just sufficient to stop movement taking place.
   b) The system is on the point of moving.
      Then friction is a maximum,
      $$f = \mu_S R_N$$
3. If moving (sliding), then
   $$f = \mu_K R_N$$
4. If (2b) or (3) is the case, determine R_N.
   a) Draw in all forces ON the body.
   b) Choose suitable axes.
   c) To find R_N, use perpendicular components.

While considerations with respect to friction are normally included in equilibrium problems or those involving Newton's second law, the above algorithm assists students in developing a planned approach when dealing with friction. The approach can then be integrated into the algorithms for Newton's first and second laws. Students are thus assisted in making decisions regarding various aspects of the same problem.

Again, it needs to be emphasized that since disadvantaged students show a particular inability to plan effectively, the above features are designed not only to make an effective procedure clear, but also implicitly to demonstrate the need for concern with specific detail if such a procedure is to be developed.

Additionally, the booklet assists students in the following ways:

i) The determination of the direction of the force of friction as opposite to the direction of motion is shown to be a non-trivial problem, not always apparent from the configuration of the system in question. To accomplish this, specific features of the data in the problem need to be carefully evaluated. This works against the blurred and sweeping perception which characterizes a number of students, as discussed in Chapter 4.

ii) A clear distinction is drawn between three possibilities as regards the frictional force. While textbooks normally highlight the critical value of the static or the dynamic friction since these can be expressed by an equation, students may overlook the fact that static friction can take on any value up to the limiting value, depending on the situation of the system. The emphasis in the algorithm on the determination of the applicable state obliges the student to make a careful analysis of the data in the problem.

7.5 Moments and the Second Equilibrium Condition

The booklet developed for this section is longer than the others and really consists of two parts:

a) Devising a plausible algorithm for the determination of the moment of a force.

b) Combining the algorithm for Newton's first law and that for the second equilibrium condition into one coherent algorithm.

7.5.1 Discussion of Mech Bl 4 : Moments and the Second Equilibrium Condition

Experience has shown that students at UWC experience considerable difficulty with the concept of the moment of a force. The first section of the booklet is devoted not only to a clarification of the concept but also to the development of a procedure for determining the moment of a force.

The discussion of the moment of a force is introduced by demonstrating that the point of application of a force relative to an axis of rotation has important implications for the effect that a force may have on a body. The moment of coplanar forces is carefully defined and it is noted that the axis of rotation will always be perpendicular to the plane of the forces and will thus be represented by a point in the plane. The calculation of the moments of some typical forces found in five different situations is then carried out by making explicit the various features related to moments of force which need to be identified.

The Second Equilibrium condition, viz.

> If a body is in equilibrium under the action of a number of coplanar forces, the algebraic sum of the torques about any arbitrary axis is zero,

is then analyzed along the lines already followed with Newton's first law. The algorithm for Newton's first law is combined with the obvious features of the second equilibrium condition to provide a single algorithm to deal with problems on the equilibrium of systems. Actual problems are then solved to provide practice for students in applying the algorithm.

It may be noted that in the approach in the booklet the concept of moments is treated within the requirements of the first year physics course, viz. that coplanar forces only are considered. Experience has shown that it is futile with disadvantaged students to define moments in terms of vector products with the moment as the vector perpendicular to the plane of radius vector and force. Indeed this approach is self-defeating in that not only is the sophisticated definition not understood, but students are left without being able to calculate even simple moments of forces since the underlying algorithm is lost in a maze of mathematical difficulties.

7.5.2 Analysis of the Approach

The following algorithm has been developed for calculating the turning moment of a force:

1. Choose the point O (axis of rotation) about which moments are to be taken.

2. Determine which forces have a moment about O.
3. a) Is it possible to draw a perpendicular from O onto each of the lines representing the forces?
   If not, extend the line of action of the force either forward or backward.
   b) Is it more convenient to use components of a particular force? Extend the line of action of these if necessary.
4. Draw the perpendicular from O to the force or the line of action of the force (or the components if the force is split up into such).
5. Determine the moment:
   i) Calculate the product : Force x Perpendicular distance.
   ii) Insert the signs of the moment.

Additionally, the general algorithm for dealing with problems on the equilibrium of systems is the following:

1. Establish that it is a problem on equilibrium
2. Isolate the body or bodies involved.
3. Draw in all the forces on the body.
4. Choose a convenient system of axes.
5. Write down the equations:

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$

6. Carry out the algorithm on Moments.
7. Write down the equation:

Σ Moments about a convenient point = 0.

8. To the three equations thus obtained add the equation $f = \mu R_N$ if and where applicable.
9. Solve the equations for the unknowns.

It was noticeable that by this stage in the course, students were easily able to appreciate the need for an algorithm as well as its nature.

The following points are noteworthy:

i) No detailed analysis is given of the second equilibrium condition since something similar has already been dealt with under Newton's first law. It is expected that by this time the method of careful consideration of the words and implications in the statement of a law or principle will have been sufficiently impressed upon students that they would by now have cultivated the ability to plan in using a concept.

ii) This is the first booklet in which actual examples are solved. The reasons for this are the following:

   a) This booklet serves to conclude the statics course and helps place the first three booklets in perspective.

   b) Fairly involved problems can now be solved by using the algorithms in booklets 2,3 and 4. It was considered useful to demonstrate to students exactly how the procedure developed does indeed help one approach problems in a more structured fashion.

   c) It serves to demonstrate how a single problem can be divided into a number of sub-problems each with its own particular goal. The entire problem is then solved by integrating these various facets.

iii) While the algorithm enables a planning procedure to be adopted, it also assists in careful analysis of the data with only that data relevant to the problem being extracted from the problem formulation. In this way a more precise and accurate data analysis by students is facilitated.

7.6 Newton's Second Law

The emphasis placed in the lectures on Mech B1 2: Newton's first law, makes it relatively easy to introduce Newton's second law since many of the features are similar. By this time the students are very familiar with forces acting on particular systems and the need to choose appropriate axes.

7.6.1 Discussion of Mech B1 5 : Newton's Second Law

The booklet is introduced in the following way :

"Before considering this unit, please review Mech B1 2 on Newton's first law. The material discussed below is an extension of that unit.

Newton II : The acceleration of a body is proportional to, and in the same direction as the resultant force acting on it, and inversely proportional to the mass of the body.

This normally reduces to : $\vec{F} = m\vec{a}$

Please look at the following questions and see if you are able to answer them by looking at the above formulation of the law:

1. Since only one body is mentioned, how will you deal with a system involving related masses?
2. a) Which forces are needed to apply the law?
   b) Do all the forces on the body influence the acceleration?
3. a) Which two directions are implied in the law?
   b) How does this influence your choice of axes?
   c) What would be the equations in these directions?
4. How can you use the law to solve problems?
5. Is the resultant force causing the acceleration necessarily in

the direction of motion?

We will now attempt to answer these questions individually and in so doing demonstrate what is required to use the law."

These questions are discussed using eight examples typically found in a Physics I course. It was found in practice that students had very little difficulty at this stage in isolating bodies and drawing in the relevant forces. As part of the discussion on the choice of suitable directions for the axes, a method is given for determining the direction of motion of the system. In point 5 (as numbered in the booklet) a distinction is drawn between the direction of motion and the direction of the acceleration. It is shown that, in the case where friction is present, care needs to be exercised in establishing the direction of motion before the acceleration is calculated.

In this booklet, as with the others, an attempt is made to maintain the interactive approach so that students are not only led to the implicit algorithm with the attendant understanding of the particular law or concept discussed, but are also enabled to conduct fruitful self-study leading to better problem solution.

7.6.2 Analysis of the Approach

The algorithm developed represents an extension of that for Newton's first law. It is :

i) Determine the mass that is being accelerated. (point 1 above)

ii) Draw in all the forces acting on the mass. (point 2 above)

iii) Determine the direction of motion, if this is not given, by using the procedure at the bottom of page 6 (of the booklet).

iv) Choose one axis in this direction, the other perpendicular to it.

v) In the case of connected masses, see how the accelerations are related.

vi) Determine if forces in the perpendicular direction influence forces parallel to the direction of motion. Thus

$$\Sigma F_{perpendicular} = 0 \text{ and } f = \mu R_N$$

vii) $$\Sigma F_{parallel} = M \,.\, a$$

viii) Repeat the procedure for each mass individually.

ix) Solve the equations.

It was noticeable that by this time in the course, relatively little time was needed to justify the steps in the algorithm. Indeed, it was apparent that students were now accustomed to a careful analysis of the words in a law and that, to them, the development of a procedure for use of the law was regarded as a necessary part of the presentation of the law in the lectures. It was also clear in watching the manner in which they approached the solution of problems that they were focusing on the details of the problem essential for application of the law. As has been discussed above with the other booklets, not only does the approach facilitate planning in using concepts, it also enables students to perform accurate data analysis.

In all five booklets, heavy emphasis is placed on establishing clearly the implications of laws and concepts (e.g.the number of bodies involved, the need for axes etc.). These are features which experience has shown are not perceptually obvious to the disadvantaged students who are the subjects of this study, and are thus not internalized as part of the understanding of the particular laws or concepts. By repeating these details in each booklet an effort is thereby made to assist students to make these features a part of their understanding of the relevant law or concept. In so doing, the aspects necessary for their developing an ability to plan in using the concepts, are supplied.

The results documented in the following section indicate that at least in the area of Mechanics, this approach has produced a fair measure of success.

7.7 RESULTS

The problems used to test the students on these sections deliberately kept data collection and interpretation to a minimum, (contrast this with those in Chapter 6 above) since the cognitive ability at issue is an elaborational one. It was therefore not deemed necessary to ask extremely involved problems - the type of questions asked were those which are standard to most first year university physics courses and are similar to those in first year textbooks or in the printed notes which all UWC first year physics students receive. No claims are being made that the development of planning algorithms automatically gives students global problem solving abilities even in the concepts considered for there is more to the solution of involved problems than an elaborational ability only.

The following are typical examples of the problems set to test the various concepts:

Newton III; First Test

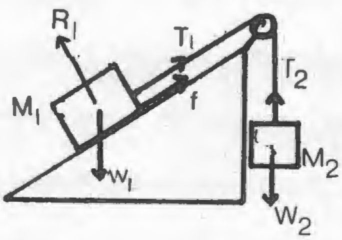
A body, weight W, is suspended from the roof by a rope as in the sketch.

i) T and W do not form an action-reaction pair because ....

ii) The reaction to W is ....

iii) The reaction to T is ....

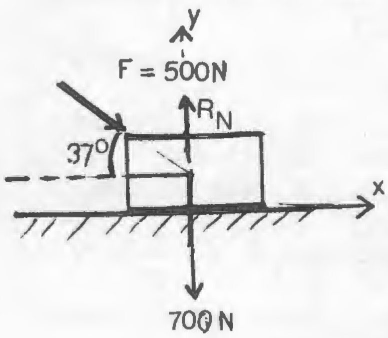
Newton III; Second Test



Masses M_1 and M_2 are held in equilibrium as shown, where M_1 rests on a rough surface. Use Newton's third law to write down the reactions to the following forces:

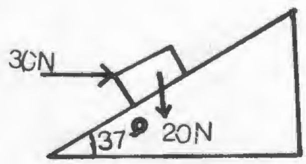
W_1, T_1, f, T_2, W_2, R_1

Friction First Test



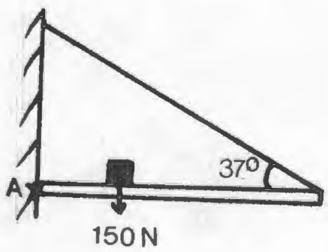
A body, weight 700N is pushed along the horizontal plane at constant velocity by a force of 500N, which makes an angle of 37° with the horizontal. Calculate the coefficient of kinetic friction (μ_k) between the body and the plane.

Friction; Second Test



A horizontal force of 30N is applied to a weight of 20N on a slope, angle 37°. If the mass is on the point of moving up the plane, calculate the coefficient of friction.

Equilibrium

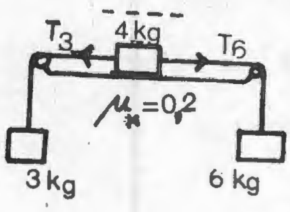


A uniform beam, weight 500N, is hinged to a wall and is held horizontal by a rope which makes an angle of 37° with the beam. A weight of 150N rests on the beam at a distance one third the length of the beam from the wall (point A).

Calculate : i) The tension in the rope

ii) The size and direction of the reaction at A.

Newton II



Given the system in the figure, calculate:

i) acceleration of the system

ii) tension, T_6

iii) tension, T_3

RESULTS:

The results obtained for each question are given below.

The "First Test" and "Second Test" mentioned below were the normal March and June quarterly tests written in the Physics Department at UWC.

The symbols used have the following meaning:

N = Number of students; $\bar{x}$ = Average; σ = Standard Deviation;

S E = Standard Error

Newton III : First Test   Second Test

	Test	Control	Test	Control
N	44	44	46	46
$\bar{x}$	1,73 ± 0,35	0,98 ± 0,29	3,72 ± 0,59	2,13 ± 0,63
%	57,6 ± 12	32,6 ± 10	62,0 ± 10	35,5 ± 10
σ	1,17	1,02	2,06	2,17
S E	0,18	0,15	0,30	0,32

Friction : First Test   Second Test

	Test	Control	Test	Control
N	44	44	46	46
$\bar{x}$	1,36 ± 0,33	0,74 ± 0,27	4,27 ± 0,86	1,27 ± 0,57
%	45,3 ± 11	24,6 ± 9	53,3 ± 11	15,9 ± 7
σ	1,10	0,94	2,92	1,97
S E	0,17	0,14	0,44	0,29

Equilibrium

	Test	Control
N	46	46
$\bar{x}$	$4,4 \pm 0,9$	$1,57 \pm 0,84$
%	$44,0 \pm 9,0$	$15,7 \pm 8,4$
σ	3,1	2,9
S E	0,46	0,43

Newton II :

	Test	Control
N	40	40
$\bar{x}$	$6,53 \pm 0,82$	$4,81 \pm 0,90$
%	$72,6 \pm 9$	$53,4 \pm 10$
σ	2,61	2,91
S E	0,42	0,46

It is apparent from these results that in none of the cases above is there any overlap between the confidence intervals of the test and control groups. The statistical difference is thus significant and so one can reject the null hypothesis that there is no advantage in the algorithmic approach. Thus one is led to conclude that the method of making explicit the algorithms associated with each of the laws/concepts has enabled students to solve the problems given much more effectively than those students who were not subjected to the approach. These differences were maintained throughout the year even to the final examinations.

Since the results on both input and elaborational phases have now been discussed, it is useful to illuminate these results in terms of the Feuerstein paradigm.

Two matters need clarification.

1. Instructional strategies adopted

It is perhaps necessary to explain the difference between the instructional strategies adopted for the test and control groups. In South African universities the large lecture format is standard: the lecturer attempts to present a particular concept with as much associated information as he sees fit, he perhaps illustrates the concept with interesting demonstrations, and solves a few typical problems. The students are left to solve more problems in the tutorial sessions, which are small-group discussions. Seldom are cognitive or meta-cognitive aspects implicit in the material being discussed, made explicit. This was the approach, called above "the traditional approach", followed very enthusiastically by the highly-esteemed lecturer of the control group. In contrast to this, the author, in dealing with the nine areas discussed above in the assigned lectures to the test group, spent considerable time helping students analyse the manner of data analysis required not only to understand but also to use the concept. In addition the sequence of steps necessary to apply the concept was discussed in detail. In this case the approach with the test group can certainly be regarded as "non-traditional". To illustrate : in presenting Newton's second law to the control group the lecturer stated the law, established the equation and then set about showing that the equation is indeed valid by using an airtrack. With the test group, the law was written on the board and a discussion then

It is worth repeating (see section 6.7.2) that independent academic staff who analysed both sets of worksheets were convinced of the fact that the test group "very obviously" knew more "physics", that is, their answers clearly indicated that they were often able to deal with the various features of the concepts tested.

It is apparent from the above that while the instructional approach was different from the two groups, the method of assessment was the same. Apart from the results both statistical and anecdotal mentioned above, the following points serve to emphasize the outstanding success of the approaches espoused in this thesis:

i) While results are only given for one year, the methods were actually employed for two years (for the purpose of this thesis) with statistics being kept for both years. It is extremely noteworthy that for both years the differences between test and control groups were almost identical. For example, the results for Newton II were $(60,6 \pm 8,4)\%$ for the test group and $(40,7 \pm 8,7)\%$ for the control group - a difference of 20%. (compare p. 227) This result is typical, but all the results have not been repeated to prevent unnecessary duplication.

   It should be noted that in both years of implementation by the author, it was only in the mechanics section of the course that significant differences between the two groups were obtained. Indeed in the second year of implementation the class average of the test group in the mechanics section in the final examination was over 60% - unheard of in the history ensued in an attempt to extract the various features of the law and to establish the steps in the algorithm on pp. 221 and 222 as inextricably bound up in the use of the law. So while the first approach can be described as establishing the law and its meaning, the second involves the mental operations in understanding and using the law.

2. Assessment of performance on test questions

As can readily be calculated from the percentages given, the actual marks assigned for the test questions range from 3 (Newton III) to 10 (Equilibrium). It must be remembered that the marks for each question were those which were required in the examinations of which the questions were a part. The marks could thus not be disproportionately high. But while the actual marks ranged from 3 to 10, the utilization of ½-marks in the marking scheme enabled the number of intervals to be extended from 3 to 20 for the various items.

The method of assessment differed between questions. Thus for example, the answer to the questions on Newton's third law are either correct or not. On all other questions students were assigned marks as is customary in a thorough memorandum. For example, on the problems involving force, their ability to draw in the correct forces as well as to assign proper coordinates to equations and the correct use of equations were assessed. The emphasis was placed on assessing the amount of physics knowledge displayed rather than on correct answers and arithmetical calculations. The tests were frequently set by the lecturer of the control group. The two lecturers always agreed on the memorandum.

department. (they usually have the most non-lecture contact with students) It can easily be appreciated why these considerations were certainly played down at the time since the very real possibility of inter-departmental professional difficulties arose.

Finally it is possible to demonstrate conclusively the power of the Feuerstein approach by illustrating how it predicts the reasons for differences not only on questions between groups but even within each group. The differences between groups show clearly that the emphasis on the cognitive and meta-cognitive functions required for correctly understanding and applying concepts with the test group produced significantly better performance in comparison to the control group. A notable exception is in the circular motion task. For a complete discussion in terms of Feuerstein theory the reader is referred to section 6.4.2, pp. 182-184. In that section it is pointed out that Mech. Bl. 8 cannot be expected to produce markedly improved results because of the requirement to visualize a 3-dimensional event drawn in 2-dimensions.

Feuerstein mentions in his writings that it is easier to acquire an elaborational ability than an input one. This is borne out by the fact that both test and control groups performed markedly better on the Newton's second law question than on the equilibrium one. Equilibrium is considered early in the course when the method of cognitive analysis of material was new to the test group. The difference between test and control group was about 30% on this topic. However, Newton's second law is considered much later (the second as opposed to the first quarter). By this time the test of UWC!

ii) It is difficult to convey in writing the enthusiasm of the students in the test group in both years of the experiment. The author had, prior to this time, taught the Physics I class for four years and the change was remarkable. Especially rewarding was to listen to the failures of the previous year (1980) who were exposed to the new approach in 1981. Many of them actually came to the author's office to comment on the new teaching strategy which they described as "new", "very clear", "very effective", "marvellous", etc. Indeed, throughout the two years other staff members frequently conveyed to the author the many positive comments of students. The fact that the lectures and the tutorial sessions (frequently conducted as full class sessions and not in small groups) of the test group remained full for the duration of the course, is indicative of student enthusiasm. It was thus not believed to be necessary to conduct interviews with students to determine how they found the approach.

Indeed, not long ago the author met an ex-student outside of UWC. After suitable introductions (who can remember all his ex-students?!) he proceeded to expand on "the marvellous way he was taught mechanics at UWC", in 1982.

While all of this is anecdotal, the weight of evidence suggests that not only did the test group do so much better statistically but the students also had a very clear perception that they were masters of their own "mechanics destiny".

The complete absence of similar enthusiasm by the control group was frequently noted by the senior technical assistants in the group had acquired practice in the methods and may have been expected to do better. The fact that the control group too did much better indicates that the elaborational ability required to deal with this type of problem was acquired to some extent simply by solving problems as they did in the tutorial sessions. It would be unlikely that the same effect would be discernible with the input phase. This was, however, not investigated.

It is a pity that in the first year of the experiment, student unrest prevented testing of the Work-Energy instrument while in the second an administrative mistake made the marks unavailable. So no statistical evidence is available on the effectiveness of the booklet. Apart from favourable student comments however, one evidence of the value of the booklet was obtained from underqualified teachers on an in-service training course. It was found that the only ones who could successfully deal with work-energy type problems were those who had carefully worked through the booklet. All the rest foundered on the correct interpretation of the data in the problems.

In summary, not only does the Feuerstein approach lead to new insights into the reasons why students have difficulty with certain topics in physics but it also supplies a teaching strategy by means of which significant improvements can be made to the achievement of students.

7.8 Conclusion

While this chapter has tended to focus on the need to develop in students a planned approach to the use of concepts - an elaborational ability - an examination of the booklets makes it apparent that it is impossible to separate completely the input and elaborational phases of the mental act. Thus, not only is it necessary to analyze carefully the data in a problem which tests a particular concept (for example, which body or bodies are in question in the case of Newton's first and third laws, respectively), but also in developing the algorithm to utilize these laws/concepts a careful analysis of the data (words and their implication) in the law or concept itself needs to be made.

The development of a planned approach in the use of concepts takes place on two levels:

i) The algorithm which is intrinsic to the concept is extracted and made meaningful to the students in the lecture presentation and in the booklets. As has been demonstrated by the analysis of the booklets this involves sequencing the steps involved in the application of the concept. This sequence is in its turn obtained by a very careful analysis of the words and features of the concept so as to demonstrate the plausibility of the algorithm. The algorithm is thus never a recipe applied to solve problems but is seen as supplying the understanding necessary for utilization of the concept. In this way, the student is heavily assisted in his/her planning. That the approach has been successful in assisting students to be more structured in the way in which they deal with problems can be seen by the very significant difference, maintained throughout the entire year, between test and control groups. In addition to this, students were most vocal in their praise, with expressions such as : "For the first time I understand this stuff", being very common.

ii) A planned approach to the use of any law or concept needs to be generated by the students themselves. It was apparent as the course continued that students were beginning to see the necessity of a careful analysis of words and features in the statement of laws etc. Thus the analyses presented in booklets 4 and 5 were done far more rapidly than those in booklets 1-3 and it was very obvious that students were now becoming more adept at the type of analysis necessary for planning.

While the results obtained with the students with the Mechanics course were very satisfying, it is abundantly clear that if cognitive deficiencies are to addressed then the approaches outlined in Chapters 6 and 7 need to be employed not only in other sections of physics but also in other subjects. Indeed, the experiences with Mechanics indicate that such an approach can make a meaningful difference to the way in which disadvantaged students think.

CHAPTER 8

CONCLUSION

8.1 Historical Development Of This Research

The research which is reported in this thesis has taken place over some five years. It began around 1978 simply with the idea that something constructive needed to be done about the very obvious learning difficulties which students at the University of the Western Cape (UWC) were experiencing with science subjects in general and with physics in particular. The main problem facing the researcher at that stage was exactly how to tackle this situation. Little existed in the research literature in South Africa to indicate that these issues had received much local attention. Indeed, the usual platitudes regarding underqualified teachers and badly-equipped schools were advanced as almost the sole reasons for the low achievements of so-called Coloured and Black students in science. Almost no attention was paid to the students themselves.

A review of the international research literature in Physics Education at the time revealed three main streams viz.

i) The application of Piaget's theories of development to teaching and learning physics.

ii) A growing interest in Information Processing theory.

iii) Accumulating evidence of the misconceptions which learners have of basic physics concepts.

Initially it was felt that Piagetian theory would provide a fruitful point of departure in assessing students and some time was spent on developing suitable tests of students' developmental levels. However, it was quickly appreciated that there was no hope of introducing a Piagetian programme into the very rigid structure of the first year physics course at UWC. The Information Processing approach of Cognitive Psychology was examined carefully and some of the first booklets were based on this theory.

The major difficulty with both the approaches was that neither paid particular attention to persons who could be classed broadly as disadvantaged. The "discovery" of the work of Feuerstein thus represented a major step forward in the development of the research reported upon here. With its componential analysis of the thinking processes of disadvantaged persons, it provided important insights into the possible reasons why students at UWC were in fact experiencing such difficulty with science-related disciplines. The Feuerstein theory helped to establish a broad framework within which the research could be conducted and thus served to provide direction for the development of additional aspects of the research programme.

8.2 The Main Results Of The Research

8.2.1 A Paradigm Within Which The Learning Problems In Physics of Disadvantaged Students Can Be Placed

The Feuerstein theory has proved very useful in placing the learning problems of disadvantaged students in perspective. As has been said repeatedly throughout this work, the power of the approach lies in the identification of cognitive deficiencies as being at the root of at least part, if not all, the "retarded performance" of disadvantaged students. The insights gained from Feuerstein's work enabled a more directed attack to be made on the problems that UWC students have in learning physics. It enabled socio-economic factors (about which very little can be done by teachers at university level) to be accepted and by-passed for the purpose of providing <u>real assistance</u> in physics to first year students.

In three main ways then, the use of the Feuerstein approach has influenced the research described in this thesis:

i) It establishes that the determining characteristics of a disadvantaged person is the manner in which information is processed by that person.

ii) Cognitive characteristics of students could be determined in the light of Feuerstein's Cognitive Map. The list of cognitive deficiencies supplied by Feuerstein proved invaluable in the following ways:

a) It gave an idea of the types of problems in physics which could be used to test students in an effort to establish the degree to which certain cognitive operations are used by students and indeed the extent to which certain cognitive functions are deficient in students.

b) It provided an understanding of why certain sections of the physics course proved so difficult for students.

iii) A match could now be made between the cognitive characteristics highlighted in (ii) and the requirements of a particular section of the physics course. Suitable remediatory materials could now be produced with special emphasis on the requirements for solving problems.

To date the application of Feuerstein's work by himself and his co-workers has been in content-free cognitive instruction in their own "Instrumental Enrichment" programme. The present work represents one of the first efforts to take features of Feuerstein's theory and to apply them in a specific discipline.

8.2.2 Identifying Student Difficulties In Dealing With Mechanics

An examination of the Feuerstein approach gave a preliminary indication of the possible reasons why students were experiencing so much difficulty with the kinematics sections of the first year physics course - a section ostensibly based on only a few simple equations. It was noted that since problem formulations in this section tended to be rich in data, students were possibly experiencing difficulty at the input phase of the mental act. It was decided to test this premise using the person-to-person interview technique as the students worked through some typical kinematics problems.

In addition to the thirty detailed interviews conducted, a specially designed test was given to students at the beginning of the academic year to determine the extent to which they had developed any ability to elaborate Newton's laws. A number of difficulties was highlighted.

8.2.2.1 Semantic Difficulties

The extent to which certain words used in the formulation of problems proved to mislead students in the way in which they interpreted data was not only surprising but also very illuminating. (Specific examples are given in Chapter 4.)

Some work in this field has already been done in South Africa with regard to the readability of science textbooks (Wegerhoff, 1981). The aspect highlighted in the present research relates to the extent to which students are able to decontextualize words which are common to them for use in a particular scientific sense. Research in this area will undoubtedly provide meaningful insight into disadvantaged students' problems in understanding science since for many such students, English (the predominant language for science instruction to the majority of these students) is often a second or even a third language.

8.2.2.2 Error Factors In Physics

Both the interviews and the written test highlighted a number of common errors which students display in the Mechanics section of the physics course. Some of these errors have already been documented by a number of researchers under the heading "misconceptions". Other types of errors relating to the choice of axes and the signs given to kinematic variables can easily be addressed by careful presentation of these ideas, given the knowledge gained from this research of the nature of the confusion which often exists in students' minds regarding such operations. Indeed, further research which has as its aim the careful documentation of such error factors which may exist in students in dealing with any section of content, can have a significant influence on the instruction given and materials produced.

8.2.2.3 Mathematical Difficulties

The interviews also demonstrated, not unexpectedly, that students also experience difficulties with certain mathematical concepts. The question with disadvantaged students seems to be not whether but rather with what, they have mathematical difficulties. Translated into practical terms within a physics course this implies that an analysis needs to be made of the mathematical requirements of any section of the course with appropriate remediation being built into, or parallel to, the presentation. Thus, for example, the test in Chapter 5 demonstrated that a number of students had difficulty with elementary trigonometrical ratios. It is therefore unlikely that such students would be able to handle the components of vectors adequately.

8.2.2.4 Cognitive Difficulties

The Feuerstein theory was very useful in giving direction to the interpretation of the wealth of data which was generated by the interviews. Two related areas of difficulty clearly manifested themselves:

a) The manner of data interpretation (Input phase)

It was abundantly clear from the interviews that the majority of students do not interpret the data in the problems clearly and accurately. Indeed, what distinguished the good students from the others was the care with which the former clarified the various features of the problems to themselves as opposed to the haphazard and frequently inaccurate data analysis of the latter. Among those who detected details only after considerable help, if at all, two groups could be distinguished:

i) Those who were simply unstructured in their approach to the problem. Where they were asked to visualize the problem carefully and to record important details they were able, with some help, to do so. They usually then proceeded to a solution fairly easily.

ii) The rest of the students seemed unable, despite help and prompting, to visualize the problems clearly or to give a qualitative evaluation of the problem. They were, of course, unable to solve the problems on their own.

The analyses of the interviews have yielded a list of deficient cognitive functions which have been related to those documented by Feuerstein (see Chapter 4). While a number of deficient cognitive functions can be mentioned, the one which proved to be of particular difficulty to almost all the students interviewed was the ability to relate two sources of information. Since this cognitive function is fundamental in establishing relationships it could well merit further investigation. Another cognitive operation which proved difficult was the ability to visualize clearly the key features of the problems. While it is one thing to know that these cognitive functions are deficient and to determine their influence on student achievement, it is quite another matter to know exactly why they are deficient. This would be another interesting area of research. This would be equally true of the other cognitive deficiencies as well.

b) <u>The manner of data elaboration</u>

The interview analyses have shown clearly that students' difficulty in carefully analysing data in a problem, influenced the manner in which they worked toward a solution of the problem. Apart from not being able to recognize cues in the problem statement which would lead them to either the necessary equations or procedure to follow to reach a solution, they also did not demonstrate any clear plan in their use of the equations, which were often apparently rather arbitrarily chosen. Thus, for example, in using one or more of the three kinematic equations, many students did not carefully search through the data in the problem nor relate various features of the problem to determine the applicable variables. Instead they would usually use any available numerical data. In effect, therefore, their approach to the problem showed no particular design evidencing some strategy they planned to follow.

Since the application of physical principles to the solution of problems requires a very definite elaborational ability (in the Feuerstein sense as in Chapter 3), it has proved possible using a paper-and-pencil test to determine whether students develop the capacity to plan effectively in using concepts such as Newton's laws. It seems clear from the results in Chapter 5 that in spite of instruction received, very little structure is discernable in the way the use of the concepts is approached. It seems possible to generalize this result: <u>if disadvantaged students are not taught a method of planning in their use of a concept, no procedure for use of that concept develops from the usual methods of instruction.</u> The reason for the generalization is the following: Newton's laws are taught to students in detail at school together with the solution of reasonably sophisticated problems. Indeed, the problems set on Newton's laws in the national school-leaving examination in Physical Science are frequently similar to those which they encounter in their first term of physics at university. In spite of this they are unable to solve the problems. The test in Chapter 5 clearly shows that a planned approach to the use of these concepts has not been developed by the methods used in teaching this material. It therefore seems reasonable to assume that formulating a planned approach to the use of physical concepts is not something readily accomplished by students themselves.

8.2.3 Physics Content Analysis In Terms Of Cognitive Demand

8.2.3.1 Content Analysis

On the strength of the list of cognitive functions supplied by Feuerstein and, indeed, of the particular ones established by the research described in this thesis, it is possible to analyse any section of work to identify the cognitive functions necessary to understand and use the concepts in that section, especially in problem-solving. The way this can be done has been demonstrated in Chapters 6 and 7.

Thus, for example, in teaching sections in kinematics it must be recognized that the ability to analyse data effectively (input phase) is crucial if students are to obtain proper solutions to the problems. In teaching any section of a syllabus, therefore, it becomes necessary to determine:

i) Which phases of the mental act (input, elaboration, output) are necessary for understanding and/or applying the particular concept or content material in general?

ii) Can the particular cognitive functions within each phase which are necessary for proper understanding of a concept, and which may be deficient in students, be isolated?

iii) How can the instruction be developed to acknowledge the necessity of these functions and to strengthen these abilities if they are lacking in the students?

iv) What is the best manner of cultivating the necessary thought-processes? Is it possible, for example, to develop suitable laboratory practical work?

v) The cognitive requirements for any testing procedure that is used need to be analysed.

The steps enumerated above may even supply one means of reducing the number of misconceptions which students develop, or of distinguishing misconceptions from cognitive problems.

It is necessary to place the claims of this thesis as respects deficient cognitive functions in perspective: the claim that is made is that areas of cognitive deficiency exist in the disadvantaged students who are the subjects of this study and that once these deficiencies are identified, it is possible to compensate for them in particular areas by the development of suitable subject-related material. No claims are made that in so doing permanent all round improvement of these specific functions will take place although in some cases it may be so. What is said is that since the application of the particular concept may require certain cognitive functions in addition to pure content, some effort must be made to assist the disadvantaged student in whom such cognitive functions may be weak, by the production of suitable.

ancillary material. The cognitive ability thus addressed, may not transfer to another section of physics where this specific cognitive ability is required. For example, if a student is assisted to do meaningful data collection in an area such as kinematics, it does not follow that he will now have acquired a cognitive function which he can use in studying electricity, although one would of course hope that this was indeed the case. However, the recognition of cognitive functions needed to deal with specific areas in physics does enable the instructor to assess more meaningfully the difficulties which students may experience with that specific area. Also the approach encourages the instructor to distinguish between content deficiencies (e.g. a weak mathematical background), misconceptions which the student may have and cognitive deficiencies which may exist. Further one may conjecture that a concerted effort to address cognitive difficulties across the entire area of physics, wherever it is possible, may indeed produce permanent cognitive improvement. The approach described here at the very least enables us to examine cognitive deficiencies which students have, provides us with the means of making a more realistic assessment of the students and goes some way to suggesting how, in context, an improvement in students' performance may be effected.

In summary, therefore, the procedures are developed to produce cognitive change in particular contexts. They demonstrate that a change in the manifest performance levels of physics students is possible using the methods. Permanent cognitive change is, of course, sought, but this will undoubtedly need considerably wider application of the principles enunciated.

8.2.3.2 The Role of Algorithms

To enable instructional material to be developed which could compensate for the lack of planning ability demonstrated by the UWC students, the manner in which Newton's laws and related concepts are applied to the solution of problems was carefully analysed. Further materials were developed to address the difficulties students experience with accurate data analysis. It was determined that despite any heuristic approaches which may be used to deal with a given problem, ultimately the problem is solved (once a definite concept or concepts have been determined as applicable) by using a discrete sequence of steps which can be formulated as an algorithm. Of course, the idea of simply giving an algorithm to a student to enable him/her to solve a problem, is not tenable, since it would perpetuate rote-learning. The view taken of algorithms in this thesis, however, is as follows:

i) An algorithm is intrinsic to normal problem solving, i.e. in using a concept it is of fundamental importance that certain features thereof be recognized, normally in some particular sequence. Without such recognition it is usually impossible to use the concept correctly.

ii) The algorithm, by highlighting details of the concepts which are both explicit and implicit and hence elaborating its content, or by providing some order in data analysis, assists toward a better understanding of the concept and thus to enhanced problem-solving ability.

iii) In no way is it implied that the algorithm provides students with a prescription for solving problems. Rather the stepwise development of the intrinsic algorithm is an integral part of the presentation of the concept.

> It supplies that component of the concept which makes it possible to use it, especially in the solution of problems.

Two types of algorithms are developed in this thesis:

A. Elaborational algorithms: these enable compensatory materials to be developed especially to address the deficiencies in planning in the students.

B. Input algorithms: these were developed to enable appropriate data collection and interpretation to be carried out.

There is obviously a degree of overlap between the two types. Again it must be stressed that the compensatory materials do not necessarily develop specific cognitive abilities in students so that these now become usable cognitive functions in other areas. All that was done was to analyse the areas of the syllabus to determine the cognitive abilities required to cope with these areas and then, because these abilities may be deficient or lacking in students, to supply compensatory materials designed in such a way that the cognitive requirements of the syllabus are met in such materials.

8.3 Further Directions For Research

Physics Education research over the past few years has just begun to be concerned with the role of the processes of thought on the way in which students learn physics. It is a field with enormous research possibilities. In the light of the research in this thesis the following may be suggested:

A. The cognitive functions which students use or fail to use, markedly influence their understanding of concepts, laws, principles etc. A number of research directions exist here.

i) The influence of language on cognition is a field in which much work has been done, but very little of it in physics. It would be interesting to see how the vocabulary of physics influences the cognitive processes which students use.

ii) No large-scale determination of error factors (as opposed to the more specific misconceptions) seems to have been done in physics. This, too, could prove to be a very rewarding study.

iii) It is apparent that more cognitive deficiencies than those already documented may exist in students. It would be very useful to determine these.

In addition, the reasons for the existence of some of the deficiencies noted, need investigation. Thus, for example, while it is important to know that students find great difficulty in visualizing a particular situation or in relating two sources of information, a very meaningful study could be to determine the (cognitive) reasons for those inabilities.

B. As demonstrated in this research, the analysis of the cognitive difficulties of students is only one aspect of the instructional package for disadvantaged students. In addition, the content-area which is taught needs to carefully analysed for its inherent cognitive demand, i.e. what cognitive functions are required for dealing with that particular area of content. Once a match is made between students and curriculum, then suitable compensatory material can be developed.

C. The entire question of the transfer of cognitive functions from one content-area to another, needs to be carefully examined. To what extent is it possible to remedy cognitive deficiencies in physics so effectively that these cognitive operations become part of the mental structure of the individual and the acquired cognitive function is usable in other areas of life?

8.4 Conclusion

This research, conducted over some five years, has shown that great care needs to be taken when instructing disadvantaged students. It is insufficient simply to have extensive knowledge of the subject we teach. It is as vital to have an insight into the very fundamental learning problems of the recipients of the instruction. And yet this care, if properly exercised, may well produce persons who have not only learnt, but have learnt how to learn, persons who not only seem to be educated but who are also in fact, educable.

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APPENDIX A : TRANSCRIPTION AND ANALYSIS OF PERSON-TO-PERSON INTERVIEWS

The interviews documented in this section are those discussed in Chapter 4. Note: Throughout these transcripts "I" stands for interviewer and "S" for student.

The three problems given are as follows:

1. At the instant the traffic light turns green, a car starts from rest with a constant acceleration of 2 m s^{-2}. At the same instant a lorry, travelling with a constant speed of 10 m s^{-1} passes the car.
   a) How far beyond the starting point will the car overtake the lorry?
   b) How fast will the car be travelling at this instant?
2. A lift, height 3 m, accelerates upward at 2 m s^{-2}. At the instant that its velocity is 2 m s^{-1}, a screw falls from the roof of the lift. Calculate:
   a) how long the screw will take to reach the floor of the lift
   b) what distance it has then fallen.
3. At the instant car A pulls away from a traffic light with acceleration 3 m s^{-2}, car B is 50 m from the traffic light travelling in the same direction as A but with speed 15 m s^{-1} and acceleration 1 m s^{-2}. Calculate how far from the traffic light B draws level with A and the velocity of each at this point.

INTERVIEW 1

No Physics but he has Applied Maths I

PROBLEM 1

He reads the problem.

I : What is the problem about?

S : Velocity and acceleration.

I : Please extract the data from the problem.

S : Acceleration of car = 2 m s^{-2} (at time t = 0)

Speed of lorry = 10 m s^{-1} (at time t = 0)

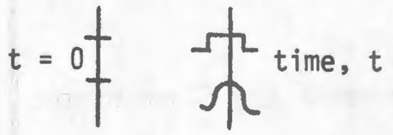
The car and lorry will have equal time = t

I : How do you know this?

S : They ask when they are equal. I can now use two methods.

I : What about distance?

S : They will have moved the same distance



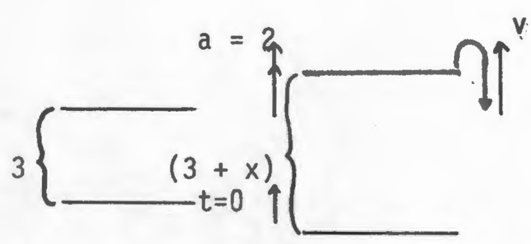
I have done something similar in applied mathematics

(He is obviously confident because he has done something similar. He can in fact relate the analogous type of problem which he has done).

PROBLEM 2

I : Would you like to give an interpretation of the problem?

S : (He draws a sketch):



The screw falls out when the velocity of the lift is 2 m s^{-1}

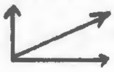
Also a force on the screw mass x g

It seems necessary to stop the one object.

I : How does the screw move?

S : Faster and faster downward.

I : Does the screw move upward at all?

S : (Thinks) (Draws)  screw (He refers to his sketch and insists that the motion is parabolic)

I : Where does the screw get its horizontal component?

(He stands for a while trying to visualize the problem as he looks at the sketch).

S : No, I have misinterpreted the problem.

(After much discussion he appears to see that the screw will first move upward before dropping to the floor).

I : What is the acceleration of the lift?

S : 2 m s^{-1}

I : What about the screw?

S : (Only after discussion does he say) : downward, g.

I : Is there any more data?

S : No.

I : What about the time?

S : Same for screw and lift.

I : Is the distance the same for both?

S : No, because the screw moves horizontally (he appears to return to his original assumption of parabolic motion).

ANALYSIS OF INTERVIEW 1

Error Factors

1) Does not see that initial velocity of screw is that of the lift.
2) Does not easily see that the acceleration of freefalling screw is g - downward.

Semantic

1) Screw "falls" - his insistence on parabolic motion.

Cognitive Deficiencies

1) Difficulties visualizing an unfamiliar problem.
2) Does not attempt any qualitative evaluation before extracting data i.e.data mean only numerical facts.
3) He is not clear in his mind about the principle to be used - e.g.talks of the force on the screw m x g, also about stopping the one object i.e.relative velocity. Does not clearly define the problem. Later introduces parabolic motion.
4) Treats the bodies separately and does not easily relate the data of both. He does not easily acknowledge implicit data e.g.in 1 he easily sees that s and t are the same. It is so obvious to him that it is hardly worth mentioning. However, in 2 it is clear that no such relationship suggests itself and he ignores it i.e.he does not easily externalize what he has internalized - he does not see what is relevant.
5) While he sees the need to draw a sketch, the quality thereof is extremely poor and reflects the haphazard and incorrect way in which he collects and orders data e.g.it is not clear

whether he is drawing the lift or the screw and how these are related.

INTERVIEW 2

The student is enrolled in physics, mathematics and chemistry. He says that he is not doing too well.

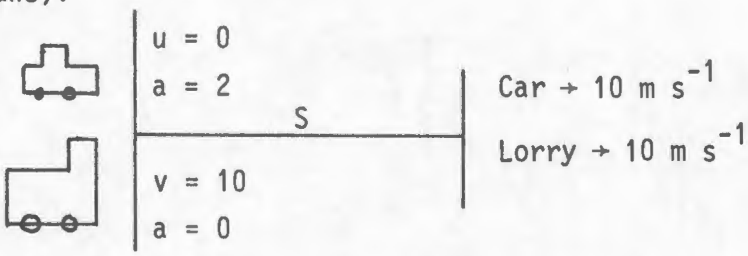
PROBLEM 1

I : Do you recognize the problem?

S : Yes, it is an acceleration problem. I have done quite a number of these.

I : Would you write down all the data in the question. Draw a sketch if you like.

S : (He draws):



The car starts from zero. At a certain point both the car and the lorry are together i.e.they have the same speed. Both have a speed of 10 m s^{-1}.

I : Why do they have the same speed when they are together?

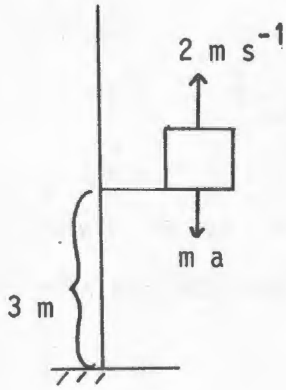
S : The car is accelerating so it will reach the lorry and then they must have the same speed.

(After some time, when it is clear that he will not proceed further on his own, the problem situation is discussed. He finally sees that the time required in the problem is the same for both vehicles).

PROBLEM 2

He reads the problem and then draws:

S :



The lift is at a height of 3 m

The screw falls when the lift has speed 2 m s^{-1}

I will try to find the height at which the lift is when the screw falls.

(As he again reads the problem)

I haven't been given the mass of the lift. I'm trying to represent the downward force on the lift (which he indicates is m a)

I : What will you do now?

S : (He writes) : $v = u + at$; $v^2 = u^2 + 2as$; $s = ut + \frac{1}{2}at^2$

I am trying to find a suitable formula to use for the height of the lift when it is travelling at 2 m s^{-1}.

Since the lift is travelling upward, the upward force is greater than the downward. (He reads the problem again)

But I should not use anything with force because I am not given the mass and therefore it will not help me.

I : Where does the screw fit into the problem?

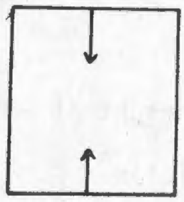
I : I am trying to find the distance of the lift when the screw falls out. Then I find how long the screw will fall to the floor.

I : Where is the floor?

S : I am speaking of the bottom as the floor. (After discussing this). The problem must be from the top of the lift to the floor of the lift. So the lift must be going to the screw and the screw to the floor of the lift.

(Draws)

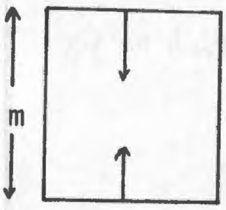
A8



Screw with 10 m s^{-2}

Taking the distance as 3m is wrong

2 m s^{-2} It is actually from the top of the lift to the bottom.



3 m

10 m s^{-2} $v^2 = u^2 + 2\ a\ s$

$v^2 = 2 \times 10 \times 3$

2 m s^{-1} $v^2 = 60$

(Even after discussion he has difficulty in comprehending that the initial velocity of the screw is 2 m s^{-1}, upward).

ANALYSIS OF INTERVIEW 2

Error Factors

1) He confuses same positions and same velocity by assuming that when the car draws level with the lorry they must have the same speed.
2) The nature of the problem is confused. In problem 2 he tries to introduce the idea of a downward force on the lift by "m a".
3) The concept of force as it relates to motion (e.g. "Since the lift is travelling upward, the upward force must be greater than the downward") is not understood.

Semantic

1) He misinterprets the word "height" in problem 2. He takes it to be the height the lift is above the ground.
2) In harmony with (1) he interprets the "floor" as being the ground below the lift. Only after some discussion does he see what is actually implied.

Cognitive Deficiencies

1) The student demonstrates only blurred and sweeping perception as he attempts to analyze the problem. No effort is made to focus on details except on the numerical data. It is apparent how this deficiency prevents any further elaboration of the problem in a meaningful way since his next step is to experiment with equations in the hope that an answer will materialize. This approach is repeated in problem 2.
2) The student displays a large degree of impulsivity which is

revealed by a very poor idea of the problem to be solved which in its turn makes his examination of the data extremely unsystematic. He does not really understand what he is supposed to find, nor does he establish interim goals in his search for a solution (Problem 2).

3) He examines the objects in problem 2 entirely on their own and makes no attempt to relate the movements of the bodies. There is no spontaneity in his use of the two sources of information. Throughout he must be prompted to do so.

4) It is clear that he also lacks verbal skills as itemized under "semantic". It is interesting how this lack impedes his ability to elaborate the problem effectively and indeed to gather data meaningfully.

5) It appears that he will visualize the problem only as far as is required to obtain data to insert into the relevant equations. Even after demonstrating that he understood the way in which the screw and lift moved, he would not analyze the physical situation in depth but was satisfied with only the numerical data. He is not concerned with precision or accuracy in the way in which he gathers implicit data.

INTERVIEW 3

He reads the problem and immediately writes down the data.

$a_c = 2\ m\ s^{-2}$. Speed of lorry : $V_\ell = 10\ m\ s^{-1}$

S : Now I am going to draw my formulas :

$V = V_o + at$

$S = V_o t + \frac{1}{2} at^2$

$V^2 = V_o^2 + 2\ a\ s$ I am going to try to use this one.

(He tries to fit values into the equation unsuccessfully).

I : What is the problem all about? Could you try to describe it?

S : (Gives a summary of the problem).

I : When does the lorry pass the car?

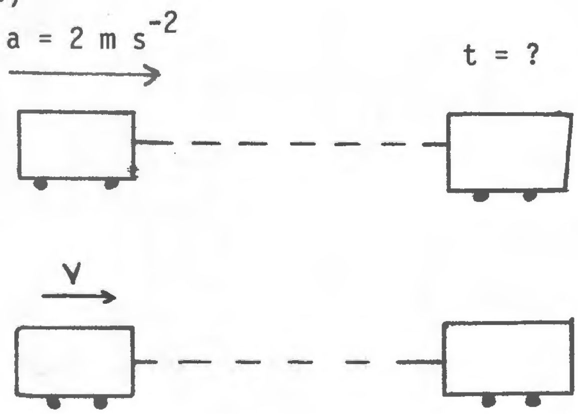
S : At the robot.

I : Does the car catch up with lorry?

S : After some time it does.

I : Why not draw a sketch of how you see the beginning and the end.

S : (Draws)



I : Would you like to try and explain what you've done on the sketch there.

S : Here are the traffic lights. And here the time is zero. Here to a certain extent this is the car. Then after some time the car passes the lorry.

I : You say after some time. What is the time? Is it the same for the two or is it different?

S : The time is the same.

I : For the car and the lorry?

S : Yes.

I : And the distance?

S : Distance is the same too.

I : The speed when they pass each other, is that the same?

S : No.

I : How do you know that?

S : Since the lorry passed the car when the car was stationary and the lorry carried on with this speed.

I : OK now, on that side, on the right?

S : It's .... they are not the same.

I : Are you sure of that?

S : Yes, the speeds are different.

I : What would you do further now?

S : I think I will find the time, the time the lorry takes for this distance.

I : How long would that be?

S : Need to work it out.

I : I mean, how would you work that out?

S : I'll use this (writes down $S = ut + \frac{1}{2} at^2$)

I : What formula applies to the lorry, with constant speed?

S : $S = vt$

I : Yes. Then what formula would you use for the car?

S : $S = ut + \frac{1}{2} at^2$

I : OK use it then.

S : (Does calculation on the board):

$s = \frac{1}{2}(2)\ t^2$ 1) for car

$s = 10t$ 2) for lorry

I : That's fine. So now you just relate the two and then you'll have the time. S = 10t and is also equal to t squared. Now you've got that:

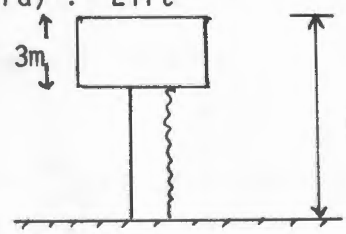
$$10t - t^2 = 0$$

$$t\ (10 - t) = 0$$

So : t is 10s

Would you like to read Problem 2, about the lift. Would you again go to the board and explain what the data is in the problem. Tell us what you're doing as you do it.

S : (Draws on the board) : Lift



$v = 2\ m\ s^{-1}$ (lift)

$a = 2\ m\ s^{-2}$

H = Height from roof to floor

I : So tell us what is going on.

S : This is the height.

I : The height of what?

S : From the floor to the roof. It is the height of the lift.

I : OK. So this 3m, what is this 3m here?

S : It is the height of the lift. At a certain height, this height, the screw falls. A screw falls from the roof.

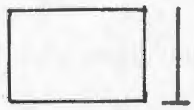
I : What is that now? H?

S : H : this height from here. The velocity of the lift is $2\ m\ s^{-1}$ and its acceleration is also $2\ m\ s^{-2}$.

I : Would you explain the big H?

S : The screw is falling from the roof of the lift to the floor (then sees that it is the floor of the lift, not the ground).

I : So which floor?

S : Of the lift (he draws :  and erases other sketch)

I : Yes, that's it, the floor of the lift.

S : Then the velocity of the screw is also $2\ m\ s^{-1}$ and its acceleration is also $2\ m\ s^{-2}$.

S : Yes. So the velocity of the screw is $2\ m\ s^{-1}$. What about its acceleration?

S : Is also $2\ m\ s^{-2}$.

I : How do you get that?

S : Since the lift is going at acceleration of $2\ m\ s^{-2}$.

I : Even if it falls out of the roof it still has an acceleration of $2\ m\ s^{-2}$?

S : When it is falling it has an acceleration of $10\ m\ s^{-2}$.

I : Why $10\ m\ s^{-2}$?

S : If the screw falls it has that acceleration.

I : So that is actually g. Is that up or down?

S : It is down.

I : Anything further?

S : Need to find the velocity of the screw when it is falling downward.

I : Do that then.

S : (Writes on the board) $S = V_{ot} + \frac{1}{2} a t^2 \ (V^2 = V_0^2 + 2 a s)$

$$v_0 = 0$$

I : Why is $v_0 = 0$?

S : He hesitates. (This is discussed and finally he sees that it is not).

I : So what is the initial velocity?

S : It is 2 m s^{-1}.

I : Oh, why?

S : It is in the lift and the lift goes up at 2 m s^{-1}

I : What is the direction of the velocity?

S : It is in the opposite direction to falling.
(Writes) : $v^2 = (-2)^2 + 2(10)(3)$
Acceleration is 10.

I : Where do you get the 3 from?

S : (Points to the sketch). It is the distance from here to here. (Shows the length of the lift).

I : Isn't the lift moving while that falls out?

S : It is certainly constant at that instant.

I : At what instant?

S : When the velocity is 2 m s^{-1}.

I : OK, but what does that mean? Does the lift stop moving while this happens? Doesn't the lift go on moving up while the screw moves down?

S : When it starts falling, the velocity isn't zero.

I : When it starts falling what happens?

S : The velocity tends to go up.

I : What happens to the lift while the screw is falling?

S : It rides up.

I : So where will the floor of the lift and the screw meet?

S : When the lift moves a distance x, say here. (Points at sketch).

I : So the lift will have travelled up the distance x.

S : (Writes) $v^2 = (-2)^2 + 2(10)(x) = 24x$.

I : OK. Perhaps I can just explain it again. You said at that point the screw falls out. So what does the screw do? The screw first goes up and then it comes down.

(The interview is concluded after further efforts to enable the student to visualize the problem).

ANALYSIS OF INTERVIEW 3

Error Factors

1) Does not see that velocity of lift and screw are the same. Even after discussion he returns to the idea of $v_0 = 0$.
2) The accelerations of the lift and screw are confused. It appears that the notion of free-fall is not well established.

Semantic

1) The floor in the case of the lift problem is interpreted as the ground, not as the floor of the lift.

Cognitive Deficiencies

1. Does not appear to relate the data to what is required in the problem i.e.he makes no goal analysis but simply puts numerical data into formulae.
2. Makes no qualitative evaluation of the problems. Extracts numerical data only for use in a formula.
3. Implicit data is only seen to be relevant after prompting.
4. Will not try a different approach unless prompted to do so. Tends to play around with numerical data aimlessly. No spontaneous reevaluation when stuck.
5. It appears that spontaneous sketches can be categorized very much as the simple recording of numerical data - the sketches show no real visualization or even interpretation of the problem.
6. In the lift problem, the entire focus is on the screw. There seems to be very real problems w.r.t.appreciating the relevance of the other body and establishing a relationship between the variables of each.

INTERVIEW 4

I : Would you like to read through the problem in italics. Tell us how you will set about solving it.

S : Yes, this is one of the problems which many students couldn't solve actually.

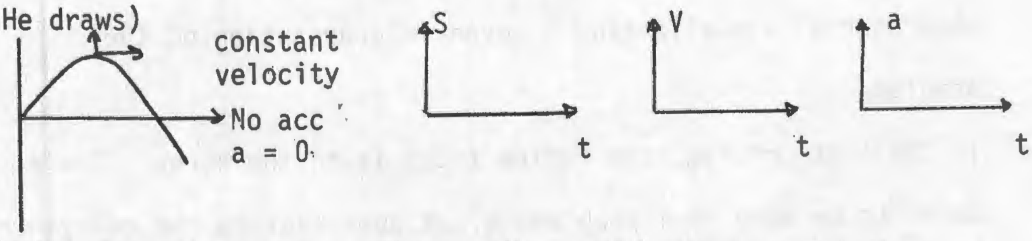
I : How would you interpret the data in the problem?

S : At the instant the traffic light turns green a car starts with a constant acceleration. At the same instant a lorry, travelling with a constant speed of 10 m s^{-1} passes the car. Firstly, I think I did the sum before, so I'll work it out and try to get the same answer as this.

I : OK.

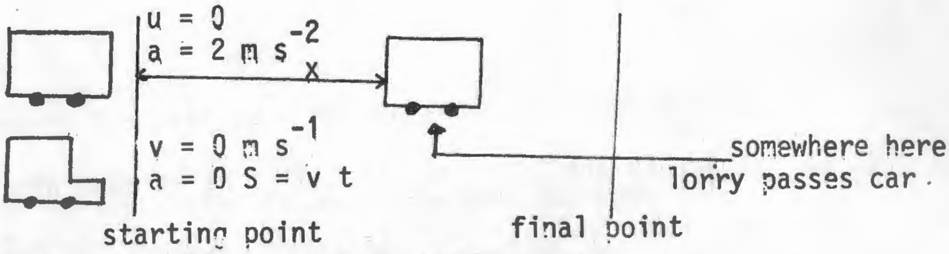
S : The traffic light turns green and the car starts with a constant acceleration. Here this relationship you told us before, I remember that still from last year. If you have constant velocity, there is no acceleration, as with this here and here's again a change in acceleration in the y-direction I'm putting up here in the x-direction. So this is a guess here. Like here if you have constant velocity the acceleration is zero. I'm not sure about this, so I thought the velocity should be zero or something.

(He draws)



I : Just before those graphs there. How could you draw out the problem and indicate all the data?

S : (He draws)



I'll draw the car here, and say a line like that to represent the starting point of the equation. That there is the starting point and here somewhere is the final point. It has a constant acceleration so the velocity is zero. So initial velocity is zero.

I : Which is that now?

S : The car I'm speaking about. Then the acceleration is 2 metres per second.

I : 2 metres per second squared you mean.

S : At some instant a lorry travelling with a constant speed of 10 metres per second passes the car. So here the lorry passes the car and then I need the question: How far beyond the starting point will the car overtake the lorry and how fast will the car be travelling then. Oh, and I thought it was the same instant. It was at the same instant. Here we have the lorry here and it has a constant speed of 10 metres per second so the velocity of the lorry is 10 metres per second and it has a constant speed or velocity. The acceleration is zero and what I interpret this to be here is using the formula : the distance equals velocity times time. Then the car problem uses one of the three equations. I'm not sure what.

I : What about the end point?

S : Oh. How far beyond the starting point does the car overtake the lorry? So then the car must have a greater velocity to overtake the lorry and that will be at some distance. We can call that s. And after say a time t, if that t is required in that equation.

I : OK. What about the time that they overtake? Is there any relationship between the time of the lorry and the time of the car?

S : Yes. That will occur after the same time.

I : Why is that?

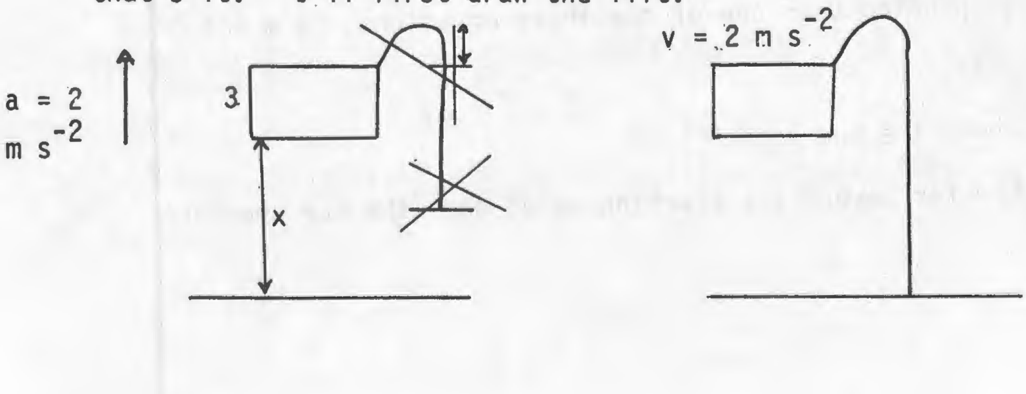
S : Oh here they left (points to previous sketch). They started at the same time and then after a distance s and with different velocities the one overtook the other. So the time must be fixed for both and I could perhaps determine that from one of the three equations here. Use this one. Say that distance one. (He writes down the three equations).

I : Very good.

S : Must I still solve this?

I : No it's right, I think you've got it. Will you look at the second problem now. You read the second one through and then see if you can see what the problem is all about and what you need to solve it.

S : We had one about the screw falling from the roof. Oh yes, that's it. I'll first draw the lift.



What is important is to get all the data from the page. The height of the lift is 3 metres. It accelerates upwards with 2 m s^{-2} when the velocity is 2 m s^{-1}. It wasn't this one.

I think it was the sandbag one. We had to calculate the time and what bothered us was how it came down. We were interested in this time here whether that time had to be calculated or just assumed. How long will the screw take to reach the floor when the screw falls from the roof.

See here, I think its the same here. You could say that 2 m s^{-1} is the velocity of the screw, and then it comes down. (He points to sketch on the right above). I'm not sure about this. Can I work this out quickly?

I : Sure. So what's the problem? Can you see the problem there?

S : I'm not sure of this distance here. (He points to distance marked x).

I : What is that distance?

S : That distance?

I : Yes.

S : I'm not sure.

I : I know you're not sure. What is that? Where does that distance come into the problem?

S : Oh, this is from the floor. Called that distance x. But I wonder if I should use the parabolic motion to solve it or the one of Newton's Second Law. I'm not sure about that. Mainly because of this dip here I mean.

I : Mainly because of what? What did you say?

S : I'm not sure of how the screw falls. The lift is accelerating upwards. The thing won't just fall down just like that (he draws : $\downarrow$) because it must fall something like this. (He draws). If I had gone over the physics problems, I would know which formula to use to solve it.

I : Tell me why does the screw fall like you say there?

S : This here? (Points to parabolic motion).

I : Does it actually fall in a loop like that?

S : It actually falls like that, like the parabolic motion because it's not stationary, not just dropping down. It is actually moving up. The screw won't just drop down. It will first move up with this lift's velocity and then come down.

I : OK.

S : Say something like that; so this is up, (he indicates on sketch on left, above), then on top there the velocity is zero. When it comes down again it has the same velocity here (points opposite starting point) but in the opposite direction so the velocity is minus 2 metres per second when it comes down. Yes that's right.

I : So?

S : Some formula is applicable to find the time it takes to drop, and I'm not sure about the distance.

I : Why do you say the floor is at the bottom? Isn't it really the floor of the lift that is important?

S : The lift?

I : Yes. If the screw falls out of the roof of the lift, where is it going to land?

S : No, I assumed it was going to fall from here, from the lift. I did a problem like this.

I : Don't you think that it would be more reasonable that it would fall onto the floor of the lift?

S : The final distance was x. I thought the distance of the lift was given and the distance it falls you subtract from that distance. No, that is also something else. So you just get this distance, x.

I : I see. Let's say the screw falls out of the roof of the lift and it falls down onto the floor of the lift. How would the problem look then?

S : Say there was some mechanism to let it go down.

I : Would there be any horizontal movement?

S : Horizontal movement? No.

I : You see, you said here that it doesn't fall straight up and down, it falls with a loop. Now where did it get the horizontal velocity from if it falls with a loop?

S : Well initially it couldn't, it doesn't have any velocity there, but after that time, yes initially there is no velocity in the x-direction, I mean the y-direction. But it falls from the roof. Otherwise it is going straight up. No. I think it does not have a velocity in y-direction, otherwise then it goes up and it comes down, but if you look at it from the side it falls - something like that. (Again points to his sketch of parabolic motion).

ANALYSIS OF INTERVIEW 4

Error Factors

1) The student experiences great difficulty in identifying one-dimensional motion. In both problems he endeavours to introduce a horizontal movement. In problem 1 he draws graphs indicating an x-direction, while in problem 2 he introduces a parabolic movement of the screw.
2) A familiarity with similar problems (e.g. a sandbag dropping from a rising balloon) leads to an incorrect interpretation of problem 2.
3) Problem identification proves difficult. This can be regarded as a molar difficulty on one level as well as implying a cognitive deficiency in that data analysis does not bring recognition of the parameters of the problem.

Semantic Difficulties

1) The "floor" is interpreted as the ground.
2) "Falls" : parabolic motion is assigned to the screw. He sees that motion is not just downward, but cannot relate the initial velocity to an upward motion.

Cognitive Difficulties

1) Does not easily identify the type of problem e.g. "I wonder if I should use parabolic motion to solve it or the one of Newton's Second Law."
2) While he attempts some analysis of the problem, he is keen to find the necessary "formula".
3) At one stage he makes an accurate analysis of the motion of

the screw, but his initial drawing of a parabolic motion misleads him. It seems that a combination of the drawing made and the interpretation of the "floor" (as in the sandbag problem) influences the direction in which he thinks. Thus he appears to make an impulsive initial judgment and then relates everything to this premise without reevaluating its accuracy. Although the student appears to identify the important terminal points in problems 1 and 2 (even though incorrectly in the latter) he seems to lack the ability to reflect on what he has done to examine its correctness.

4) He focuses entirely on the motion of the screw without relating it to the relative movement of the lift.
5) There is no orderly progression of thought in developing the problem. It is never clear what the student is attempting to determine at any point.
6) There is a marked difference in the sketches of problems 1 and 2. While 1 shows a perception of the situation, the sketch of problem 2 merely gives some numerical data without any clear visualization of the movements of the two bodies.
7) There is no order in the manner of data analysis. Ideas in the problem are not carefully analysed for their meaning.

INTERVIEW 5

I : Please read through the first problem. Try to extract the data from problem. In other words how would you categorize the problem, what are the data which are there, data which are stated and implied. Please stand at the board and write down and explain what you are doing.

S : Well Physics is not one of my best subjects. (Writes). Car not starting from rest, acceleration of 2 m s^{-1}; the lorry has a velocity of 10 m s^{-1}

Car	Lorry
$a = 2 \text{ m s}^{-2}$	$v_0 = 10 \text{ m s}^{-1}$
$v_0 = 0 \text{ m s}^{-1}$	$s = v\,t$

So .... s = ut. So ....

I : You say car is not starting from rest but you write down Vo = 0. So is car starting from rest?

S : Yes I think it is, the car is starting.

(Writes) $v = u + at$

$$v^2 = v_0^2 + 2\,a\,s \;; \quad v^2 = 2\,a\,s \quad v = \sqrt{2\,a\,s}$$

I : So you have $v^2 = 2\,a\,s$. What is that? What does it apply to?

S : I'm trying to find an equation where I could put S in here and in here s = vt for lorry, so that I can equate the two.

I : Are the two S's equal?

S : They have to be if they are to overtake.

I : How do you see this problem? What is it all about?

S : I see the car starting at the robot. The truck is coming from behind and it passes the car, but the car is continually picking up speed at 2 m s^{-2}, so obviously it has to pass the truck which is going at a constant speed.

I : You didn't think you need a sketch to picture it better? What are you trying to do now? (She writes $S = \frac{1}{2}\ a\ t^2$)

S : I'm trying to find t, but I can't because I have two unknowns. (Focuses entirely on one object).

t will be the same.

I : How do you mean, will be the same?

S : Time at which they overtake each other.

I : Why would you say that time will be the same?

S : No it won't be the same.

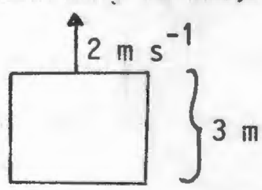
(Then writes) : $vt = \frac{1}{2}\ at^2$, then $v = \frac{1}{2}\ a$. No!

then $v = \frac{1}{2}\ a\ t \quad \therefore \quad 10s = t$

I : Would you read the second problem please. (She starts by immediately drawing a sketch and recording values):

$a = 2\ m\ s^{-2}$

$v = 2\ m\ s^{-2}$



She writes : $F - mg = m\ a$

I : Would you explain what you are doing? Do you recognize this kind of problem? What is it about? What part of the work is it about? Is this Newton's law? Have you come across anything like this?

S : Yes, the type of problem where you must calculate the tension in the cable. But this has no relevance.

I : Can you explain how do you see the problem?

S : Yes, well the lift is accelerating upward at $2\ m\ s^{-2}$. The distance is 3 m. That is the distance of the lift.

I : What distance would that be?

S : The height S. This is the distance that the screw has to fall. Obviously the screw is also travelling with this acceleration because it is in the lift.

(Writes) $S = V_o t + at^2$

I : Are you applying these equations to the lift or the screw?

S : To the screw.

I : What is the screw doing as you see it?

S : The screw is at rest at the moment.

I : Why is the screw at rest?

S : Doesn't it start from rest?

I : What do you think? Its part of the lift.

S : Then it must be moving at 2 m s^{-1}.

I : If you were to draw how the screw moves, how would you do it? Say there the lift goes while you stand on the ground at the side and can see into the side of the lift, how would the screw travel?

S : As it falls it hits bottom here. The floor of the lift. Final velocity is 0. (Writes) $3 = 2t + t^2$.

I : What is the 3?

S : Height through which it falls.

I : And the 2?

S : Its velocity it has in the lift. (Writes) : $t^2 + 2t - 3 = 0$

$(t + 3)(t - 1) = 0 \quad \therefore \quad t = 1s$

(Compares with answer on sheet)

I don't get the right answer - I must be wrong.

I : If you were to explain the situation to somebody how would you do it?

S : Lift going up, screw in lift, stuck to lift and has the same velocity as the lift obviously, lift has acceleration and when screw falls it has to fall a distance of 3 m.

I : Think about that distance 3 m .....

Only after much discussion does she see that the distance moved by the screw is not 3 m.

Further discussion follows on the meaning of the word "falls" in relation to the initial motion of the screw as upward. Student again seems to be misled by the word "falls". Only after considerable discussion does she appear to see that the motion of the screw is first upward and that the distance travelled is -(3-x). She also has difficulty in perceiving that the acceleration of the screw is "g" with a negative sign.

ANALYSIS OF INTERVIEW 5

Error Factors

1) Has difficulty in recognizing the type of problem in No. 2. Lifts evoke the idea of tension in cables as applied in Newton's second law.
2) Does not relate the velocity of the screw to that of the lift, but assigns the acceleration of the lift to the screw, and uses 2 m s^{-2} as the acceleration of the screw in an equation. She clearly has little perception of freefall.
3) She pays no attention to signs for variables. Thus she writes the equation for the screw : $3 = 2t + t^2$, after she explains that the 2 m s^{-1} is the upward velocity of the screw while the 3 m is the distance through which the screw falls.
4) Assigns a final velocity of zero to the screw - "it hits bottom here".

Semantic Difficulties

1) The word "falls" produces the conception of a motion entirely downward.

Cognitive Difficulties

1) No qualitative evaluation of the problem. Only numerical data is extracted from the problem and suitable equations are then sought wherein the numerical values can be placed.
2) Even after prompting, little effort is made to visualize the situation. Experimenting with equations takes place constantly until the correct combination is found for a solution more-or-less by trial-and-error. Within seconds she says both that

t will be the same for car and truck and that it will be different. However this in no way influences the fact that she uses "t" for both vehicles in problem 1.

3) Focuses on one body at a time. While she establishes the relationships in problem 1 ("S's are equal"), she is completely unable to do so in problem 2. Even with problem 1, she tends to use the equation relating to the car ($s = \frac{1}{2} a t^2$) to find t. She has great difficulty in understanding that the distance through which the screw moves, is not 3 m.
4) Related to point (2) above, she seems to have no planned approach to the problems. The answer comes as a surprise in question (1). The equations evoke no structured thought pattern as for example, which equation is applicable for a specific situation. All the equations are tried in an attempt to get the answer to "fall out".
5) The lack of clarity of detail is seen by the meaningless sketch drawn.
6) Lack of accuracy in interpreting the variables in the equation is shown by the use of the height of the lift, 3 m, as the distance S. She says "That is the distance of the lift".

INTERVIEW 6

Problem 1

He reads the problem and then writes down:

Car : $a = ms^{-2}$; $v_o = 0$ Lorry : $v = 10\ ms^{-1}$

He says that this is the only data in the problem.

When prompted, he says that the car and lorry would have travelled the same distance when they are alongside each other.

He makes no effort to write down distance and time explicitly, but it is clear that he realizes this implicitly. He writes down:

Car : $s = u\,t + \frac{1}{2}\,a\,t^2$

Lorry: $s = v\,t$

He is stuck for a while as he compares equations because of writing them as:

$$0 + \tfrac{1}{2}\,t = 10t$$

He erases this and then writes : $0 + t^2 = 10t \quad \therefore\ 10\ s = t$

Throughout, he makes no attempt to visualize the problem or to draw a sketch.

Problem 2

He spends a reasonable amount of time reading the problem. As he starts to write down the data he is asked to explain what he is thinking: "The distance that the screw will fall is 3 m".

"The screw has the same initial velocity but will fall downward under acceleration".

He writes down:

Lift : $s = 3\ m, v_o = 2\ m\ s^{-1}$, $a = 2\ m\ s^{-2}$

Screw: $g = 10\ m\ s^{-2}$, $v_o = 2\ m\ s^{-1}$

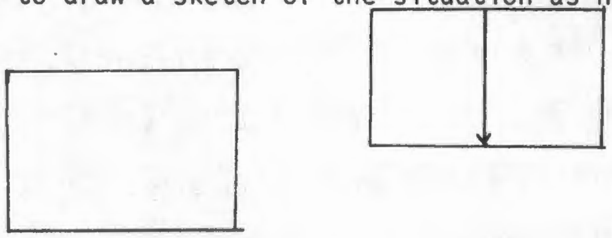
He does not refer to signs for any of the quantities. He says: "Time for the screw to move downward is the same as the time for the floor to move upward toward the screw. It is thus possible to calculate the time".

He calculates: $s = v_0 t + \frac{1}{2} a t^2$

$3 = 2t + \frac{1}{2} 2 t^2$ $t^2 + 2t - 3 = 0$

$(t + 3)(t - 1) = 0$ $\therefore t = 1$ or -3

He is asked to draw a sketch of the situation as he sees it:



He says: "The lift is 3 m and the screw will also fall 3 m."

ANALYSIS OF INTERVIEW 6

Cognitive Difficulties

1) In the first problem he sees implicitly the relationship between t and s for two bodies. He sees no need to explicate it. Thus in the second problem where t is the same for both objects but s has to be related, he seems to be unable to find the relationship, despite prompting.
2) He misreads the data because of misrepresenting the problem (that is, s = 3m). It appears that he is able to visualize the first problem without much difficulty, but is not able to do so with the second.
3) No effort is made to give a physical representation of the problem either with a sketch or by articulating the various features in a qualitative manner.

Semantic

1) When asked to draw a sketch in the lift problem, he shows the the screw falling downward only. This seems to represent either some misinterpretation of experience or difficulties with the word "falls".

Error Factors

1) No effort is made to choose axes and hence signs are not attributed to quantities such as g, a, v_0 in problem 2.

INTERVIEW 7

Problem 1

He reads the problem and immediately writes down:

Car : $a = 2 \text{ m s}^{-2}$ $v_o = 0$ Lorry : $a = 0$; $v = 10 \text{ m s}^{-1}$

He is asked for the significance of the expression "at the instant" : "It is the point in time when everything starts to happen".

When asked to draw a sketch:

0 ⟶ x Point where car catches up with lorry

When asked to solve the problem:

Car : $s = v_o t + \frac{1}{2} a t^2$ Lorry : $x = 10 t$

$x = 0 + t^2$

He then solves it. He never articulates the fact that s and t are the same for both, but seems to see it implicitly.

Problem 3

He reads the problem and immediately writes down:

Car A: $a = 3 \text{ m s}^{-2}$; $v_o = 0$

Car B: $a = 1 \text{ m s}^{-2}$; $v_o = 15 \text{ m s}^{-1}$

He makes no distinction between the symbols for the different cars.

He is asked to draw a sketch of the situation:

A —— 50m —— B —— x A catches up with B.

He has the physical situation incorrect because of not reading the problem correctly. He seems to focus only on the first section of the problem and places the cars in the order in which they appear in the problem. The problem is then solved correctly:

$$A: s = v_0 t + \tfrac{1}{2} a t^2 \quad B: x = 15 t + \tfrac{1}{2}$$
$$50 + x = \tfrac{1}{2} 3 t^2$$

t is seen to be the same and the distances are correctly related.

Problem 2

He reads the problem and then writes:

Lift: $a = 2\ m\ s^{-2}$; $v = 2\ m\ s^{-1}$ Screw: $g = 10\ m\ s^{-2}$;

$v_0 = 0$

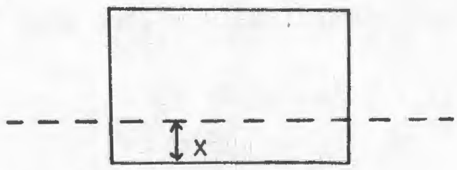
No attempt is made to visualize the problem or to verbalize it. The statement of the problem is simply read and figures are written down.

There is also no effort to insert signs where necessary.

No indication is given whether "v" for the lift is the initial or the final velocity.

He is now asked to solve the problem:

He draws a sketch:



He says: "Assume that the lift moves upward a distance x before the screw strikes the floor, that is, the screw falls a distance (3 - x)".

He then writes : $s = v_0 t + \tfrac{1}{2} a t^2$

Screw : $(3 - x) = 0 - 5 t^2$

(Note that a sign is inserted for g different from that written above. However, the initial velocity of the screw is still taken to be zero). At this point he was unable to proceed further. He could not decide whether the $2\ m\ s^{-1}$ was the initial or the final velocity of the lift. When told to assume that it is the initial velocity, he then writes:

$$x = 2\ t + t^2$$

ANALYSIS OF INTERVIEW 7

Cognitive Difficulties

1) The student does not always read the problem carefully enough to make sure that the data are correct and that the problem situation is correctly identified. There is always haste to record the numerical data and to search for a suitable equation to apply.
2) No attempt was made, without prompting, to visualize the situation carefully and to explicate broad, qualitative features. This is apparent from his giving the initial velocity of the screw as zero.
3) He does not easily relate the two moving objects so as to obtain a coherent picture. This is seen from his apparent inability to determine whether 2 m s^{-1} is the initial or the final velocity of the lift.

Semantic Difficulties

There seems to be limited understanding of the expression "at the same instant". Although "t" is given as the same in the equations used, there is apparently no clear perception of what the time period is for each object since there is no certainty of initial or final velocities.

Error Factors

There was again no attempt to consider signs for the various quantities in a consistent manner.

INTERVIEW 8

Problem 1

He correctly writes down the data in the problem:

Car: $v_0 = 0$ $\because$ from rest; $a = 2\ m\ s^{-2}$ $\therefore$ $s = x$

Lorry: $v = 10\ m\ s^{-1}$; $s = x$

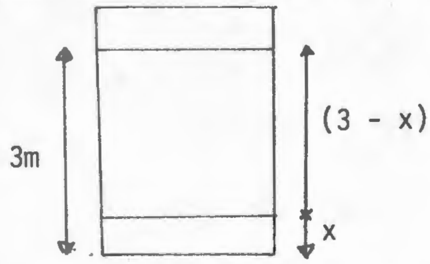
He writes down the same distance explicitly. When asked why he does not draw a sketch, he responds:

"I can see the situation in my imagination".

He has no difficulty in solving the problem.

Problem 2

After reading through the problem, he immediately draws a sketch:



He writes alongside the sketch:

"As the lift moves up x, screw has fallen (3 - x).

Lift: $s = x$; $a = 2\ m\ s^{-2}$; $t=?$"

He asks: "If the lift moves up is the value of g + or -?"

He does not write down the initial velocity of the lift. After some prompting he writes : $v = 2\ m\ s^{-1}$.

(The reason why a prompt was used at this point was because he wrote down the equations : $v^2 = v_0^2 + 2\ a\ s$; $v = v_0 + a\ t$. After thinking for a while, he then writes: $s = v_0\ t + \frac{1}{2}\ a\ t^2$ and then immediately erases it. He was clearly stuck here and unable to proceed further).

After the prompt , he writes : $\therefore$ $x = 2\ t + t^2$

Screw : $v = 2\ m\ s^{-1}$; $a = 10\ m\ s^{-2}$; $t = ?$

I : Is the time the same for the lift and the screw since you write 't' for both?

S : No. After thinking for a while, he says: "Yes, they are the same because the screw and lift meet each other".

I : Explain what in the question shows that the time is the same.

S : From the question, how long will it take?

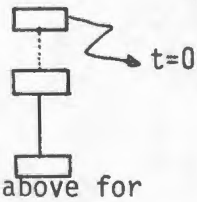
I : How do you understand the expression 'at the instant?'

S : At the same instant that the lift is 2 m s^{-1}, the screw falls out.

I : How would you represent the physical situation?

S : The lift moves up while the screw falls down.

Asked to draw the motion of the screw, he draws:



In spite of the fact that he has written v = 2 m s^{-1} above for the speed of the screw, when asked what is the velocity of the screw, he responds: "Zero". It took considerable discussion before he saw that the initial velocity of the screw was 2 m s^{-1}, upward.

ANALYSIS OF INTERVIEW 8

Error Factors

1) In spite of being asked, no effort was made to relate the data to an axis for direction.

Cognitive Difficulties

1) There is difficulty in visualizing the motion.
2) While the motion of the lift is correctly drawn, the motion of the screw is not related to it. This can be seen from the fact that the time for each motion is not readily related.
3) It is unclear what the meaning of the distance (3-x), given in the sketch drawn, seems to be. It is not related by the student to the distance travelled by the screw at any stage, although he explicitly states it initially. It is as if he forgets about it.
4) While he makes a good qualitative evaluation of problem 1 with which he appears familiar, he proceeds immediately with numerical data and equations into which the data can be fitted, in problem 2 i.e. his evaluative strategy does not appear to be an essential part of his cognitive repertoire.

Semantic Difficulties

1) The idea of the screw falling from the roof was interpreted as a downward motion.

INTERVIEW 9

Problem 1

He reads the problem and immediately writes:

Car : $v_0 = 0$; $a = 2\ m\ s^{-2}$; Lorry: $a = 0$ (constant velocity)

$v = 10\ m\ s^{-1}$

He writes down: Distance which the lorry covers: s = ; then stops and erases it. He is asked if there are additional things given. He responds by writing down : $v = a\ t$; $10 = 0\ t$. He then erases it. He then experiments with the three kinematic equations and says: "No, those are all the facts given".

I : Is the time the same for the two bodies?

S : To calculate the distance, the time is the same.

I : How do you understand the expression 'at the instant' and 'at the same instant?'

S : The lorry was already in motion at the instant that it passed the car.

I : Can you give a representation of the problem?

S : (Stands for a while) At one or other point in time the car passes the lorry.

I : Where does the problem begin?

S : When the car pulls away from rest.

I : What happens to the lorry?

S : It has a constant speed of $10\ m\ s^{-1}$.

I : Where are the two relative to each other at the beginning? (the student hesitates). Are they alongside each other or separated?

S : They will be alongside each other.

I : So what is the problem you have to solve then?

The student tries to show with his hands how movement takes place. He clearly has great difficulty in visualizing the physical situation. After discussion he sees that the distance, s, is the same for both bodies.

I : What about the time?

S : Lorry will take longer to reach the point.

It is only after considerable discussion that he appears to understand that the time is the same for both objects.

ANALYSIS OF INTERVIEW 9

Cognitive Difficulties

1) No sketch is attempted. The simple data (four numerical facts) are written down and then the equations are immediately attempted.
2) There is no effort to analyse implicit data. He does not spontaneously see that s is the same for both and has extreme difficulty understanding that t is identical.
3) There appears to be difficulty in visualizing the physical situation.
4) Each object is dealt with separately and seldom is the motion of the one related to that of the other.
5) The student does not isolate the important points in the problem. He does not appear to see spontaneously what the initial and final points are or, indeed, their relevance.

INTERVIEW 10

Problem 1

The student reads the problem and immediately writes:

Car : $a = 2\ \mathrm{m\ s^{-2}}$; $t = ?$; $V_0 = 0$; $s = ?$

Lorry: $v = 10\ \mathrm{m\ s^{-1}}$; $t = ?$ $a = ?$

He then writes down the three kinematic equations.

The student acknowledges that the lorry is travelling at constant speed but is still undecided about the value of the acceleration of the lorry. After discussion he changes it to $a = 0$.

When asked whether the time t (the same symbol was used for both bodies) is the same for both, the student was unable to answer.

The student is encouraged to solve the problem: he writes down the equation $v = v_0 + a\,t = 10$, for the lorry. When asked why he is doing this, he responds: "Because I want to calculate the time".

He now writes: Car: $v = v_0 + a\,t = 0 + 2\,t$ and

$$s = v_0\,t + \tfrac{1}{2}\,a\,t^2$$

The same symbol is used for the velocity of both car and lorry.

The student now attempts to relate the two equations for velocity and deduces that $t = 5$ s. Discussion reveals that the student has not necessarily made the assumption that when the two vehicles draw level they have the same speeds. What has happened is that because of not distinguishing between the symbols for the velocities of each vehicle, two equations with the same symbol are simply equated.

ANALYSIS OF INTERVIEW 10

Cognitive Difficulties

1) The student has difficulty in visualizing the problem. When asked to draw out the physical situation, he proved incapable of doing so. In discussion it was apparent that the student did not understand that the lorry passes the car at the robot.

2) Explicit data are noted and then there is an immediate attempt to relate it to known equations.

3) Implicit data are ignored or confused (e.g. whether t is the same for both bodies). This is related to the apparent inability of the student to visualize the physical situation.

4) Data of the two objects are not necessarily explicitly differentiated. This resulted in the velocity equality confusion.

Semantic

1) The significance of the expression 'at the same instant' was not perceived.

2) The physical meaning of constant velocity was not readily perceived.

INTERVIEW 11

The student immediately writes down:

Car: $a = 2\ m\ s^{-2}$ (constant) : $u = 0\ m\ s^{-1}$

Lorry: $v = 10\ m\ s^{-1}$; $a = 0$

After writing this the student continues reading the problem and finally says that she is not able to continue. It was decided to help her analyse the problem in detail:

I : What does the expression 'at the same instant' mean?

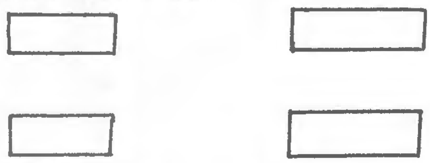
S : It happens at the same time. When the car moves off, the lorry passes at 10 m s. I am not quite sure what the word 'instant' means.

I : What is the point of the problem?

S : Where the car has picked up enough speed to catch the lorry?

I : Could you draw out the problem?

S : Where start out



must find point where they are equal.

I : What will be the same when they are equal?

S : Time and distance.

I : So what is the problem?

S : Distance at which car overtakes lorry.

She now has no further difficulty in applying the equation to find the correct solution.

ANALYSIS OF INTERVIEW 11

Cognitive Difficulties

1) Implicit data are not readily made explicit. The ability to do so appears to exist but the necessity to do so is not felt.
2) No physical representation is readily given nor is much effort made to visualize the problem clearly.
3) Only numerical data, the perceptually obvious, are considered. No goal analysis is attempted and neither is an effort made to analyse the point of the problem in any depth.

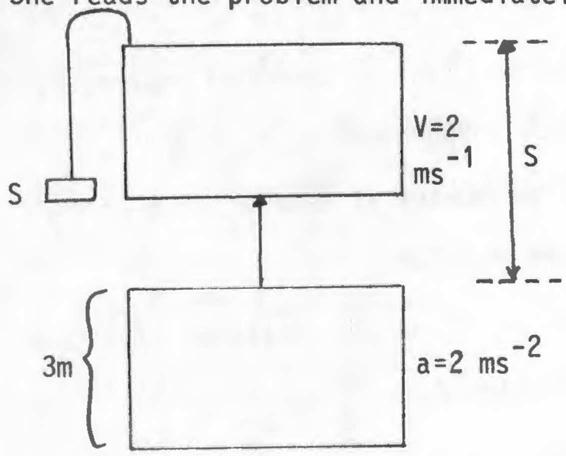
Semantic Difficulties

1) Words e.g. "at the same instant" do not appear to suggest any particular meanings or carry special significance in attempting to understand the problem.

INTERVIEW 12

Problem 2

She reads the problem and immediately makes the following sketch:



Student: The screw has to fall 3 and s.

I : Where is the screw in the lift?

S : On the floor probably. Oh no! The roof.

I : How will the screw fall?

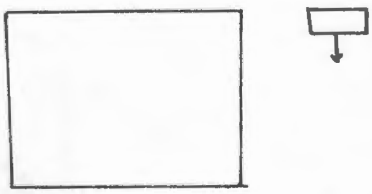
S : Like a projectile. (Draws):

I : What is the velocity of the screw?

S : 2 m s^{-1}

I : How would you draw the screw's path?

S : (Draws)



I : What is the acceleration of the screw?

S : That is zero. (She thinks for a while and then says:)

It falls under the influence of gravity $\therefore$ acc = -10

The student continues to experience difficulty in understanding the physical situation. She is finally able to see that the distance covered by the screw is (3 - s).

She gives no thought to directions until asked.

ANALYSIS OF INTERVIEW 12

Cognitive Difficulties

1) The two objects are not related. Each is examined separately and conclusions drawn for each independently.
2) An immediate search is made for numerical data without concern for qualitative aspects of the problem.
3) The student proved unable to comprehend the physical situation and to visualize the problem clearly.

Semantic Difficulties

The word "falls" again misleads. Thus the motion of the screw is always pictured as being downward.

Error Factors

1) The idea of parabolic motion is again introduced.
2) Signs for variables are not regarded as essential.

INTERVIEW 13

Problem 1

He draws a line and then writes down:

Car: initial velocity = 0; acceleration = 1 m s^{-2};

Lorry: $a_L = 0$

S : We want to find the distance, therefore the two distances are the same.

He writes down: Equation for lorry s = v t, then erases it.

For lorry:

$$s = \frac{v^2 - u^2}{2a} \qquad \text{For car:}$$

At this point he is stuck and unable to proceed further.

I : Why not read through the problem again and note every single fact given, not just numerical.

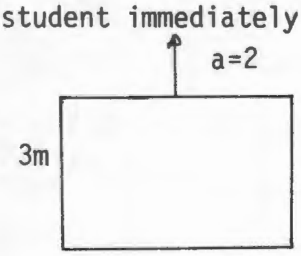
S : At the start t = 0. Car: a = 2 m s^{-2}; $u_c = 0$

Lorry: $u_L = 10$ m s^{-1}; $v_L = 10$ m s^{-1} ; $a_L = 0$

After some discussion he sees that the distance is the same for both objects and t is the same. After this he easily solves the problem. No prompting was given to the student to draw a sketch. No attempt was made.

Problem 2

The student immediately draws a sketch:



For lift: a 0 2; a = 2

For screw: v = ? g = 10 m s^{-2}

u = 2; s = ?

I : The velocity v for the lift, what velocity is this?

S : Final velocity i.e. velocity the lift reaches at that time.

I : What does the expression 'at the instant' mean?

S : Does it mean anything else?

S : No, it means nothing else.

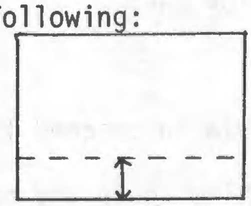
I : Won't the screw still accelerate upward?

S : No.

I : Why do you have an acceleration of $\pm$ 10?

S : No, that's wrong. I didn't picture it correctly.

He now draws the following:



I : What is the distance given by the arrow?

S : Distance the lift will move.

Further discussion indicates that he still has not grasped the relationship between the two moving bodies, especially as regards the time for each and the distance moved by the screw.

ANALYSIS OF INTERVIEW 13

Cognitive Difficulties

1) No effort was made to picture the situation. The sketch drawn is meaningless insofar as an understanding of the system is concerned. Numbers only are recorded.
2) Implicit data are understood only with difficulty.
3) No spontaneous comparisons are made between the screw and the lift. He is not able to establish any relationship between the two moving bodies.

Semantic Difficulties

1) In spite of the fact that "u = 2" is written down for the screw, the student does not see that the screw will first move upward.

Error Factors

1) Directions are not regarded as essential. Signs for quantities are only inserted after prompting.

INTERVIEW 14

I : What we would like to do here is to see how you read through problems and what sort of data you extract from it. Would you like to stand at the board and write down the data which is in the problem. All the things that you think are important for solving the problem Try to explain as you go along.

S : I'll first tackle the diagram to visualize the problem. (She draws):

s,t the same

Here's the starting point. Here's the car starting from rest. Here's the truck moving along. So this one is moving already when this one starts. And somewhere over here it has to overtake. The distance over here from this point to here has to be the same. So the distance would be the same and the time would be the same.

I : How do you know the distance and the time are the same for both?

S : To be able to pass there must .... If two cars pass each other and they have to meet at a certain place, and the distance you measure from where they pass each other is the same.

I : Can I ask you, do you always draw sketches when you solve problems?

S : Yes, not electricity problems, but for motion problems I do. (She writes):

Car	Lorry
$v_o = 0 \text{ m s}^{-1}$	$v_o = 10 \text{ m s}^{-1}$
$a = 2 \text{ m s}^{-2}$	$v = 10 \text{ m s}^{-1}$; $a = 0$
s,t	s,t

I : Would you talk aloud while you write on the board.

S : Yes OK. Well the lorry has been moving while this one started from rest. The lorry's initial velocity would be 10 m s^{-1}. But what we deduce from this is that the lorry is moving at a constant velocity all the time. So its final velocity is also 10 and its acceleration is zero and the distance would be the same and the time is the same too. And then I write down the equations which I think are relevant to the problem.

(Writes):

$$s = v_0 \ t + \tfrac{1}{2} a t^2 \qquad v \quad u + a t \qquad v^2 = v_0^{\ 2} + 2 a s$$

Lorry : $s = v_0 \ t + \tfrac{1}{2} a t^2$

$= 10t$

That's for the lorry. And for the car:

$u = 0$ and $v = v_0 + a t$

$$s = v_0 t + \tfrac{1}{2} a t^2$$

$$= \tfrac{1}{2} (2)t^2 = t^2$$

Now we can equate the two, because the distances would be the same and these times t are also the same for the two.

So: $10t = t^2$ so $t = 10$

So the time taken would be 10. And they ask: How far beyond the point; we just plug this value in there and:

$s = 10 \times t = 100m$

I : That's OK. Very nice. So when you read a problem through, what do you do first?

S : I try to recognize the problem; see what or classify where it comes in. If you don't even know where it fits in then that's very bad.

I : So you try and recognize it first.

S : Yes.

I : How would you catagorize this problem?

S : Equations of motion.

I : Yes.

S : Mostly I catagorize it by formulae; which section - say motion, it is a big section. So this is motion.

I : Right. Would you read the second problem through?

S : Well, in this problem we have two items accelerating upwards and downwards. I can still use the equations but with g. This 3m doesn't play any role in the acceleration of the lift itself, but it will play a role in the distance that the screw will fall. So putting the height by the acceleration of the lift ....

I : Sorry, what did you say ... I didn't get that point.

S : The height of the lift has got nothing to do with the screw that falls. So a person can picture .... one can use s in that lift's equation and you tend to put a three in there.

I : Yes. So it's a matter of understanding the problem itself.

S : Yes. The direction of this lift. I will take up to be positive.

(Writes on the board).

Lift : $a = 2 \text{ m s}^{-2}$ <u>Screw</u>: $V_o = 0 \text{ m s}^{-1}$

$v = 2 \text{ m s}^{-1}$ $S = 3\text{m}$

$g = -10\text{m s}^{-2}$

If take up as positive than this must be negative. Since no masses are given, Newton would be out, because to be able to use, Newton's laws there must be masses given. So it's just equations of motion. The screw would albo be travelling at this acceleration, at 2 m s^{-2}.

I : So how will all this help us with the problem?

S : I'm just wondering - with respect to the lift the screw is actually standing still, so I think it should be zero.

(She writes)

$$s = V_0 + \tfrac{1}{2} a t^2$$

$$3 = 0 + \tfrac{1}{2} (-10) \times t^2$$

$$3 = 5 t^2$$

(She is stuck here)

I : Just before you go on, let's think a bit about this. You were saying with the previous problem that you want to visualize the problem. If you were trying to do the same thing here, how would you describe the problem.

S : Actually, I know what the lift looks like, I want to try and see it as it moves upward .... (She appears stuck at this point).

I : The word fall, how do you interpret it?

S : Free without being thrown down, or so, under the influence of gravity alone.

I : Can a thing fall upward?

S : No, it could if you had to .... If there is an attracting force, it won't fall up.

I : Fall up is the wrong expression. Does the screw move up at all?

S : Initially the screw moves up.

I : OK. Let's just think about this. Try and picture in your mind. Here's the lift - right? Now at the instant that the screw comes loose from the lift what happens to it? If you jump out of a car, what happens to you? Are you moving at all?

S : You will move at the velocity of the car.

I : Precisely. So what is the inital velocity of the screw?

S : (She writes):

$v_0 = + 2 \text{ m s}^{-1}$ (for screw)

I : Why do you write + 2?

S : Because upwards is positive and I've been moving upwards.

I : What about the distance that it moves?

S : This 3 should be -3 because it falls from top to bottom. (She points to the data previously recorded).

I : Let's just think about the problem a bit further. How do you come on the fact that S is -3 regardless of sign?

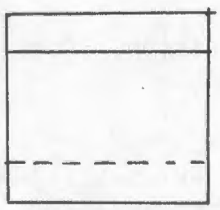
S : Because the distance it has to fall to reach the floor it has to cover 3 metres.

I : Are you sure?

S : Yes.

I : Let's think about it. (Gestures with hands). Here's the lift. Now the screw falls from the roof and the lift moves up. So what now? Try to draw a sketch of the lift continuing to move up and the screw falling out, moving up first and then dropping down.

S : (Draws)



The bottom, the floor also moves up.

I : Yes, the floor moves up. (Indicating on sketch). Here's the screw and at that instant it becomes loose and falls. Now what happens to the screw? It has a velocity of 2 m s^{-1} up, and so it will do this: 

S : That's where it comes down.

I : What happens to the lift in the meantime?

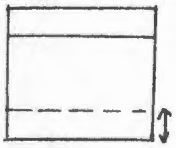
S : It moves up.

I : Will the lift get away from the screw, will it move up further than the screw?

S : Yes.

I : So the screw is going to do that. Now if the screw started here and the lift was stationary, the screw would have fallen that amount. (on sketch). Now how far did the lift move in the time that the screw fell down?

S : That distance



I : Call that something say x. That distance there is x. So how far did the screw fall?

S : 3 minus x.

I : Good. So what would be 3 minus x?

S : (Indicates correctly on sketch).

I : Now if you think in terms of your signs, is it up or down?

S : It is down.

I : So that is?

S : x - 3

I : Yes. OK. You don't have to work it out any further. You see the answer, how it works?

S : Yes.

I : It's been very helpful talking to you because it is good to see the way you draw sketches after trying to visualize the problem first. What do you think went wrong with this one?

S : I don't know.

I : Did you perhaps get an imcomplete picture of the problem?

S : I suppose so. I did not actually consider what the lift was doing. I was wondering why you did'nt give any information about the lift.

ANALYSIS OF INTERVIEW 14

Semantic

1) The word "falls" is interpreted as downward only. It takes considerable discussion before she sees that the screw will move upward also.

Error Factors

1) The student has great difficulty understanding that when one moving body is loosened from another, both will have the same speed initially. This seems to be related to the difficulty students have in appreciating the significance of instantaneous values. This is also seen from the fact that students in this study very frequently give the final velocity of a body which falls vertically and strikes the ground as zero.

Cognitive Deficiencies

1) The student shows well-developed ability to analyse data effectively. This is related to the ability to visualize the problem. With problem 2, however, the difficulty she experiences is related to an inability to consider two sources of information at once. She focuses entirely on the screw without considering what the lift is doing.

INTERVIEW 15

I : Will you please read though this first question and get the data and details from the problem. Try to indicate (by doing and saying) how you would solve such a problem. How do you start to think about the problem as you read through it now?

S : Looking at the problem it seems they want to know how far from the starting point will the car overtake the truck. I would therefore call the distance covered by them x.

(He draws :

Suppose the starting point is A. Look where they will overtake each other, i.e. where the one will catch up with the other. Name the point B and the distance they covered x.

Now in the case of the car moving at a constant velocity of 10 m s^{-1}, if that distance is x, you can get an equation for x in terms of the velocity of the car.

Now I want to use the equation S = v x t. Then x is 10 times the time but the time is still unknown. If I consider (take) the details of the truck : initial velocity.

I : Does the truck move away or the car.

S : The car. That's right. That one is now the truck and this is the car. He started from rest. Therefore, say his initial velocity is zero and then you are also given its acceleration - therefore a = 2 m s^{-2}.

If I now go to the equations of motion, $V = u + a\ t$;

$s = u\ t + \frac{1}{2}\ a\ t^2 \quad v^2 = u^2 + 2\ a\ s$

Now we consider the second equation. The initial velocity is zero - therefore that part will fall away. Then $x = \frac{1}{2} \times 2 \times t^2$

The acceleration is two, so that is one and then $x = t^2$.

I : Is the distance the same for the car and the truck?

S : They started from the same starting point so the distance is therefore the same. I will find the time first.

I : Is the time the same for the two?

S : They started from the same point and they are at the same point here. So the time must be the same.

I : Yes. Are their speeds the same at this point?

S : Not necessarily. Because the one will be constant and the other from rest. So it cannot be the same.

Now $10t = t^2$

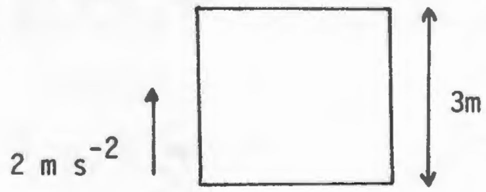
$$t^2 = 10\ t$$

So now you are actually going to get two answers. The reason for this is that in the beginning when they start out you could say they were together. Then, after a while they will overtake one another. So the answer they are actually looking for here is a time = 10. Now you substitute it into one of your equations. Then you get: they will overtake each other 100m from the starting point.

I : Good. Please read through the second problem.

S : If I read through it now, I will make a sketch first.

He draws:



This is a lift accelerating upwards at 2 m s^{-2} and at a certain velocity, a screw falls from the roof. Now they want to know how long it will take the screw to actually reach the floor. But while the screw is falling the lift is also moving up. So this must be taken into consideration in this problem. The best will be to make a sketch. Now you look at the lift on that moment that its velocity is 2 m s^{-1} upwards and they also gave the dimensions of the lift - it's 3 metres. So you know the distance which the screw will fall is actually 3 metres. Now you will have to make a few suppositions. As I mentioned before, suppose the distance travelled is x. Then perhaps you can now ... they want to know how long it takes it. So you are dealing with time. Now, while the screw is falling, the lift is also moving upwards. Now you are dealing with those two times.

When the screw is up here in the roof and falls down, we must remember that it is not really falling from rest. At that moment it actually moves with the lift at a velocity of 2 m s^{-1}. So its initial velocity is $u=2$ and then you have the distance which it falls - it is s. This is now the dimensions of the lift, it is 3m and you can now write an expression in terms of the time i.e. $s = ut + \frac{1}{2} at^2$. This is equal to 3. Its initial velocity is not zero, but 2 since it moves upwards with the lift. So it is 2t plus .... Now: The screw is a freefalling body, so the acceleration acting upon it is the gravitational acceleration. We call it $g = 10 \text{ m s}^{-2}$.

Hence : $3 = 2t + 5t^2$. This is in the case of the screw.

Now for the lift. During the time that the screw fell, we call this t, the lift was still busy moving upwards. You have the velocity of the lift as it move upward and you were given its initial velocity because they said that at that moment it was moving at a certain velocity and now you have its acceleration. Now you must put this into an equation of motion. (Student does the arithmetic on the blackboard - he appears to be stuck).

I : Your presentation of the screw is excellent. I think there is just one point where you are a bit uncertain; let's see if we can clarify this. If you were to draw the screw on its own - how would you illustrate the motion of the screw?

S : Before the screw fell, the lift was busy moving upwards, but when it was released the screw moved down.

I : Will the screw move down immediately?

S : No, because look it was busy moving upwards. The lift is busy moving upwards. Correct? It is in fact something like .....

I : Correct, now where will the lift be? Look here - this is where the lift was at the moment that the screw fell ..... (gestures with hand). The word fall is a bit deceiving because it initially moved upwards - "fall" as you illustrated it here. Now what happens to the lift in the meantime?

S : The lift is still busy accelerating upwards.

I : So where will the screw hit the floor of the lift?

S : Well, actually it will not hit the lift where it was originally. So you can't exactly say that the screw moved a distance of 3 metres.

I : Good!

S : So that is where I went wrong. If the lift is busy moving upwards, you come to the point where it will hit the floor. Then the lift is something like this. The screw definitely does not fall through a distance of 3 metres. So if you call the distance through which it fell S then the other distance is 3 minus S, is it not?

I : Correct. So then your equation for the screw is wrong. How did it change now?

S : (Writes on the blackboard). $S = 2t + 5t^2$. I said we call the distance through which it falls S, so it remains S.

I : Could I just ask a question. How does the screw move initially? Direction?

S : Immediately after it is released it moves upwards.

I : Upwards. Correct, and now?

S : Till the moment its velocity becomes zero and it turns to move down.

I : So you have two directions.

S : Yes.

I : So if you have different directions?

You see, you have g there as 10; plus 10. In other words you say that g and the initial velocity of the screw have the same directions, is that so?

S : Yes.

I : Is it? What is the initial velocity of the screw? What is the direction of its initial velocity?

S : It is positive, i.e. upwards.

I : And g?

S : That is actually a problem that we missed in class. When to regard g as positive and when as negative. At school we had that when a body moves down, then it accelerates. I think we took g downwards as positive , because it is busy accelerating if it goes upward, it is negative because it moves slowly. When we came here we were told to look at a system of axes. When a body moves upward you take g as positive and when it comes down, g is negative.

I : So in this case, i.e. as soon as you have two directions, you need to choose a system of axes. Correct. How did you choose your system of axes here? You did not write it down, but you have chosen it.

S : To determine the signs.

I : Yes. Look, what is the direction of g, not what is the sign, but what is the direction, always..

S : Down, towards the centre of the earth.

I : Always down. Now when will g be positive if you think in terms of a system of axes?

S : When will g be negative?

I : No, when will g be positive in terms of a system of axes?

S : Is it not when it is upwards?

I : No, think now, you can choose a system of axes in any direction. Let me show you. You can say this $\longrightarrow$ is X, normally we choose this as positive and this $\longleftarrow$, as negative. So if a vector has that direction then we say?

S : It is positive.

I : And if it has this direction?

S : Negative.

I : Correct. Now in this case, how would you choose a system of axes? You said that the initial velocity is u. The initial velocity is 2.

S : Yes.

I : But actually you said plus 2. So in other words?

S : Everything that goes upward is positive?

I : That is what you said. And everything that comes down is negative. Now does g come down?

S : Yes.

I : g always comes down, now in this case g must ....

S : Be negative.

I : Correct. Is there anything else that comes down?

S : No. The lift is going up.

I : No. Look, we are just considering the screw.

S : The screw initially goes up and then it comes down. There is a stage where velocity will be negative again.

I : Yes, but its acceleration is always negative. Even if it is here, when it is busy moving upwards, but what is the direction of the acceleration?

S : Downwards.

I : Always downwards, so the acceleration is always negative. Its velocity changes as you have said. Initially it is upwards - so this means?

S : Positive.

I : Positive. Now it is downwards. What about S? What is sign of S?

S : So we are actually going to write S as negative.

I : For the screw one actually has to put a minus sign here, when putting in values. Do you see?

S : Yes.

ANALYSIS OF INTERVIEW 15

Error Factors

1) There seems to be no compulsion to insert signs where motion takes place in two directions. While the student perceives that the initial velocity of the screw is 2 m s^{-1} upward, the acceleration "g" while acknowledged to be downward, is not given the appropriate sign. This seems to be related to the tendency to regard the values s, a, v and u in the kinematic equations as scalar rather than vector quantities.

Cognitive Deficiencies

It is extremely interesting to compare the way in which this student analyses the data in the problem in comparison with many of the other students interviewed. He is careful about visualizing the problems and in searching for qualitative data. Even in the case where he does not see the manner in which the floor of the lift moves toward the screw, a short explanation suffices to help him. He displays the ability to relate the information from two sources of information at once.

He was the top student in the matric examination of the Department of Internal Affairs the previous year.

INTERVIEW 16

I : What we want you to do is to read through this. Look at the first problem. Read it carefully. Do you recognize the problem? Have you seen it before?

S : No.

I : What would you say it is about?

S : Equations of motion.

I : Yes. Would you like to stand at the board and tell us how you would think in working out the problem. Talk as you work so that we know what you're up to.

S : I'll start by writing down the given things. For the car acceleration = 2 m s^{-2} and the initial velocity of car = 0 as the car starts and in order to eventually reach the lorry, the car has to reach a speed of 10 m s^{-1} and as the lorry is turning, the car is still at a constant speed of 10 m s^{-1} so the lorry will accelerate. The time for the car is t and the first question is how far. So here are two equations (she writes): $s = ut + \frac{1}{2} a t^2$; $v^2 = u^2 + 2 a s$; $v = u + a t$ First equation is $s = u t + \frac{1}{2} a t^2$. You need the time, but you don't have the time. And in the second equation you need the final and the initial velocity and you've got the final velocity of the car which is 10. (Calculating) $(10)^2 = 0 + 2. 2. S$

I : I see.

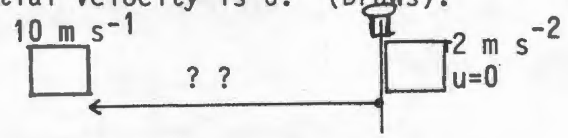
S : You don't have the time.

I : Why is that?

S : The car moves much faster and passes the lorry.

I : Why don't you try and draw a sketch of everything, how do you see the situation? Use blocks for the lorry and car.

S : The lorry is travelling here, the car stands here and its initial velocity is 0. (Draws).



Something is the same somewhere.

I : Where is the starting point?

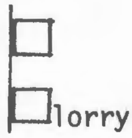
S : Where the car starts to move.

I : Where is the lorry?

S : Driving here, it is passing the car.

I : Where is the lorry?

S : Next to the car. (Draws)



I : So what happens after that?

S : This is the time ....

I : Which time?

S : At the moment the car passes the lorry and at that moment in time. So, but we only have ....

I : First think about it. We start here; what happens here?

S : The car is moving at 10 m s^{-1}.

I : So what happens to the lorry?

S : It passes the car.

I : Yes and what happens to the car?

S : It catches up with the lorry.

I : So what is the next important point?

S : Where they meet.

I : Now you said earlier the lorry and the car will not leave at the same instant. Look at it this way. Where do they start?

S : At the robot.

I : OK. Is there anything that you can relate here?

S : The distances are the same.

I : Are you convinced? You say the distances are the same. What equation applies or what equations apply?

S : This one. ($v^2 = u^2 + 2 a s$)

I : For the car or the lorry or both?

S : Well we don't have the final velocity of the car. The initial velocity is zero and the final velocity .....

I : What equation applies for that? How is the lorry moving?

S : Constant velocity.

I : So what is the equation?

S : First one. ($S = u t + a t^2$).

I : OK. If a is zero, what is the equation. Write it down.

S : $s = u t + 0$.

I : What don't you know about the equations for the car?

S : I don't know V.

I : Now for the lorry you've got S. Is there any possibility of something similar for the car?

S : The car will have ....(Writes) $V^2 = u^2 + 2 a s$; $s = v^2 - u^2$ The distance will be the same so we can equate the two. (Writes) : $S = \frac{1}{2} a t^2$.

I : Are the t's the same?

S : No, the car will take a shorter time to reach that distance.

I : Think about it. They both start at the same instant. When the car passes the lorry, what happens to the lorry?

S : No, the car will take a shorter time to reach that distance.

I : They both start at the same instant. When the car passes the lorry, what happens to the lorry?

S : No, the car will take a shorter time to reach that distance.

I : They both start at the same instant. When the car passes the lorry, what happens to the lorry?

S : The car pulls away.

I : So say for example the lorry is moving for 20 seconds and now the car comes along. How can the time not be the same?

S : Yes, but to reach this the car might take a shorter time to move this distance.

I : Let's say that this here is the robot. Then here is the car and here is the lorry. Here comes the lorry; it passes the car, it keeps going. The car pulls away and here they catch up. Can you see that they moved the same distance and they moved the same time?

S : Yes.

I : You're confusing the fact that the car can move faster than the lorry. The car takes off and it picks up speed more rapidly, but when they get here .....

S : But the car will get there sooner.

I : Well, think about this, if they had both been stationary and pulled away, then the car would obviously have got here long before the lorry, but they don't start stationary. The lorry is already moving. It is like two runners. See here I've got two runners. The one is in the starting box and the other one comes running pass, but they get to the tape exactly at the same time. Have they run for the same time?

S : Yes.

I : Yes. Well, the one will pick up speed more quickly, so left on his own he'd be able to get there much quicker. So if you start a stopwatch here where they're both starting, but this one is already moving, when they get here you stop the stop-watch. Must you stop two stopwatches or what?

S : Only one.

I : So, same time. So now if the time is the same, see what you get.

S : (Writes on board) $ut = \frac{1}{2} a t^2$ $t = 10s$

$$s = u t = 10 \times 10 = 100$$

I : Are you convinced of that or did I just talk.

S : No, I'm convinced.

I : How do you start off a problem? Do you try to picture the whole thing first or what do you do?

S : No, I try to, but sometimes I can't visualize it. I just try and take down what is given and use equations.

ANALYSIS OF INTERVIEW 16

Error Factors

1) The common error of same position meaning same speed, is made.
2) She insists that the car will reach the meeting point in a shorter time than it will take the lorry. This seems to rest on the intuitive feeling that the object which moves faster must take a shorter time regardless of the parameters of the problem.

Cognitive Deficiencies

1) There is no spontaneous attempt to visualize the problem. Numerical data are immediately recorded and fitted into the relevant equation with very little concern for accuracy.
2) Her perception of what the problem is about seems blurred. She gives herself no time to analyse the details of the problem, as, for example, relevant important features such as the initial and final points.
3) She tends to focus entirely on one object in the hope that the answer will "fall out". She has extreme difficulty in dealing with simultaneous equations and even in establishing these relationships.
4) Implicit data in the problem are not regarded as very relevant and are also misinterpreted when discussed. She has great difficulties in perceiving the equality of distance and time for the two objects.
5) It is striking that if the data collection is defective, it is impossible to elaborate the problem and produce any planned approach to a solution.

INTERVIEW 17

I : Would you read through the first problem and explain how you would work it out. Write on the board if you like.

S : I have not seen this problem before.

I : Well think about it. Do you recognize it?

S : This problem will be guided by using the three equations.

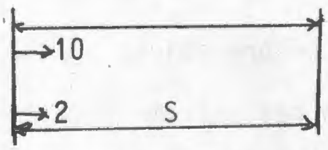
I : Would you like to stand at the board and show how you would work out a problem like this.

S : We have objects in motion like the car and the lorry. They give you the acceleration for the car and then the speed, that's their starting point here, that's the acceleration of the lorry. This is the car and the lorry passes the car at the same instant with the speed of $10\ m\ s^{-1}$.

(He writes and draws):

$v^2 = u^2 + 2\,a\,s$ $\quad a = 2\ m\ s^{-2}$ $\quad$ Car $\qquad S = V_o\,t + \frac{1}{2}\,a\,t^2$

$v = V_o + a\,t$ $\quad v = 10\ m\ s^{-1}$ $\quad$ Lorry



And that's now the lorry. The first thing I always do is to draw a picture, a diagram of the problem. So the starting point, the car starts with this acceleration up to here and the lorry passes this car with the velocity of $10\ m\ s^{-1}$. Now they want to know how far we are from the starting point when the car overtakes the lorry. Of course the car will overtake the lorry. The lorry is moving at constant speed of $10\ m\ s^{-1}$. How far? I do not know how far by using this equation

$$s = u\,t + \tfrac{1}{2}\,a\,t^2$$

We have to find S which is the distance they give you. U = 0 of the lorry. They don't give you a of lorry but they give you time and distance. Right. The car's initial velocity is zero. So v = 2t.

I : This one that you wrote here which is it? ($s = v_0 t + \frac{1}{2} a t^2$)

S : This is what I wrote for the lorry, to find S. So you can use this one ($v = v_0 + a t$) for the car to find the final velocity of the car. This equation here so, because they give you a and you know that the car starts from rest and u = 0. We are going to find the final velocity of the car here, at this point. (he writes): v = 2 t. But now that I am thinking about it, is it really necessary to find the final velocity of the car?

Let us see, they give you a, they do not give you t. In this case ..... (student is thinking about problem, working on board). From this point to that point this is s and t. This is for both the car and the lorry. The distance and time will be the same because from this point to that point it is the same distance so it is going to be the same time for the car to cover the distance. The distance will be the same but the time, the time ... For the lorry, I say at the constant speed. So there is no acceleration in the case of the lorry. So if we take (student doing calculation) $s = v_0 t = 10 t$. So if we find the time then we can find the final velocity of the car.

I : Why do you want the final velocity?

S : No, but I mean if we get the final velocity from this equation you can find here, (e.g. v = 2 t) and if you put t in there (s = 10 t) then you can find the distance.

They ask you in (b) how fast the car is moving when it over-takes. So I have to find velocity, but first of all they ask me how far from the starting point. So you've got to find S and from S you can find t. You can use this equation:

$$V^2 = v^2 + 2\ a\ s$$

We see that S for both cases is equal. We can say that V = 2t = 10t.

So 2t = S. No (Student rubs out). Can use : $V^2 = 2 \times 2 \times s = 4\ s$, so $S = v^2/4^2$ is equal to this 10t.

I : So the problem seems to be to relate these different equations.

S : That is the problem.

I : How many equations have you got?

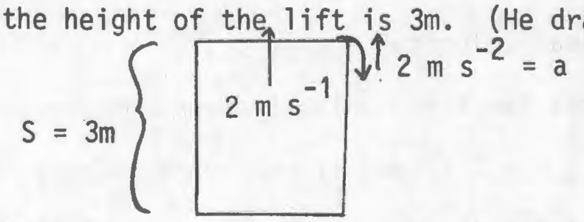
S : We have 3 in all.

I : Which are they?

S : These three are V = 2t for the car and the other one also for the car is $v^2/4$ and s = 10t. This is for the lorry.

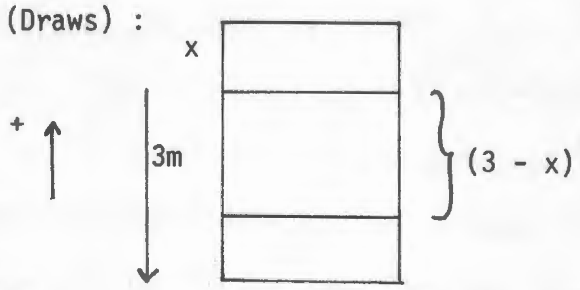
I : (Shows how to solve the simultaneous equations). You seem to play around with the equations until you get the right thing; it's obvious here that you have problems with simultaneous equations as well. Would you read the second problem there and then tell us how you would set about solving it.

S : The lift has an acceleration upwards. First of all let's draw a diagram of what's going on actually. Let's draw the lift and the height of the lift is 3m. (He draws) :



The lift is accelerating upwards at an acceleration of $2\ m\ s^{-2}$.

At the instant that the velocity is 2 m s^{-1} the screw falls down and its velocity is 2 m s^{-1} up, so from the roof of the lift the screw if falling down so its velocity is 2 m s^{-1}. So this screw's initial velocity is then also 2 m s^{-1} upwards, then falling down. So it follows a sort of circular path when it falls down. Right. So take the screw; initial velocity is then 2 m s^{-1}. Now the problem is : what they mean here is when the screw falls down, it falls down from the roof top, down to the bottom, the floor of this lift. So as this lift is moving up, the screw falls down. So as the screw falls down, the lift moves up, and they give you the height of the lift and you can deduce from that fact, seeing the lift is moving up still, the screw falling down, the distance which the screw must fall to the floor is going to be less than 3 metres. So here the screw is falling down while the lift is say moving that distance there. (Draws) :



The distance - call it x. The lift is moving upwards and this distance here will be 3 minus x for this is the distance s then for the screw. So the distance the lift moves up is going to be x, the distance the screw will fall down is going to be 3 minus x due to the height of the lift given. Now we've got x; we've got $V_o t$ for the screw and we've got a upwards for the lift. Now we have x, first in the case of the lift. Using $s = V_o t + \frac{1}{2} a t^2$. The distance that the lift will move up is x and in this case the lift is moving upwards, so we have $S = V_o t + \frac{1}{2} g t^2$, and then in this kind of problem we have to choose the direction for positive and the negative direction. So I choose up as positive and down as negative. This for g. So g in the case of the screw will be a minus 10 m s^{-2} and g in the case of the lift is then + 10 m s^{-2}. So now we've got g for the lift, we've got to find t, we know V_o. But I can't see how to use this - they give you the acceleration of the lift upwards. The first question is how long will the screw take to reach the floor. If you take this as the equation for the screw (3 - x) is the distance the screw will fall; Vo is 2 m s^{-1}. You've got to find t and you know g as -10. So by pulling S = 3 ....

I : That S; V_o, t is that for the screw?

S : This here is for the screw so the distance for the screw is 3-x and V_o,t, so $(3-x) = 2t + \frac{1}{2}(-10)\ t^2$

$3 - x = 2t - 5\ t^2$

That's in the case of the screw. Now we're interested in this x. That enables us to solve t for the screw, so this x is the distance that this lift will move upwards and use the same equation $s = vt + \frac{1}{2} gt^2$. (Writes on board). For lift:

$v = 2$ m s^{-1}

$a = 2$ m/s^2

$g = + 10$ m s^{-2}

and so using this information you are able to find the distance that the lift will move up.

I : Well, which is the acceleration of the lift - g or a?

S : Acceleration of the lift is a.

I : What has g got to do with the story?

S : g is acting downwards on lift. So we can leave out g and use a for .....

$s = V_0 t + \frac{1}{2} at^2 = 2 t + t^2.$

By using $a = 2 \text{ m s}^{-2}$

I : What is S?

S : S is the distance that this lift moves up and the distance the screw moves. But they're not the same. You can't say that this 3-x or this S is equal to that S.

I : Why not x here?

S : This is x (correcting)

You can say x here

$-S = - x = 2t^2 + t^2$

I : Why do you put minus?

S : To eliminate x. Let me just check it again. $3 - x = 2t - 5t^2$.

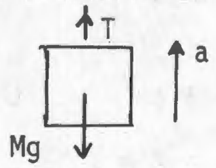
I : Oh,I see. OK don't worry further. Remember this is a distance, if you choose direction you must also choose an origin, so that that distance would be actually negative because it is going down. But for the rest you've got it quite well. Tell me, have you seen this type of problem before?

S : Yes.

I : Anyway, thank you. It has been very helpful talking to you. This gives us a better understanding. One gets some interesting thoughts. This idea on g. I couldn't work out what you're up to.

S : I know it has something to do with the lift or throwing objects upwards.

I : But you see, the screw has acceleration g because of its free fall. But the lift is being pulled by cables and all sorts of things. So actually the lift has an acceleration of 2 m s^{-2}. You see if you take this one (draws):

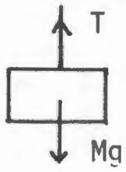


that's the lift, there's your tension in the cables. This is the weight of the lift and T = Mg must equal Ma and that a is 2. So g doesn't play a role in this except on the screw.

ANALYSIS OF INTERVIEW 17

Error Factors

1) While signs are given to g in problem 2, it is not assigned to S.
2) He has a very strange view of g for the lift : "g" is acting downwards on the lift". He has difficulty in differentiating acceleration due to gravity and an acceleration from other mechanical causes. He seems to confuse this problem with that of tension in the cable of a lift:



in which case
$T - Mg = Ma$

Cognitive Deficiencies

1) He tends to concentrate on a single aspect of the problem (the velocity v in problem 1) and experiments with equations in an effort to get an answer. This is another example where insufficient data analysis leads to difficulty in elaborating the problem. No simple approach to a solution using suitable equations seems to suggest itself.
2) His lack of constructive thinking causes him to search around for equations which look the same, resulting in his writing mathematical nonsense : $v^2 = 2t = 10t$.
3) He appears not to be aware of the fact that equation $s = V_0 t + \frac{1}{2} at^2$ can be used for the car after stating that it applies to the lorry. Hasty and inaccurate categorization of details are subsequently accepted without questioning. The student does not seem to realize the need to review steps taken to check his accuracy.

4) There is little qualitative evaluation of the solution steps. The entire problem centres around the equations. There seems to be no spontaneous comparative behaviour in the elaborational phase.
5) He demonstrates peculiar inaccuracies in data collection - after stating that the lift has a upward acceleration of 2 m s^{-2} and recording it, he then says it has an acceleration of g = 10 m s^{-2}.

Mathematical Difficulty

1) Finds difficulty in solving simultaneous equations.

INTERVIEW 18

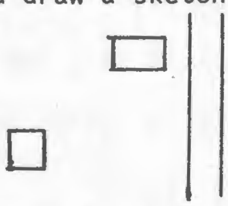
I : Would you like to explain how you set about solving problem 1?

S : I would write down all the information that is given. First a was given for the car. (Writes : Car Lorry

$a = 2\ m\ s^{-2}$ $V = 10\ m\ s^{-1}$

Hence for the car we have acceleration $2\ m\ s^{-2}$. And we have a lorry; speed of $10\ m\ s^{-1}$. That was given to us. And it was also said that it was at the same instant for .....

Let's try and draw a sketch. (Draws):



$acc^{-n} = 2\ m\ s^{-2}$
2-
$v = 10\ m\ s^{-1}$
10-
Distance = meters

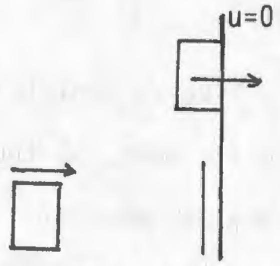
I have the car here, the robot there and the lorry here. At the same instant one there and one there. So how far beyond: that means we have to calculate the distance, therefore that is this distance in meters and the units is meters per second. So I will isolate the car. Acceleration $2\ m\ s^{-2}$. So from rest - that means $u = 0$. We have the equations $V^2 = u^2 + 2\ a\ s$, and $V = u\ t + \frac{1}{2}\ a\ t^2$. Write down the equation for distance which one has. $V = ut + a\ t^2$, so $u = 0$ so we find the velocity of the car.

$$\left.\begin{aligned} v &= u\ t + \tfrac{1}{2}\ a\ t^2 \\ &= \tfrac{1}{2}\ 2 \end{aligned}\right\} \text{This does not work.}$$

So we change our mind, to this one. So it will be $V^2 = 2 + 2\ a\ s$. We have the lorry on this side and we have a speed $V = 10$ m/s, then the equations are $V^2 = u^2 + 2\ a\ s$ and $V = u\ t + \frac{1}{2}\ a\ t^2$.

I : Before you go further how did you picture that all? What's happening?

S : We have 2 objects (draws)



but we must isolate it to get equations for the car and the lorry. They start from here and they are both there at the same instant.

I : What does that mean : at the same instant?

S : That means that what happens to the one here is happening to the other one at the same moment. So I try to write equations out for both and see what I get for either sides and then I have to see what I can relate.

I : If you were to draw a sketch of how they move what would you draw? Let's say they are here at the robot, here.

S : Yes. There is the robot and the car is right there and the lorry is here. The car is moving. The car starts here and goes there so that the lorry is on the left. It comes with a speed of ten, you don't know the distance.

I : So at the instant that the car leaves the robot where is the lorry?

S : He is also here at the robot. u is not nought for the lorry since it is moving already.

I : OK. Now what is the problem? That is where it starts so what happens now?

S : There will come a time that the lorry will overtake the car, go pass it. The problem asks how far beyond. We must know in the first place it will pass the car. The question was how far beyond. So that means we assume that it goes past.

I : What will go pass?

S : The lorry will pass the car since they ask how far from the starting point it will overtake the lorry.

I : Which one is ahead?

S : The lorry. I don't know, sorry .... just assuming.

I : OK. Here they are at the robot, what happens at the robot? The car pulls away and the lorry?

S : Oh yes. The lorry is there, its speed is ten and it goes past the car.

I : OK, so the lorry moves ahead of the car. Now where do they catch up?

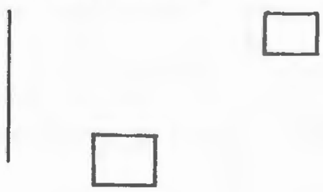
S : Yes. The lorry will be going there and the car will be here. We give the distance a variable x, so then ..... (Student appears to be stuck). Somehow I don't know, we have to find an equation.

I : Never mind the equations. Lets look at the problem again. What are you asked?

S : The distance from the robot, the starting point to where the car will overtake. So we must find the distance. So here we .... If this is the starting point we want the distance there. How far beyond this. We can call that x. If we do calculation we can find x; the distance from the starting point to here where the car overtakes.

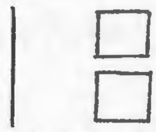
I : OK, now where the car overtakes, just draw that. How will it look?

S : There's a car and there's a lorry. (Draws)



I : Now the car is just about to overtake the lorry, how will they move with respect to each other?

S : They will be next to each other. Like this.



I : Yes, now? Is that the position that you're looking for?

S : Yes. Then we can draw the x for the car there.

I : What do you notice about x?

S : x is common, the same for both the lorry and the car. If this is the car I'll call it x_c and the other for the lorry x_ℓ. But I'll remember that $x_c = x_\ell$.

I : Why don't you just call it x?

S : OK.

I : Now write down an expression for x_c. The distance that the car will move.

S : U = 0 there, and a was given as 2 m s^{-2} and so if I had been given all this, I can use it all in the equation and see if I can work out something.

I : Acceleration constant means that you can use one of the three equations. (Writes) : $v^2 = u^2 + 2\,a\,s$

$$s = u\,t + \tfrac{1}{2}\,a\,t^2$$

$$v = u + a\,t$$

They are called the constant acceleration equations. Which one do you want to use? Have you got initial velocity?

S : Yes. I've got u and a and I want S. The first won't work since there is no t given.

I : Talking about t, what can you say about t? The time that the car takes to go from the robot to where the car overtakes, how does that relate to the time that the lorry takes to go from the robot to where the car overtakes?

S : It's the same. They don't tell us that, but I would assume that and also they wanted where they overtake, where they would be together. So now I would assume that the time is the same.

I : Would you just assume that? Is that a convenient assumption or is it actually so?

S : I don't exactly know.

I : Well think about it, here are these two objects, here's the lorry and here's the car. If the car is standing at the robot, what's the lorry doing?

S : It's coming on at 10.

I : Here the lorry reaches the car, what is the car doing?

S : It is starting to move.

I : The lorry goes ahead so what does the car do?

S : It catches up.

I : Right, there they catch up so they have moved the same distance, OK?

S : Yes.

I : That's what you said over here. Have they moved for the same time? From the time here at the robot? Say you were to have a stopwatch and you stood at the robot at the instant that the car pulled away and the lorry passed it. And you clicked the stopwatch. Would you need to have two stopwatches, one for the lorry and one for the car or would one be enough?

S : I'm not sure.

I : OK. Think about this. You've got two runners. The one is in the starting blocks and the other one comes and at the instant this chap gets out of the starting blocks, this chap passes him, and say they both come to the tape at the same instant. Is there any difference in the time?

S : No.

I : Ah!

S : So that means we definitely have the time the same.

I : OK. So the time is the same and the distance is the same.

S : Yes.

I : And now? Is there an equation for the lorry?

S : Yes.

I : What? How's the lorry moving?

S : It is moving at a constant speed of 10.

I : What does that mean? What is the acceleration if it is riding at a constant speed?

S : Nought.

I : Yes.

S : So v = 10 and a = 0. So we can use the top one.

I : OK if you use the top one that say $V^2 = u^2 + 2\,a\,s$ and a is 0. It tells you that $v^2 = u^2$ or v = u. Final velocity is equal to initial velocity. The velocity is constant so won't the final velocity equal the initial velocity?

S : Will you just help me with the third equation?

I : Look at the second equation, for the lorry.

S : $S = ut + \frac{1}{2}at^2$ but acceleration is zero.

I : So distance = velocity times t.

S : So S = 10t.

I : Now that is for?

S : That is for the lorry.

I : OK. Now what does it look like for the car?

S : For the car : $u = 0$; $a = 2$; $S = ut + \frac{1}{2} at^2$

$s = 0 + \frac{1}{2} \times 2 \times t^2$

So : $S = t^2$

So s is the same for both so we have $10t = t^2$

So : $t = 10$.

We put it in here. For the lorry $s = 10 \times 10 = 100$

And for the car also : $s = 100$

I : OK, that's been useful. As I say, what we're interested in is to see what problems you have, you seem to have a difficulty in visualizing the problem. We find that to be so for others too. Thank you.

ANALYSIS OF INTERVIEW 18

Cognitive Difficulties

1) Examines the problem only to the extent of extracting numerical data. She immediately attempts various equations. Data collection is governed by the equations but the problem statement itself is not carefully analysed. In that sense the data gathering process is unplanned and unsystematic.
2) Each object is considered separately. She seems to have great difficulty relating the motion of the two objects.
3) The drawings indicate that the student has difficulty visualizing the actual initial situation implied in the statement of the problem. She insists that the lorry passes the car sometime after leaving the robot.
4) Because of not clearly visualizing the entire situation, she is not able to focus clearly on the initial and final points implied in the statement of the problem. Consequently, she has no clear idea of what is required to solve the problem and her elaboration of the problem shows no particular goal-orientation.
5) Implicit data are only extracted with great difficulty. She sees, for example, that the distance travelled by both objects is the same (after some discussion), but cannot immediately see that the time taken by each vehicle to cover this distance, is the same.

INTERVIEW 19

I : What we're going to ask you to do is to read through the first question there and to explain how you would work out a problem like that. We're trying to determine how students think about these things, the sorts of ideas that come into your mind as you work through the problem.

S : I'll try and draw a diagram (She does not actually make a sketch). Here we see a car starts with a constant acceleration when the traffic light turns green. Here's the car and the acceleration 2 m s^{-2} and then at the same instant a lorry is travelling at a constant speed of 10 m s^{-2}. Then they want to know how far beyond the starting point will the car overtake the lorry. Now we have to associate these with the equations of motion. They give you the acceleration 2 m s^{-1}.

(She writes): Car Lorry

Car	Lorry
$a = 2$ m s^{-1}	$v = 10$ m s^{-1}
$v = ?$	
$s = ?$	
$t = ?$	
$u = 0$	

So acceleration is 2 m s^{-1}, velocity is unknown and the time is also unknown. Then for the lorry they only give the velocity. Then we sort of have to try and solve the problem for the car and the lorry simultaneously to get the solution. The initial velocity is given so we can put $u = 0$. I can use the equation $s = u\,t + \frac{1}{2}\,a\,t^2$, but then we haven't got time so that means something .....

I : You were saying earlier that you want to make a diagram. How would you describe what's happening by making a sketch.

S : Say this is the traffic light and the car is stationary and then it starts with an acceleration of 2 m s^{-2} and then at the same instant a lorry travelling with constant speed of 10 m s^{-1}. (She draws) :

$$\begin{array}{l} \quad\quad\quad\;\; * \;\; x \\ \longrightarrow L \; V = 10 \text{ m } s^{-1} \end{array}$$

I : So where would the lorry be when the problem starts?

S : It just passes the car.

I : So where would it be on that sketch?

S : Here .....

I : OK. And then what is the next thing to happen?

S : Say the lorry passes then the car starts.

I : And now what happens? If you were to see such a thing at the robot. Say you were standing here with the car there. The car is standing at the robot and here the lorry comes along. Now the robot turns green, what's going to happen now?

S : The lorry will be moving and then the car starts and at a certain point they sort of will have travelled the same distance.

I : Right. So where will that be?

S : It will be say here, I don't think quite far. (On sketch)

$$\begin{array}{lll} & x & x \\ \longrightarrow L & & x \end{array}$$

I : So what happens at that point?

S : At this point the velocity of the car and the lorry will be equal.

I : Why should the velocities be equal?

S : Let's look at the equations of motion.

I : Just before you do, what happens at that point where the car and the lorry as you say, are equal? What is equal about them?

S : The velocity.

I : The velocity?

S : OK, the time and the distance they travel is equal.

I : The distance from where?

S : From where they come together at the certain point, and the distance they travel together for some time.

I : OK, what is the distance you are talking about. Could you indicate that on your sketch?

S : Say they come together at that point and they travel together for quite a distance, up to here (indicates beyond $\overset{x}{\underset{x}{\longrightarrow}}$)

I : Why do they travel together.

S : They travel the distance there.

I : What has that got to do with solving the problem?

S : Now they want to know when the car will overtake the lorry.

I : So when will that be according to your sketch.

S : Say the lorry is travelling a certain speed then the car's speed has to exceed that of the lorry to overtake it.

I : So where is that going to be? The car and the lorry start off here and now they meet over here ...OK. So these variables that you have written down here a, v, s, t and u where do they come into this whole thing? What is a for example?

S : a is the acceleration of the car.

I : OK, so that is it.

S : v is the velocity of the car.

I : Which velocity is that?

S : Final velocity.

I : OK. Now where is the final velocity. When you say final, what do you mean?

S : (She studies things for a while) Say I want to work out the distance and the time that it takes for the lorry and the car to move until they have the same speed. Then I can use the equation s = ut. Then I can equate the two and then I think time is 10 minutes.

I : Try that.

S : I've got an initial velocity of nought, so : $s = t^2$ and then for the lorry I can use s = ut, because the lorry is travelling at a constant speed. Then that will equal to 10t, right, now, I can see that these two would be equal.

I : Are you sure that s and t are the same for both? What is s?

S : s is the distance I'd say they're equal, because it is the point where they come together, so these are equal.

(Doing calculation) :

$$S = ut + \tfrac{1}{2} at^2 = \tfrac{1}{2} at^2 = \tfrac{1}{2} (2) t^2$$

$$\therefore s = t^2 \quad ; \qquad S = v\,t = 10t$$

$$t^2 - v\,t = 0 \qquad t\,(t - v) = 0 \qquad t = 10s \text{ or } t = 0$$

I : What does this mean t = 10?

S : The time when they will meet.

I : And time t = 0, why is that a solution? Where is t=0?

S : Here when the car starts with constant acceleration. It means than the car is at the traffic light.

I : Yes, and obviously when t=0 then the two pass each other. Just on this point, where you keep saying that the velocities are equal. Are the velocities equal? If the velocities are equal then what must be the velocity of the car at this point here.

S : 10.

I : But is it?

S : For the car .....

I : $V = u + at$, right.

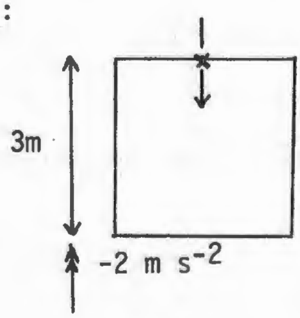
S : $u = 0$ and $a = 2$ and $t = 10$. $V = u + a\,t = 0 + 2 \times 10 = 20$

I : The fact that they meet over here doesn't mean that the velocities are equal. It means, as you correctly said, that the distance is equal. If they both start off here and they both go that distance. And they also move for the same time. That's what's equal. OK, do you agree with this. Think about that. Say for example: my two fingers, this one is the car standing at the robot and here comes the lorry. Now what happens, when the lorry gets here, it will go pass the car and the car will start accelerating and it will catch up. Now when they catch up over here are they going to move at the same speed?

S : No.

I : What will happen, what will the car do? It will go faster than the lorry. So sometimes with this type of problem it helps just to see what is the problem all about. Would you read the last problem there about the lift and would you make a sketch of the way you see the problem?

S : We're given here a lift of height 3m and the acceleration is $2\ m\ s^{-2}$. (Draws):



Lift	Screw
$a = 2\ m\ s^{-2}$	$a = 10\ m\ s^{-2}$
$v = 2\ m\ s^{-1}$	

Then they want to know how long it will take for the screw to fall from the roof of the lift to the floor when the lift is moving at a velocity of $2\ m\ s^{-1}$. I'd say that it's acceleration is negative because the lift is moving upwards, and the velocity is given as $2\ m\ s^{-1}$.

I : Has velocity also got a sign?

S : No.

I : How do you see that problem? If somebody were to ask you to describe what is happening there what would you say?

S : A lift is accelerating upwards at $2\ m\ s^{-2}$ and when it is at a velocity of $2\ m\ s^{-1}$ a screw falls from the roof and they want to know how long it will take to reach the floor.

I : If you were to draw how the screw moves, how would you do it?

S : The screw comes down and the lift continues to move up.

I : So would you say the screw falls down in a straight line? Does the screw move up at all?

S : No, only the lift is moving up. Say the screw moves down, doesn't it have an acceleration of 10?

I : Plus or minus?

S : Plus.

I : Why is it plus?

S : It is in the direction of the force of gravity.

(She is quiet for a long while as she studies the problem).

I : What are you trying to do now?

S : I'm trying to get an equation of the screw to associate it to that of the lift and equate the two.

I : Oh. What is the initial velocity of the screw?

S : It is zero.

I : v which you wrote here, is that for the screw or the lift?

S : For the lift.

I : Is that initial or final velocity?

S : Final.

I : Why is it final?

S : I wouldn't say it's final, because there it says at the instant when the velocity is two.

I : What does that expression mean : "at the instant"?

S : It means that the velocity is changing.

I : Correct, so is that initial or final, what would you say?

S : Initial.

I : Yes. Let's not take up more of your time, I know you're busy in any case. Let me just show you very quickly how to do this problem ......

ANALYSIS OF INTERVIEW 19

Error Factors

1) She confuses the same position of the objects in problem 1 with the same velocity. She even postulates that the car and lorry will travel together for some distance.
2) She ascribes signs haphazardly without first choosing axes. She only gives a sign to the acceleration, but not to the velocity. Acceleration due to gravity is always regarded as positive.

Semantic

1) The word "falls" is interpreted as downward motion only.

Cognitive Difficulties

1) She does not easily visualize the actual details in the problem. Her insistence that the lorry and car have the same speed causes her to lose sight of the current features of the problem she has enumerated up to that point.
2) She makes no qualitative problem analysis thus leading in some ordered manner to a method of solution. Rather she seems to choose the equation $S = u\,t + \frac{1}{2}\,a\,t^2$ rather randomly, apparently based on previous experience with a similar problem.
3) There is no spontaneous evaluation of problem details. She is keen in both problems, to fit values into the relevant equations. The focus of attention is always on an equation.

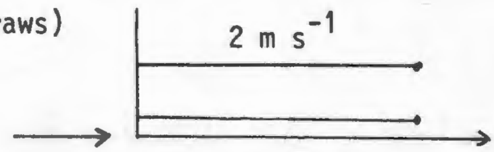
INTERVIEW 20

I : What we would like you to do is read this problem and explain how you would work it out . (gives problem).

S : The time this takes from the robot will be the same. Must I write it down?

I : Whatever you want to. If you wish to draw a sketch, do so.

S : Time will be the same. (Draws)



I : How do you know that?

S : By the time it overtakes, it will be the same time because both start at the robot. The distance will be the same. (Writes) : $s = v\ t$ The distance will be the same.

I : These two points, what do they represent?

S : That one is the car accelerating at 2 m s^{-1} and the lorry overtaking the car. (He appears uncertain as to what to do next).

I : What does the expression "at the same instant" mean?

S : While the car is pulling away from the robot.

I : Where is that on your sketch?

S : At the points. The car is pulling away while the lorry is passing.

I : So how would you describe what happens further?

S : Lorry is going to pass and then the car will catch up.

I : So where will they actually catch up?

I : They catch up here, some distance from robot.

I So this will have the same distance and the same time.
So this expression $s = vt$, what does that apply to?

S : That will apply to lorry, no the car, no the lorry because it has constant acceleration.

I : What about the movement of the lorry makes you say that s = vt? How is the lorry moving?

S : With constant speed.

I : What does that mean i.t.o. acceleration?

S : No acceleration.

I : What does this expression here mean to you that the car "starts with constant acceleration of". What does that bring to your mind?

S : Not a jerk; steadily upward to a certain maximum velocity.

I : Does it bring any equations to mind?

S : (Calculates and write down equations on a page):

$$v = u + a\,t$$
$$s = ut + \tfrac{1}{2}\,a\,t^2$$

I : So what is s now?

S : S is the distance. Not starting from rest. It doesn't say so. (Reads). Oh! A car starts ..... so it is from rest and the initial velocity is 0. So (writes $S = \frac{1}{2}\,a\,t^2$ then thinks)

I : So you have $S = v\,t$ and $S = \frac{1}{2}\,a\,t^2$. Is it the same s?

S : No .... Yes it is the same distance.

I : So can we equate the two?

S : Yes, but we still don't know the speed, only that the speed is constant. (He then calculates and stumbles over $v = 10\ \mathrm{m\ s^{-1}}$ and finds the correct answer. He is surprised when the answer falls out). It does not look so easy when you start out.

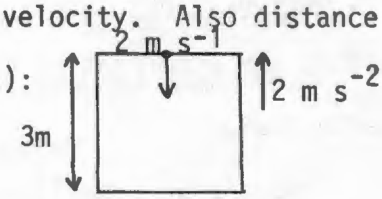
I : You seemed to understand the problem quite well - what it was about. Most of the students don't do that. They write down the equations first. You did it the other way around.

You first did the evaluation of the problem to understand what it is all about. You're quite a peculiar case. Most of the students take out the numerical data. You first evaluate, which is the correct thing to do, you try to understand what the problem is about. But then you do not write down the data accurately.

OK. Read through the second problem now please.

S : (He reads the problem and as he reads he mumbles some of the data):

$a = 2\ m\ s^{-2}$, it is going up. Velocity of lift is $2\ m\ s^{-1}$. Going up. Still needs its final velocity. Also distance is 3. So s is equal to 3 m. (Draws):



(Writes) : $s = u\ t + \frac{1}{2}\ a\ t^2$

$V = U + a\ t$

0   2

I've got to find acceleration of the screw as it falls down. But now I can use g for that. Can I? Gravitational force. So now (filling in equation $S = u\ t + \frac{1}{2}\ at^2$) $3 = 2\ t + \frac{1}{2}$

The screw falls this distance 3m, right? It's going upwards at $2\ m\ s^{-1}$. Acceleration is going to be down.

We can use (writes) = $v^2 = u^2 + 2\ a\ s$

$0 = 2^2 + 2\ a^8\ 3$

$-4 = 6a$

$a = -\frac{2}{3}$ and it will be a deceleration.

Now we can put that in there $(3 = 2t + \frac{1}{2})$

I : Which acceleration is that?

S : That is the downward one because it is coming down - deceleration of the screw.

I : What is the force on the screw?

S : Force? (Surprised)

I : Yes. If a thing accelerates it must have a force on it. What is the force on the screw? Remember it is dropping.

S : Oh. "g" pulling it down. So .... does that matter? So you shouldn't calculate the actual acceleration. You should just use g. Is that it?

I : What do you think? Look, when a thing falls under its own weight, the acceleration is g. Let's just take a look at your sketch again.

S : I see there's no propelling force.

I : Just think about this. The expression "the screw falls", how do you understand that?

S : It just falls, there's nothing pushing it; it just falls.

I : It just falls under its own weight. OK. Does it move up at all?

S : No. Yes .... I mean, yes, before it is falling it moves up with the lift.

I : At the intstant it comes loose in the roof, what happens?

S : It will just fall down.

I : Think about this. If you are sitting in a motorcar and you decide to get out while it is moving, what happens to you?

S : You will move with the motorcar at the same speed.

I : So what will happen to the screw?

S : It can't move up; at this instant it will move at the same speed as the lift.

I : And what is the speed of the lift?

S : It is 2 m s^{-1}

I : What direction?

S : Up.

I : So what happens to the screw?

S : Oh, I see what you mean. So the screw will also move up.

I : What's the acceleration of the lift?

S : 2 m s^{-2}

I : Which way?

S : Upwards.

I : Right. What's the acceleration of screw?

S : Also moving upward.

I : But what's the acceleration?

S : 2 m s^{-2}. Oh no, that's the lift.

I : It's free from the lift now. (Gestures). Lift is moving like this at the instant the screw falls out. OK. What happens to the roof of the lift?

S : It moves up.

I : And what about the screw?

S : It will move up.

I : Will it catch up with the roof?

S : No.

I : Why not?

S : The lift is moving faster.

I : OK, but what is its acceleration?

S : (Not certain).

I : Is it attached to anything?

S : No.

I : So its moving freely like this (indicates). So what's its acceleration?

S : g

I : Which way?

S : Down.

I : So the acceleration of screw is down. The acceleration of the lift is up. So the roof and the screw will move away from each other. The screw will move up and then down. What's the initial velocity of the screw?

S : It's moving with 2 m s^{-1}.

I : Which way?

S : Up.

I : What's the acceleration of the screw?

S : g

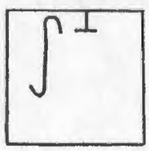
I : Which way?

S : Down.

I : So one direction is up and the other down, what must be done?

S : (Not sure)

I : So one must be negative. How would you draw out how the screw moves?

S : (Draws a sketch): 

So lift moves up. E.g. screw is here, then will move like this. So then it drops.

I : How far does it drop?

S : Drops 3 m. Not from this point though. It drops ..... Yes it drops. Yes, 3 m.

I : (Indicates with hands). Say this is the 3 m height. And here's the screw. And at this point the screw falls out. Now what happens to the screw? There goes the lift. Let's say we started on the desk. Here's the screw. Now what is going to happen?

S : The lift moves up. The screw will move up and fall down to there (desk).

I : To where you started?

S : No.

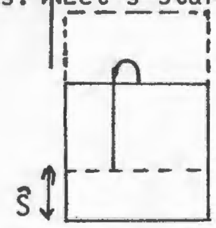
I : What has happened to the floor of the lift?

S : It has moved up.

I : How much?

S : This distance here. (Indicates the correct distance). But that is 3 m.

I : Look at it like this. Let's start off with the lift as you have it. (Draws)



The lift starts here. Now the screw moves up as you have drawnit and then moves down. Now what happens to the lift?

S : Up.

I : Let's say it moves up to here, the dotted lines. (Draws). Where is the screw?

S : To the floor.

I : So the lift has moved from that position to this new position of the floor. Let's call that S. How much has the lift moved up? The floor has moved from there to there. How much has the screw moved then, from there to there.

S : (3 - s)

I : Ah! But don't forget direction.

S : Oh -(3 - s)

I : You write down the equations now.

S : (He writes) $a = 0$ $\quad s - 3 = 2\,t + \frac{1}{2}\,(-g)t^2$

$s = s - 3$ $\quad s = 2\,t + \frac{1}{2}\,2 \times t^2$

$u = +\,2$

ANALYSIS OF INTERVIEW 20

Semantic

1) The word "falls" is interpreted as downward movement only.

Error Factors

1) He shows no awareness of the need for signs in problem 2.
2) He tends to attribute the acceleration of the lift to the screw. His idea of the concept of freefall is vague.

Cognitive Difficulties

1) While he makes a good qualitative evaluation of problem 1, he is inaccurate in the manner with which other data are gathered. For example, he does not easily see that the initial velocity of the car is zero. He speaks of the lorry as having constant acceleration. He is surprised to find the velocity of the lorry actually given in the problem after stating that it is unknown.
2) In the second problem, he makes immediate (and incorrect) decisions as respects values (such as s) and immediately inserts these into equations. The visualization attempted in the first problem does not seem to be a practice but is engaged in only if he cannot see an equation which can be used.
3) In problem 2 his entire attention is taken up by the screw. He does not relate the movement of the lift to that of the screw at all. He experiences difficulty in dealing with the two sources of information together.

INTERVIEW 21

I : What I would like to ask you to do if you don't mind, is to read through the first problem and then try and tell us how you would set about solving the problem.

S : I'll start with the equation of motion. Try to find a relationship. There are two equations involved. Try to find one for the lorry and one for the car.

I : Would you like to write on this paper and then you can show us how you do it . Explain what you're doing now.

S : (He writes) Car Lorry

$a = 2\ m\ s^{-2}$ $v = 10\ m\ s^{-1}$

I : What is the problem all about, would you say?

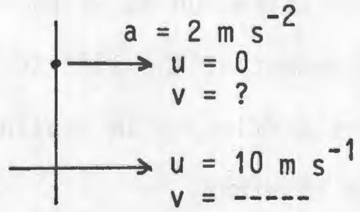
S : The car is moving at constant acceleration of $2\ m\ s^{-1}$. So its initial velocity is zero. But the lorry is already moving so at some time the lorry is going to pass the car and we want to know the distance.

I : Which distance?

S : Between the starting point of the car and the point where the lorry passes the car.

I : If you are asked to draw a sketch of the problem, how would you?

S : (Draws)



The lorry is already moving, and at this point will pass the car.

I : What is the acceleration of the car?

S : The acceleration is constant.

I : Initially?

S : Initially it is zero.

I : On your sketch what happens further?

S : They both move on. (Discussion of the significance of the initial point).

I : So now here the lorry goes and what happens to the car?

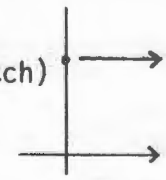
S : The car should overtake the lorry.

I : Where does the car overtake the lorry on your sketch? They start here, so this is the robot I suppose. So the lorry goes and the car goes and at some point they catch up. Is that an important part of the problem?

S : Where they catch up?

I : Yes.

S : No. (Thinks for a while). (Draws on previous sketch)



Where they catch up there are going to be some things which are equal for the car and the lorry. I think if you can equate two equations and then you make assumptions depending on what the equations are.

I : You say some things are equal. What are they?

S : Velocity (not certain).

I : At the point there where the car overtakes the lorry, both the car and the lorry are there. So you say things are equal at that point. What things are equal?

S : Here nothing is equal. And here the time.

I : Sure of that?

S : Yes.

I : What else is equal?

S : This distance equals to that distance.

I : Yes, distance is equal. So what do you do now?

S : (Writes) : $v = u + a\,t$; $S = ut + \frac{1}{2}\,a\,t^2$; $v^2 = u^2 + 2\,a\,s$

(He now solves the problem without too much difficulty)

I : Would you like to read through the second problem and explain how you would solve it.

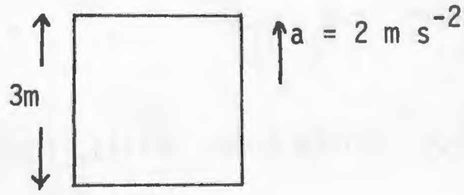
S : About the lift?

I : That's right, I'll give you some time to read it .....

How would you describe what's going on there?

S : The lift is going upwards with acceleration of 2 m s^{-1}. Then a screw falls from roof when the velocity of lift is 2 m s^{-1}.

(Draws and writes):



Lift	Screw
$v = 1\ m\ s^{-1}$	$u = 2\ m\ s^{-1}$
$a = 1\ m\ s^{-2}$	$a = 10\ m\ s^{-2}$

I : What is the initial velocity of the screw?

S : Zero.

I : Are you sure about that?

S : No, since the screw was travelling upwards with the lift.

I : So does the screw fall upwards? Can one say the screw falls up?

S : No.

I : If you were to draw the way in which the screw moves, how would you draw?

S : 

I : That's it. Yes so it has an initial velocity of 2 m s^{-1} up.

And the acceleration of the screw, what is it?

S : g

I : Which way is g?

S : Down .....

I : So u = 2 and g is 10. Values are right. What about their signs? Is there a sign there?

S : g is down, positive.

I : So is up negative?

S : Yes.

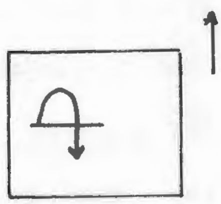
I : So should you indicate it there somewhere? (He does not). What happens now? What is the distance that the screw falls?

S : The screw falls ...... You can't say what distance.

I : Why not?

S : Because the lift is moving.

I : Think about it. Draw it out. (He draws):



Well that looks great. So how far does the lift move?

S : The lift?

I : Yes.

S : It moves up a distance.

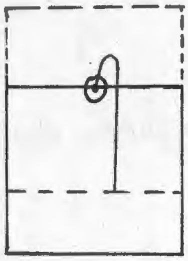
I : What do you think?

S : We must work out what distance the screw falls.

I : Why do you think they gave you the acceleration of the lift if it is not necessary.

S : We divide the distance the lift moves by that certain velocity ....

I : OK. Let's just draw a sketch here. Let's say that's the lift



and at this instant the screw falls out and moves upward.

So what is the lift going to do in the meantime?

S : It will move upward.

I : Right! (Dotted line on sketch).

The lift is now here. The screw goes up and then it comes down and here it collides with the lift. So how far has the screw moved?

(Only after considerable discussion does he determine the distances moved by the lift and the screw).

With this type of motion you need to choose an origin and you then choose a system of axes. There is your origin. What's your system of axes?

S : I'm not sure.

I : Let's say you make up positive and down is then obviously negative. OK? Then the lift moves up S but the screw moves down 3 minus S. The screw moves up with speed plus 2, but g is minus 10 because it is down.

S : Why is it plus 2?

I : Is the initial velocity up or down?

S : It is up.

I : Therefore positive. What about g?

S : g is negative.

I : Yes. Now you have $s = ut + \frac{1}{2} at^2$ for the lift and for the screw. Just write it down for the lift there.

S : Now we've got to get the time it takes for the screw to fall to the floor.

I : What do you think? What is t?

S : I'm not sure.

I : Let's look at the equation for the screw. What's the initial velocity?

S : Zero. So can't we use the freefall equation?

I : Why, what is the freefall equation?

S : $s = \frac{1}{2} g t^2$

I : The freefall equation is only $S = \frac{1}{2} g t^2$ when u is 0. But u is not 0 here. It is still freefall. It is still falling under its own weight, but it is not starting with an initial velocity zero.

ANALYSIS OF INTERVIEW 21

Error Factors

1) The student seems to feel that if a car has an acceleration it must be moving. This is related to appreciating the physical significance of the expression "at the instant".

2) While he states that the velocities of car and lorry are equal when they draw level, he is not too certain about this.

3) The signs for the values s, u, v, a in the equations in problem 2 is a source of confusion. It is apparent that he has been drilled to view g always as positive when an object moves downward. He does not think in terms of a positive and negative vertical axis.

4) He does not succeed in appreciating fully the fact that the initial velocity of the screw, $2\ m\ s^{-1}$, is that of the lift. Even after clearly acknowledging it at the beginning he returns to the freefall equation, $s = \frac{1}{2}\ g\ t^2$ again.

5) His idea of freefall is limited to the case $u = 0$.

Cognitive Difficulties

1) He has difficulty visualizing the features of the problem. Thus he insists initially that the distance required relates to the lorry passing the car - when asked which distance is required he says : "Between the starting point of the car and the point where the lorry passes the car".

2) His problem analysis does not enable him to focus on the two important points in the problem viz. when the vehicles pass at the robot and when they draw level again. He does not accurately extract the other than numerical data from the problem

to enable him to see that it is the car which must reach the lorry.

3) Data analysis in both problems consists almost exclusively of recording numerical data and then searching for suitable equations. He is able to perceive implicit data in problem 1 only after being helped to draw a sketch.

4) While he writes down the data of each object separately in problem 2, he makes no effort on his own, to relate the two sources of information but deals with each entirely in isolation.

INTERVIEW 22

I : Would you read through the first problem. You'll probably recognize it. Just read through it and then tell us how you would solve such a problem.

S : How would I solve it?

I : Yes, what do you think, what kind of problem is it? (She sits for a long while)

S : One's given the acceleration of the car. One can find his velocity .....

I : So how would you start such a problem?

S : Well, I think that acceleration can give the time. From time one gets the distance and then find the velocity.

I : What is the first thing you would do when you see a problem like this?

S : I'd write down the formulae.

I : Write it down. (She writes : a = distance x velocity

$2\ m\ s^{-2}$ = distance x velocity

$2\ m\ s^{-2}$ = distance x $10\ m\ s^{-1}$)

That 10 metres per second, what does that apply to?

S : The lorry.

I : And the 2?

S : It is the acceleration of the car.

I : That's a little bit odd isn't it?

S : Yes. (Scratches out the $10\ m\ s^{-1}$ and inserts "v").

I : Why don't you try looking first at what is actually happening. How would you describe what's going on here?

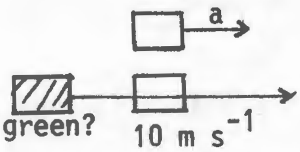
S : Well, as the car starts moving from the robot after the light turned green, the car starts moving and the lorry shoots pass.

I : And then?

S : And then the car. The lorry still has a constant speed, and the car is only starting.

I : How would you make a sketch of it?

S : From zero. The car .... (She draws)



I : So where are they when the robot turns green?

S : The car is just starting when the robot turns green and the lorry passes the car.

I : So where is the lorry in relationship to the car?

S : Here when it passes the car.

I : OK, there. So what happens now?

S : The lorry is still moving and the car starts to move.

I : And then?

S : The car will catch up with the lorry some time.

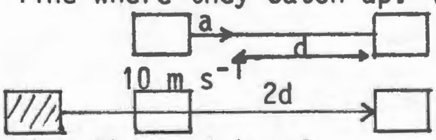
I : What is the problem? What do they want to know?

S : They want to know how far beyond the starting point will the car overtake the lorry.

I : So where is that?

S : Somewhere down there.

I : Draw that in. Draw a line where they catch up. (She draws) :



So what happens there when they catch up?

S : Well, the lorry still has the same velocity and the car will be accelerating in time.

I : OK. Does it help you? What must you find out?

S : How far beyond the starting point will the car overtake the lorry?

I : So what must you find out?

S : The distance.

I : What distance?

S : The distance from here up to where the car overtakes the lorry.

I : That distance is unknown so give it a symbol. What would the distance be for the lorry? (Now only does she insert d and 2d on sketch above).

S : More than d because the car starts here and the lorry is already moving along. I think about two d.

I : About 2d. Why? Say here we've got the car, at the robot and here comes the lorry. When you look at the situation where are they relative to each other?

S : The lorry comes along and passes the car when it starts.

I : Where is the lorry when the car starts?

S : Next to the car.

I : OK. So they're both here. What now?

S : So the car starts and the lorry is already moving.

I : The lorry keeps moving and the car starts. Here's the lorry and there they catch up from the starting point.

S : That distance d.

I : How far has the lorry moved?

S : Also that distance d.

I : So?

S : The distance is the same for both.

I : Right. Where do you get that equation, acceleration equals distance times velocity? (She does not answer). What sort of acceleration is this?

S : Constant acceleration.

I : Are there any equations for constant acceleration that you can think of?

S : I don't know.

I : Do you remember that $v = u + a\ t$

S : Yes. (She writes): $v = u + a\ t$, $S = ut + \frac{1}{2}\ a\ t^2$,

$$v^2 = u^2 + 2\ a\ s.$$

I : Those equations will apply to which of the two? The car or the lorry?

S : The car and the lorry.

I : What's the acceleration of the lorry?

S : $2\ m\ s^{-2}$

I : Acceleration?

S : For the car yes, but there is no acceleration for the lorry, but for the car.

I : Yes, so what equation applies to the lorry?

S : This one? (points to $v = u + a\ t$)

I : What's the value of a?

S : There is no acceleration.

I : So what does this equation work out to be for the lorry?

S : Nought.

I : $V = u$. Is that so? What is V?

S : $V = 10$.

I : It starts off with 10 and it ends off with 10, so what will the distance and acceleration be for the lorry?

S : The distance is d, and the acceleration is 2 for the car.

I : For the lorry.

S : Oh, for the lorry. The distance is d and the acceleration is nought.

I : So there you've only got one equation for the lorry $s = u\ t$.

S : Yes.

I : You called $S = d$, so if you put in the value what do you get?

S : (She writes) $S = u\ t$

$$v^2 = u^2$$

$$d = 10t$$

I : That's for the lorry, what about the car?

S : (She writes) : $v = u + 2 \times t$

$$s = u\ t + \tfrac{1}{2}\ 2\ t^2$$

$$v^2 = u^2 + 2.2.s$$

Substitute this value into that equation.

I : Which of these equations compares with $d = 10\ t$?

S : This one. ($S = u\ t + \frac{1}{2}\ 2\ t^2$).

I : OK. What is u by the way?

S : It's initial velocity - the starting velocity is nought.

I : Yes. Is the time the same for both? The distances are the same. Is the time for the lorry the same as for the car?

S : What time are you talking about, what time is that? t?

I : What is t?

S : Time from there to there.

I : Does the lorry take as long as the car takes to move that distance?

S : No.

I : Is it? Here's your car, here's your lorry. So that one pulls away and that one catches up. Did the one move for longer than the other one from the robot?

S : No.

I : One is faster, but they both take the same time. But the time is actually the same. (She solves equations)

ANALYSIS OF INTERVIEW 22

Cognitive Difficulties

1) After reading the problem, she immediately seeks an equation and regardless of how farfetched it is, inserts numerical values given in the equation even though these apply to different bodies.
2) She does not attempt to visualize the problem and even when helped she does not focus on details. Her perception is extremely blurred and sweeping.
3) The manner in which she approaches the problem is unsystematic and she makes no analysis of words. For example, she does not appear to perceive the significance of the fact that constant acceleraion implies the use of the three equations which are known to her. It is apparent that her lack of proper analysis of the problem affects her ability to elaborate it further especially as far as a planned approach is concerned.
4) Implicit data are not carefully determined. Thus, for example, the distance moved by car and lorry are confused. She also does not appear to appreciate that the time taken by both vehicles is the same.
5) She makes no goal determination in the problem. She does not appear to have a clear idea of what she must determine and this in turn affects the way she elaborates the problem.
6) She does not readily relate the two sources of information in the problem.

INTERVIEW 23

I : Would you mind looking at the problem - the first one. Read it through and then talk about it. Do you recognize this type of example?

S : Yes, I recognize it.

I : How would you try to solve such an example?

S : I usually start my problems by writing down all that is given.

I : Try it on the page and explain as you go along so that we know what you are doing.

S : (Writes) : $a = 2 \text{ m s}^{-2}$ $\quad V_V = 10 \text{ m s}^{-1}$

$$u = 0 \text{ m s}^{-1}$$
$$V = u^2 + 2 a s$$
$$V^2 = 0 + 2 a s$$
$$2 a s = V^2$$

As I see it here, at the moment that their velocities are the same the lorry will overtake the car.

I : Why do you say that?

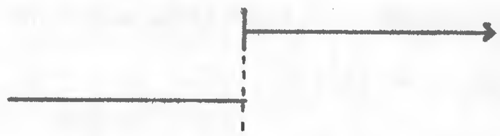
S : Since the car is accelerating and the lorry is going at a constant speed, their velocities will at some time or the other have to be equal before it can overtake. The car must accelerate until its velocity is equal to that of the lorry before it can overtake the lorry.

I : How would you solve the problem? Here you already have that $2 a s = v^2$. Why is it here? For the lorry or the car?

S : The lorry, I mean the car. Yes the car. This problem involves time.

I : Would it perhaps help if you draw a sketch. How do you see the problem? Where do they start etc.

S : (Draws)



I : Where is that?

S : At the robot.

I : Oh, at the robot. OK. And the car?

S : This is now the car that pulls away.

I : OK. What is happening now? The other line that you drew there, what is it for?

S : The car is now coming, already at a speed of ....

I : The lorry ....

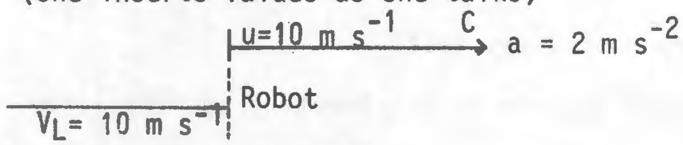
S : Yes, the lorry at a speed of 10.

I : So where does it overtake the car?

S : Over here.

I : Where is this?

S : At the robot. (She inserts values as she talks)



I : Good!

S : They are at the robot at the same time.

I : OK.

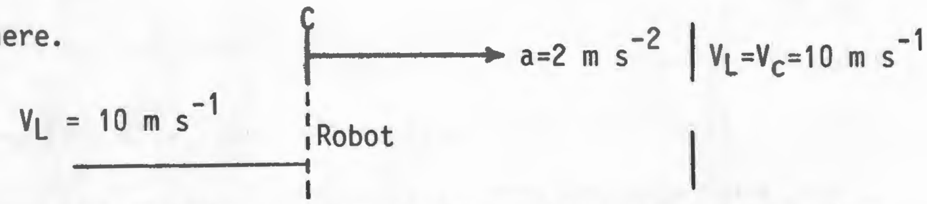
S : So you consider each vehicle on its own.

I : What happens next?

S : The lorry moves at a constant speed and the car - its initial velocity is zero - will accelerate till it goes at 10 metres per second and then it will move faster than the lorry because it is accelerating while the lorry moves at a constant speed. At a certain point the car will have a velocity of 10 and then only will it overtake the lorry.

I : So where does the one overtake the other? Indicate it on your sketch.

S : Over here.



I : You mentioned something about the time. What did you say about the time at the robot?

S : That at the same time that the one has a velocity of zero the other has a velocity of ten.

I : Let us have a look at your sketch again. This is the starting point. This is the first point. What is the final point of your problem? What is the other point of importance for the problem?

S : Over here.

I : Right, so you start here and you end up here. Now what can you say for the car. It moves from this point to that point. It accelerates from rest and over here it is next to the lorry. Now look at the lorry. It moves at a constant speed between this point and that point. Think a little about this. Let's say this is the car and this is the lorry. Here is the car while the lorry moves until they are level with each other. The stationary car now also pulls away from the robot, but the lorry overtakes it and now they move along until over here they will again be level with each other. Is that how they move?

S : Yes.

I : Right, so the car moved from this point to that point, as well as the lorry.

S : So they cover the same distance.

I : Yes, so their distance is the same. Is there anything else that is also the same.

S : Naturally. Their velocities at this point.

I : Say for instance you stand here at the robot with a stop-watch. When will you stop the stopwatch?

S : When they get here.

I : Will there be different times for the lorry and the car?

S : No.

I : Hence, what is the same?

S : Their distances and their times.

I : Is that clear?

S : Yes.

I : What will you do now? What does this word tell you?

S : Constant. So you must apply the constant formulae.

I : What is it?

S : These ones: $v = u + a\,t$ $\quad s = v\,t$

(writes) : $v^2 = u^2 + 2\,a\,s$ $\quad s = u\,t + \frac{1}{2}\,a\,t^2$

I : What else do you have as constant here?

S : Constant velocity.

I : What is the acceleration for constant velocity?

S : Zero, no acceleration. Oh, it is the lorry, no acceleration.

I : Yes. So what equation do we use for the lorry?

S : This one.

I : Which one?

S : The first one. ($v = u + a\,t$)

I : What is the acceleration of the lorry?

S : It is zero.

I : So $v = u$. What does it tell you? What is v and what is u?

S : $v = 10$ and $u = 10$

I : What does it tell you?

S : a is still zero and v is still 10. Then you can determine s?

I : How can you determine s?

S : By using this formula. ($v^2 = u^2 + 2as$)

I : What do you obtain?

S : Oh no, it becomes zero. I'll use the other formula ($s = ut + \frac{1}{2}at^2$). I can determine S from this because this becomes zero.

I : So what is s equal to for the lorry moving at a constant speed?

S : U t. (Writes) :

Lorry : $s = vt$

$s = 10t$

I : Right. What about the car?

S : The car has an acceleration. It has an initial velocity equal to zero; hence this half falls away.

(Writes) : $v = u + at$

$v = 0 + 2t$

$v = 2t$

I : How does it help you? There is a relation between S and t for the lorry. Is there also a relation between S and t for the car.

S : There is none. No, so I can use this formula.

I : What do they ask you?

S : They want s and I have s in the other one.

(Writes) : Car:

$s = ut + \frac{1}{2}at^2$ $\qquad 10t = t^2$

$s = 0 + \frac{1}{2}at^2$ $\qquad 10 = t$

$$s = t^2 \qquad s = 10\,t$$
$$= 10.10$$
$$= 100 \text{ m}$$

I : What is the velocity of the car when it overtakes the lorry?

S : Ten.

I : Why? In the equation. V = 2 t; what is t?

S : t = 10

(Calculates) : $V = 2.10 = 20 \text{ m.s}^{-1}$

ANALYSIS OF INTERVIEW 23

Error Factors

1) She very persistently insists that the two vehicles will only pass each other when their velocities are equal. This is the familiar confusion of same position and same speed.

Cognitive Difficulties

1) Her data collection shows a degree of impulsivity, dictated by a chosen equation (in this case $v^2 = u^2 + 2 a s$). The investigation of the available data is systematic only insofar as it satisfies this.
2) Because she does not appear to have any planned approach to her use of the kinematic equation, which she obviously knows, she is not forced into any detailed data analysis.
3) Her relating of the sources of information simultaneously ends when she states that the velocity of the two bodies are the same when they draw level. She does not however, use this fact in spite of having reduced one equation to $v^2 = 2 a s$.
4) While she obviously has a reasonably clear picture of the physical situation, she sees no need to sketch it out. As a consequence implicit data are not easily apparent to her.

INTERVIEW 24

I : Would you care to read through the problem and explain how you would set about solving it.

S : I would start off with the car; the car would be at the green light; its acceleration would be equal to 0. It will then travel with a constant acceleration of 2 m s^{-2} and the lorry does not stop so it does not have an acceleration of nought. When the acceleration of the car is nought, then the lorry at the same instant pass it at 10 m s^{-1}. So that's for the car and that's for the lorry. (Writes) :

Car a = 0   2 m s^{-2}

Lorry 10 m s^{-1}

Just taking down the facts; the question is how far beyond the starting point will the car overtake the lorry. Could you say that if the lorry is travelling at 10 m s^{-1} to overtake the lorry the car has to travel twice the speed of the lorry to overtake? Is there anything I'm not seeing?

I : Is there anything that you're not seeing?

S : Yes.

I : Why do you ask?

S : From the starting point, that lorry is travelling at 10 m s^{-1}.

I : Yes.

S : The acceleration, the constant acceleration of the car is from nought to 2 metres. So the distance from there ......

I : What is this problem about?

S : Acceleration.

I : Yes, acceleration or more generally, kinematics.

S : Yes.

I : If you were to explain to somebody what is actually happening here, how would you explain it?

S : We have 2 bodies and we're trying to determine the acceleration of both bodies.

I : Why do you say you're trying to determine acceleration of both bodies?

S : Because by working that out, I'd be able to determine at what distance ...... If we work out each acceleration for each body, then the difference between the two .....

I : Aren't we given the accelerations of the two bodies?

S : Isn't our definition of the acceleration, initial velocity minus constant velocity over time? You take it from that or V over t.

I : Now if you read through the problem don't they give you the accelerations of the bodies?

S : The speed is given here and that acceleration. Yes for the car it is given.

I : What is it?

S : 2 metres per second for the car. For the lorry .....

I : What is it for the lorry?

S : I'm just trying to think here, the constant speed and the acceleration would give the same thing.

I : What does constant speed mean?

S : That it's travelling that displacement without accelerating.

I : Yes.

S : Not accelerating.

I : So, what is the acceleration of the lorry?

S : Zero.

I : Yes.

S : There is no acceleration for the lorry.

I : Just describe the problem, describe this in your words; the way they give it to you. What's happening there?

S : You have a car in a stationary position.

I : Yes.

S : And then it accelerates. In other words there is a constant acceleration. But there is a lorry that is moving in the same direction; I would assume because it has to overtake. But it is not accelerating. It is moving at a constant speed of ten metres per second.

The car then, as this thing travels. Because car has nought acceleration at the robot, the lorry I would assume is travelling at that constant speed, so there is no acceleration but it was travelling all the time. So the car has to continue accelerating to pass it, because at nought for the car, the lorry is still travelling at ten metres per second.

I : Yes.

S : So the problem is how far would the lorry get, or how far, what distance would the car have to travel to overtake the lorry. How long must it accelerate. At what distance and after what acceleration would it pass.

I : So how would you draw that out?

S : (Draws) Car $0 \xrightarrow{\quad a \quad}$

$\xrightarrow{\quad \text{Speed} \quad} x$

That's the acceleration of the car; the lorry would be travelling at that speed.

I : So where does the car overtake the lorry, or where does the lorry overtake the car?

S : It would overtake the car at that nought position.

I : So mark it there. (He does by drawing a vertical line). So there they overtake. Where does the car overtake the lorry?

S : Much further.

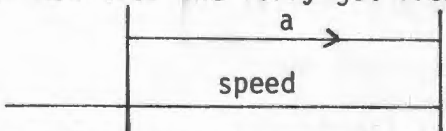
I : Where exactly? Any arbitrary quantity just to indicate.

S : It is going to overtake it at the point beyond the lorry.

I : How do you mean it overtakes it beyond the lorry? Where is it when it overtakes the lorry?

S : It has to be there. It has to overtake at that point.

I : Yes, so the two are level. OK. Let me draw it in for you. At that point they are level. So here they start and here the car overtakes the lorry. How does the lorry get from here to here? (Draws):



S : Travelling at a constant speed of ten metres per second.

I : So is there any equation that suggests itself to you for that? What is the equation for constant speed? Speed is equal to?

S : Distance times second; per second.

I : Per second?

S : Distance over ....

I : Over time. What is that equal to?

S : That gives you your speed.

I : So write down your expression.

S : Writes : Constant speed : $\frac{d}{t}$ = Is that what you require?

I : Well, what is d?

S : That's the distance.

I : On your sketch?

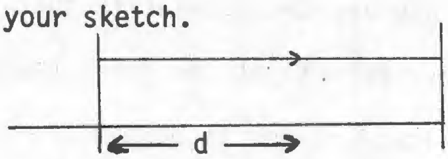
S : On this sketch.

I : Yes.

S : There in the form of axes or just the speed.

I : No, just the distance, what is d? You wrote d there, now indicate it on your sketch.

S : This is d.



I : To where?

S : From there to there.

I : To there.

S : Yes.

I : Now make it to that point. (He extends arrow to the line). And what is t?

S : t is time taken.

I : What time?

S : The time taken from there to there.

I : Yes. So what is that value there, what is the constant speed. You put an equal to sign; what were you going to write there?

S : For the car, as the problem has it.

I : Is that the car or the lorry?

S : The lorry. Would it be ten metres per second?

I : OK. So if you were to write it as one expression what does it look like?

S : For this?

I : Yes.

S : I would say constant speed is equal to ten metres per second.

I : Then write this as an equation d. You don't normally leave an equation like this.

S : You have to write this thing as an expression. Then it is 10 m over one second. (He writes = $\frac{10m}{1sec}$).

I : No, look, here you've got d over t is 10. So if you were to multiply that out, one would surely write d = 10 t
So that's for the lorry. What would you write for the car?

S : (Thinking)

I : What are you looking for - what's the problem really?

S : Distance.

I : So d, OK. You're looking for d. Can you find it from that expression d = 10 t?

S : Yes, we have t.

I : What is t?

S : t is a second.

I : Is what?

S : A second, one second.

I : No, t is time, surely. Didn't you say t was the time to go from there to there?

S : Yes.

I : Do you know what that is?

S : It is 10 times that.

I : Yes, but what is t? Do you know?

S : No, you don't know the time.

I : So you don't know. So here you've got an equation with two unknowns.

S : Two unknowns, so you can't solve it.

I : So what do you want to do now?

S : Find the time.

I : Yes, find it. Now how are you going to do that? Presumably by looking at the car, because that is the only other equation, the other moving body. So what do you know about the car?

S : The car has a constant acceleration from nought, of 2 metres per second squared.

I : So write down the facts you know about the car. What else?

S : (Writes) : $a = 2\ m\ s^{-2}$. $V_0 = 0$

It's initial velocity is nought. Do we put that here?

I : Yes, we'd say that's important. What else do you know? What distance does the car travel?

S : d.

I : So put that in.

S : It's going to be 10 of that.

I : That's 10t. You're writing down what you know about the car. So you wrote d. Now you wrote a is that, V_0 is that, so what would d be?

S : d is the distance travelled.

I : So what do you call it normally in the equations that we use?

S : Displacement, s.

I : s.

S : $s = d$

I : What else do you know about the car? Do you know anything else?

S : Nothing that is given.

I : How long is the car moving?

S : It's going to move the same time.

I : Are you convinced about that?

S : It has to be. It's going to accelerate and move for the same time.

I : So t, the time is the same time that you've got for the lorry.

S : t. I don't know.

I : No you don't know t. What are the equations that are applicable? $V = u + a\ t$. What are the others? Can you remember the equations?

S : No.

I : $s = ut + \frac{1}{2} at^2$; $v^2 = u^2 + 2\ a\ s$. So now you have a, s, v_0, t $s = u\ t + \frac{1}{2}\ a\ t^2$

He now solves the problem.

ANALYSIS OF INTERVIEW 24

Semantic Difficulties

1) The implications of the word "constant" as it is interpreted in kinematics, is obviously lost on him. For example he says of the lorry that "the constant speed and the acceleration would give the same thing".

Error Factors

1) He does not seem to realize that a stationary vehicle can nevertheless have an acceleration. This is related to the difficulty in interpreting the exact point in time that the problem begins.

Cognitive Difficulties

1) He seems to have a very blurred perception of the problem statement. He writes that the car has acceleration from 0 to 2 m s^{-2} and then a short time later he says that the acceleration of both bodies needs to be determined.
2) He attempts no sketch without prompting. The data recorded are all numerical.
3) When asked to describe the problem, he gives no qualitative evaluation. Rather he deals vaguely with accelerations. He has apparently made some goal analysis because he says that by determining the accelerations he will be able to calculate the distance.
4) His sketches show no clear definition of the important points in the problem. Both the initial and final points need to be made explicit to him.

5) It is apparent throughout that he does not "see" the problem. His data analysis is haphazard and clearly affects his elaborational capacity to the extent that he appears unable to see how to proceed on his own at any stage without considerable prompting. His analysis does not appear to generate any meaningful related idea using the three kinematic equations.

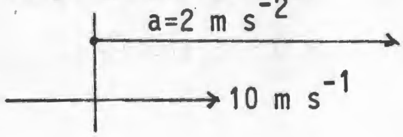
Mathematical Ability

1) He has difficulty assigning symbols to unknown quantities and then utilizing these in equations.

INTERVIEW 25

I : Read the problem and then try to solve it. Try and tell us what you are doing.

S : I'll try and make a sketch of this. (Draws)



Writes : $U = 0$

$a = 2 \text{ m s}^{-2}$

$s = ?$

$t = ?$

It will reach a constant acceleration there. It starts from rest. So this is a motion problem. So I'm going to look at all my motion formulae for something which starts from rest. We have a lorry travelling with a constant speed of 10 m s^{-1}. I'm going to work out separate things. So somewhere along the line it must have the same velocity. Am I not going to need anything more in this direction or something?

I : There's enough in the problem to solve it.

S : (Writes) : $s = \frac{v}{t}$ $\quad v^2 = u^2 + 2as$

$v = 10 \text{ m s}^{-1}$

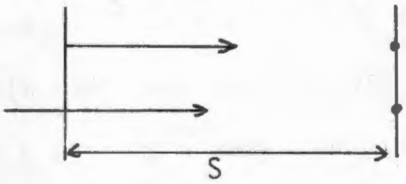
$a = \frac{v}{t}$ $\quad v = u + at$

I : So what are you doing?

S : I'm trying to find some relationship between the acceleration and the velocity.

I : If you were to analyze the problem - what is actually happening? If you were to describe this to somebody, what will you say is actually going on there?

S : They're racing. They're stuck at a robot and they both move off. The one has got an acceleration or is given an acceleration and the one has got a velocity. At some point this one is going to be there and this one is going to be there. (Draws):



I : OK.

S : And this is then the distance.

I : Is the distance the same for both of them?

S : Yes. When the car overtakes the lorry, yes.

I : You said the one is given an acceleration and the other one is given a velocity. What do you mean?

S : No it hasn't; it starts off with that acceleration, and this lorry as it comes to that section there, it already has an acceleration when this one starts to accelerate.

I : So where are we now? What's actually happening now? How is this one moving and how is that one moving?

S : This one is constantly accelerating. This one is going at a steady constant velocity. This one is constantly changing its velocity.

I : How does that help you?

S : If we can bring this to a velocity then I could compare the two. (He points to car).

I : You are going to bring it to a velocity. What are the variables in the problem? What must you find? What's the problem?

S : What I want to find is : I've got u for the car; I've got a for the car; I've got v for the lorry. (Writes):

$v^2 = u^2 + 2\ a\ s$

I : So how do you set about finding s?

S : I can't use this ($s = u\,t + \frac{1}{2}\,a\,t^2$) because I don't have t.

I : We have the car moving. You have the values $u = 0$; $a = 2\ m\ s^{-2}$. You don't know s and you don't know t. You say you must find s. You've written down the formula too; to what does that formula apply?

S : This is motion from rest.

I : I mean the one $s = u\,t$. Is it the lorry or the car?

S : It is with the car we've got something like $v = u + a\,t$.

I : Is there anything that applies to the lorry?

S : What I think I should do is get this s equal to the s of the lorry so that these two S/s are equal and I can equate the two. This s equals to this s.

I : What do you think you should do?

S : Because where they pass they're going to be at the same distance from where they started.

I : OK. Now try it then. What is the expression s for the lorry? What are you trying to do?

S : To find the distance at which the two of them will meet one another.

I : OK. So now you've experimented with all the formulae that there are. You've just dealt with the car. You said just now you must equate the distance for the car with the distance for the lorry. Now wouldn't it help to write down the expression for the distance of the car and the expression for the distance of the lorry?

S : You say it wouldn't help?

I : No, I said wouldn't it help. What is your expression for the distance that the car will move?

S : But I don't have t.

I : OK. What about a for the lorry?

S : For the lorry ..... I don't have a for the lorry.

I : You don't need t to get a. (Silence)

I : How is the lorry moving?

S : The lorry is moving with constant velocity.

I : So what is happening?

S : It has got no acceleration.

I : Yes, so a is 0. What is the expression for constant velocity?

S : $s = v/t$.

I : Here you have the expression for something which moves with constant acceleration. (points to $s = u\,t + \frac{1}{2}\,a\,t^2$). Now if there is no acceleration what is the value for a?

S : $a = 0$.

I : So what are you left with?

S : $s = u\,t$.

I : OK. So that is the expression for the lorry, that $s = ut$. What's the expression for the car? Is t the same by the way? You've got t there and you've got t there, is it the same t?

S : Yes.

I : Are you sure?

S : If they start from there to a certain point, yes.

I : Yes. So previously you said you wanted to equate those two.

S : (Writes) : $u\,t = u\,t + \frac{1}{2}\,a\,t^2$

I : What u is this? You actually have u for both.

S : I don't have s ....

I : You don't need s. What is that u there?

S : It is the starting velocity of the lorry.

I : OK and what is it?

S : It is not given. This one is ....

I : You see if you were to write in the values it would help. That's s for the car. What is u for the car?

S : 0.

I : OK, and what is a for the car?

S : $a = 2\ m\ s^{-2}$ for the car.

I : So write that below here. So what will that reduce to, that equation?

S : (doing calculation) : $s = u\ t + \frac{1}{2}\ a\ t^2$

$$s = \tfrac{1}{2} \times 2\ m\ s^{-2} \times t^2$$

$$s = 1\ m\ s^{-2}\ 1 \times t^2$$

I : Yes it is t^2. What is it for the lorry?

S : You want the value of t?

I : You don't need it. Rewrite the equation s = u t in terms of what you know. What do you know for the equation?

S : I don't know s, I don't know u, I don't know t.

I : Don't you know u?

S : (Thinks a while; writes) $s = 10\ t$

I : OK. So here you have $s = 10\ t$ and here you have $s = t^2$. What now?

(He now does the correct calculation)

ANALYSIS OF INTERVIEW 25

Semantic Difficulties

1) The word "constant" in "constant velocity" is not interpreted to mean that the acceleration is zero.

Error Factors

1) The student seems to see acceleration as he does velocity. For example he says that the car will "reach a constant acceleration there".

Cognitive Difficulties

1) His initial analysis of the problem, and the sketch which he draws, are not in an effort to visualize the related movements, but rather to fit values into an equation.
2) He does not determine the second important point in the problem but confines his attention to the robot and the numerical values there. His use of equationsand data is then an attempt to let the answer "fall out".
3) He does not consistently use the data he has determined. For example, he writes 10 m s^{-1} for the velocity of the lorry on the sketch, but when it must be used in an equation he says that it is not given.
4) He does not relate the two sources of information very effectively, or indeed, see the need to do so.
5) His elaboration of the problem consists of playing around with various formulae. He clearly has no structured, planned manner of using the formulae. He does not seem to isolate any goal in attempting a solution.

Mathematical Difficulty

1) He has difficulty assigning a symbol to an unknown and then using it in an equation. He is also reticent about using a formula in which there is an unknown.

INTERVIEW 26

I : Just tell us how you set about solving this problem.

S : At first I would realize that there are two moving bodies. The one is being accelerated and the other one has a velocity. Therefore we are not speaking of exactly the same thing. So firstly I will write down acceleration of the motor vehicle from rest = 2 m s^{-2} and velocity of the lorry, at the same instant of time is equal to 10 m s^{-1}.

The time the lorry passes the car, the time factor is exactly the same. And further along when the car overtakes the lorry the distance covered by both, or the displacement of the car and lorry must be the same. So I can calculate the displacement as if I was considering one moving body, or one body moving which would have an initial velocity of 10 m s^{-1} and then accelerated by 2 m s^{-2}. If I take displacement is equal to .....

I : Which displacement - of the car or the lorry?

S : The displacement of the lorry first. Write down the kinematic equations and first see what information I have relating to one of the two bodies which is moving. (Writes) :

$v = u + a t$

$s = u t + \frac{1}{2} a t^2$

$v^2 = u^2 + 2 a s$

The initial velocity of the truck. We don't know the time, we don't know the acceleration of the truck, we don't know the final velocity. And for the motor-vehicle - I will know its initial velocity which is zero, and the rate of acceleration. And we can regard the instant then that the two vehicles pass each other as being at t = 0.

I : How is the truck moving?

S : It started with a constant velocity.

I : Constant velocity.

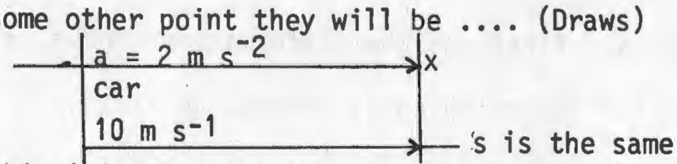
S : At this time there is constant velocity and acceleration for the truck would be zero. That would be at t = 0. The final velocity should be the same as the initial velocity of the lorry. When the car moves off, the truck passes it at that instant and the car before reaching a constant acceleration of 2 m s^{-2} would first have an initial velocity that it should have had at the time the truck passed it.

I : What was the initial velocity of the car?

S : Oh no. Initial velocity of the car was zero.

I : Yes. Wouldn't it help if you had to try and draw out things. If you had to describe to somebody what happens, how would you?

S : They are at a point where they pass each other virtually and at some other point they will be .... (Draws)



At this initial point they were next to each other and at some other point they will again be next to each other. And at this point - this is the car and at this point the car was moving at an acceleration of 2 m s^{-2}. The truck passed at a constant velocity of 10 m s^{-1}. At this point called x, the displacement should be the same. Then we can equate the displacement of the truck with that of the car and if we do that you get $u\,t + \frac{1}{2}\,a\,t^2$.

I : What is that for now?

S : (He writes) : $s = u\,t + \frac{1}{2}\,a\,t^2 = o\,t + \frac{1}{2}\,2\,m\,s^{-2}\,t^2$

S : This will be for the car.

I : What would be the displacement of the lorry?

S : It would be the same as the displacement of the car.

I : What is the expression for that?

S : Again $u\,t + \frac{1}{2}\,a\,t^2$ (Writes) : $s = u\,t + \frac{1}{2}\,a\,t^2$

I : Is it the same $u\,t + \frac{1}{2}\,a\,t^2$ as for the car?

S : No, it will not be exactly the same since initial velocity of lorry is $10\,m\,s^1$. Writes : $s = 10\,m\,s^{-1}\,t + \frac{1}{2}\,o\,t^2$.

I : Leave out the $m\,s^{-1}$ since it is confusing and just write s in terms of the time.

S : So 10 t must be equal to t^2. Therefore $t^2 - 10\,t = 0$

$$t - 10 = 0$$

So time is 10 seconds.

I : Yes. So you got that. You can see here that once you do a sketch, it seems to be clear.

S : That's right.

ANALYSIS OF INTERVIEW 26

Semantic Difficulties

1) The word "constant" in "constant velocity" is not readily interpreted as meaning that the acceleration is zero.

Error Factors

1) He appears to feel that the body must be moving before it can have an acceleration. He says, for example : "the car before reaching a constant acceleration of 2 m s^{-2} would first have an initial velocity."

Cognitive Difficulties

1) His analysis of the data seems to be rather haphazard, not related to any goal.

2) Although he writes down the three equations he does not demonstrate any structured way of applying them and his numerical data analysis does not suggest any particular approach.

3) He does not readily relate the movement of the two bodies with the aim of obtaining a complete picture of the situation. His initial approach is to look at details in isolation. It is only after he is encouraged to draw a sketch that he seems to realize the significance of the implicit data.

INTERVIEW 27

(This student is obviously familiar with problem 1. He writes down variables without a sketch, says immediately that the vehicles have covered the same distance in the same time and that the two equations in s need to be equated. He solves the problem without difficulty).

I : Please read and explain problem 2.

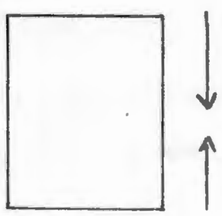
S : It is a question of acceleration. I take my acceleration upwards as g positive. It is going upwards at 10 m s^{-1}. Which means acceleration which is active upon it is ($g - 2$ m s^{-2}). Gives 8 m s^{-2}. So initial velocity is negative. So velocity is -2 m s^{-1} upwards. s = 3 m.
As soon as this thing starts falling it does not gain acceleration. Distance travelled is 3m upwards and initial velocity is also 2 m s^{-1} upwards (writes) $v^2 = u^2 + 2\,g\,s$

$$s = u\,t + \tfrac{1}{2}\,g\,t^2$$
$$+3 = 2t + \tfrac{1}{2}\,g\,t^2$$

Now the lift is going upwards at 2 m s^{-1}. So the actual acceleration is 12 m s^{-2}. The screw and the lift are going to meet each other there. The screw is going to hit the lift at an instant of time there. (He thinks for a while and then draws a line through everything he has written).

I : If you were to explain the problem to somebody, what is actually going on there? How would you describe the problem?

S : Quite simple and easy. You have the lift going up, accelerating upwards. (Draws)



$v = u + a\,t$

It starts with an initial velocity of 2 m s^{-1} and is accelerating upwards, with a total acceleration of g + 2 m s^{-2}, that is 12 m s^{-2}. The lift's accelerating upwards at 12 m s^{-2} starting with an initial velocity of 2 m s^{-1}. Thus, in order to determine the lift's velocity with an acceleration upwards g and ..... (writes the three kinematic equations).

I : How do you get acceleration 12 m s^{-2}?

S : Forces upwards is equal to forces downwards. (Writes):

$\uparrow$

mg ma

$\downarrow$

I : So?

S : The force upwards has to be equal to its weight, that's m times g for it to be travelling at a uniform velocity upward. It is also accelerating which means there is an additional force m times a upwards. Gives a rough acceleration of g plus the 2 m s^{-2}.

I : Let's think about the total upward acceleration. A thing is moving with an acceleration if there are unbalanced forces acting. If you want a body not to change then the upward force would equal the weight. T = m g. If you want it to accelerate upward then you've got to use a force bigger than T = m g.

S : In order for a body to stand still it must have an upward force g.

I : For a body to stand still the forces must be balanced.

S : So we've got a screw which has an initial velocity - 2 m s^{-1} since it is going upwards. It accelerates downwards;

it accelerates at g which is positive (writes) :

screw : $u = 2$

$a = 10$

$t = ?$

The time which is taken to reach the ground is unknown. The lift is going upwards at 2 m s^{-1} and accelerating at 10 m s^{-2} and the time it takes to reach the screw is .....

(writes) : lift $u = 2$

$a = \cancel{10}\ 2$

$t = ?$

(student thinking) (deletes 10 and writes 2)

I : Looking at the way in which the screw and the lift move, try and get a picture of what is going on.

S : Visually, we've got a lift, the floor of the lift is moving upwards to meet the screw, and the screw is moving downwards to meet the floor of the lift.

I : OK.

S : But now I need something equal to something else because I don't know the distance it covers, I can give it an arbitrary value. Distance covered equal to x. Now how do I relate that distance to the distance covered by the lift, cause it won't be the same. They may be the same, but they don't have to be the same.

I : Let's say this box is your lift, accelerating upwards. (gestures with hands). At this point this little screw falls out. Now if you were to focus on the screw, what is it going to do?

S : It's going to fall for 3m.

I : Is it? What's going to happen to the screw while the lift is going up at 2 m s^{-1}?

S : Now initially the screw will go upwards with the lift.

I : It will go up. The lift of course goes up faster. Why?

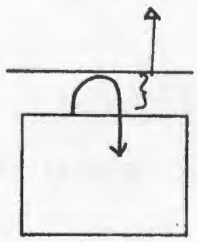
S : Because the lift is accelerating and the screw is retarding already.

I : With g. So here it is, what happens to the lift?

S : The screw goes up, but then it comes down.

I : So if you were to draw a sketch of how the screw moves what would you do?

S : Well if that's your lift going upwards the screw initially goes upwards and then starts falling. (Draws)



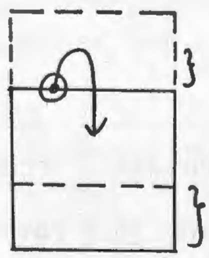
I : What happens to the lift?

S : The lift goes upwards.

I : OK. Let's say this is the lift at the point where the screw falls out and the screw actually falls up. So the screw moves up like that and it moves down. What happens to the lift in the meantime?

S : The lift goes up.

I : Yes. (Draws)



$s = u\,t + \frac{1}{2}\,g\,t^2$

(S writes)

$s = u\,t + \frac{1}{2}\,a\,t^2$

(S writes)

S : Now that distance there .....

I : What would that distance be?

S : $s = ut + \frac{1}{2}\,a\,t^2$ (bottom s)

I : Yes.

S : And that distance = $ut + \frac{1}{2} gt^2$ (top s)

I : Those two t's are they the same?

S : This t and that t isn't the same.

(He continues to experience difficulty in relating the motions simultaneously).

He finally writes :

$$S_s = 3 - ut + \tfrac{1}{2} at^2$$
$$= 3 - (2t + \tfrac{1}{2} 10t^2)$$
$$-5t^2 + 2t + 3 = S = -2t + \tfrac{1}{2} 10t^2$$
$$= 2t + 5t^2$$

ANALYSIS OF INTERVIEW 27

Error Factors

1) He attributes an acceleration of 2 m s^{-2} to the screw even after it is in freefall. There seems to be the idea that the acceleration is somehow residual in the body. Later he changes from (g-2) to (g+2).

Cognitive Difficulties

1) While the student was able to approach problem 1 mathematically because of having made the correct evaluation (probably because of being acquainted with the problem), this approach is not at all successful with problem 2. He shows a lack of accuracy in the manner in which he gathers the data. This can be attributed to the haste displayed in fitting values into equations.
2) He seems to display no particular need to make a qualitative evaluation of the problem. This can be seen from the fact that he immediately assigns a displacement of +3m to the screw.
3) He has difficulty visualizing the situation clearly.
4) It is interesting that while he deals with the two bodies as he thinks about the problem, he always seems to examine each independently. Thus he does not spontaneously relate the two sources of information.

INTERVIEW 28

I : I wonder if you would mind reading the first one through here and explain how you would set about solving it.

S : I want to find the velocity of the car.

(He writes) : Velocity of car = $0 \xrightarrow{2\ \text{m s}^{-2}}$

I : Would you explain that to me?

S : I am trying to work out the velocity (long silence).

I : If you were to explain the problem to somebody what would you say is happening?

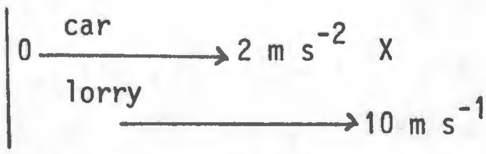
S : First I would like to know the distance.

I : That is what you are asked, but what is the problem all about? How would you explain that to somebody?

S : The car stands here and then the lorry passes it. The car starts up and seeing that the car is faster than the lorry it will pass it sometime.

I : Now would you draw a sketch of that?

S : Yes. (Draws)

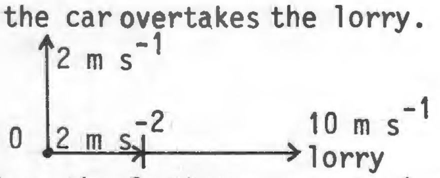


I : What is the problem actually - you have described the situation well, but what is the problem?

S : They want you to find the distance but you can't get a distance from this sketch.

I : What distance do they want?

S : Where the car overtakes the lorry. (Draws)



I : What does the 2 metres per second refer to.

S : The velocity of the car.

I : Is the car moving perpendicular to the lorry?

S : Oh yes that's another problem - it is moving this way too. (Scratches out 2 m s^{-1}).

I : What does the 2 metres per second squared refer to?

S : That is the constant acceleration.

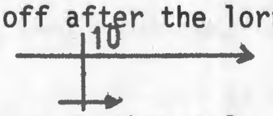
I : Good. What is the initial velocity of the car?

S : 0.

I : If you were to draw what happened at the robot, how would you do it? Lets say this ink mark that I made on the paper is the robot. Now what happens there?

S : (Draws) :

Well the car moves off after the lorry passed it. (Draws):



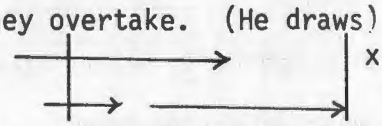
I : And then, what happens further on?

S : The car will increase speed.

I : OK, and then?

S : Then the car will overtake at point x.

I : Draw a line where they overtake. (He draws)



Now what do you notice - here they start, there they end?

S : The car's velocity should have been faster than the lorry's.

I : Is the car's velocity the same as the lorry's?

S : No, it increased.

I : From?

S : From 0 to more than 10.

I : How long did it take?

S : 2 seconds.

I : Where do you see that? (He does not answer). What is this

2 here?

S : That is the constant acceleration.

I : What is increasing?

S : Speed.

I : Yes speed is increasing. So what must you find.

S : The starting point where the car overtakes the lorry.

I : So what must you find?

S : The increasing in velocity, is that what you mean?

I : Is that the only thing you found?

S : The distance.

I : Which distance?

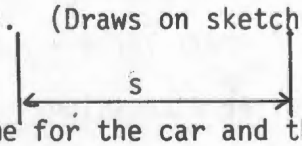
S : Where the car overtook the lorry.

I : Now what will that be on the sketch?

S : It's the x on the line.

I : That's not the distance, that's a point. What is the distance?

S : It's this, s. (Draws on sketch)



I : Is s the same for the car and the lorry?

S : It is the game for the car and the lorry.

I : Do you know the expression for s?

S : s is the metres the lorry travels.

I : How does the lorry travel?

S : 10 metres per second.

I : Now what is the velocity of the car here?

S : It is 0.

I : Right, and what is the velocity of the lorry?

S : 10.

I : So the lorry must cover this distance from there to there. What sort of travel is it?

S : What if it does increase speed?

I : Speed remains the same.

S : Oh! The car is increasing.

I : What is the expression?

S : So the lorry also travels 10 m s^{-1}.

I : What's the expression for constant velocity, constant speed?

S : $V = \frac{\text{distance}}{\text{time}}$

I : Yes. So what is the distance?

S : The distance we don't know yet.

I : No, give it a symbol.

S : So it is x. (Writes) $V = \frac{x}{10 \text{ m s}^{-1}}$

(Only after discussion does he replace the 10 m s^{-1} by t).

I : What is the value of V?

S : That is 10. (Writes) 10 m s^{-1} = x/t lorry x = 10 m s^{-1}

Deletes then writes x = 10 t after discussion.

I : That is for the lorry, now what happens to the car.

S : The car is increasing.

I : Just write down what you know about the car.

S : (Writes) Car : $v = \frac{\text{distance}}{\text{time}}$ Acceleration = 2 m s^{-2}

I : OK, what else do you have.

S : I got the distance where it is going to overtake.

I : What is that?

S : That is X again. (Writes) distance = x

And the time. (Writes) : Time = t.

I : What is the time?

S : The time will be the same.

I : Anything else you know about the car? Where does it start?

S : It starts 0.

I : So , what do you know about that? How does it start with 0?

S : It stopped.

I : So therefore, what is your initial velocity?

S : 0, car is increasing again.

I : It increases from 0. What would you do now? Is there any equation that you can think of that covers this?

S : $v = u + a\,t$.

I : And s equal?

S : (Writes) : $s = u\,t + \frac{1}{2}\,a\,t$

I : a t Squared. Use that equation and put in the values for the car. (He writes) : $s = o\,t + \frac{1}{2}\,(2)\,t^2$.

What did you call the distance?

S : I called it the distance.

I : What is the x here for the lorry and this s on your sketch? (Only after discussion does he change s in the above equation to x. He then solves the problem).

ANALYSIS OF INTERVIEW 28

Cognitive Difficulties

1) He makes only a very limited analysis of the problem initially, his attention being taken up by the last question in the problem viz. determining the velocity.
2) He is able to give a reasonable description of the problem situation but he appears to feel no necessity to do so as he attempts a solution.
3) He seems unable to select the relevant cues in defining the problem. In spite of the obvious indications that the three kinematic equations are required, he uses vector addition in some vague attempt at a solution.
4) His thought pattern is quite unstructured with no particular planned approach. He does not seem to see the direction in which a line of argument is proceeding.
5) His data gathering is very imprecise. For example, he gives the car a velocity of 2 m s^{-1} which he indicates is perpendicular to that of the lorry. Neither does he use symbols consistently (he confuses x and s).
6) He does not recognize the important points in the problem nor does he relate it to the requirements of the problem.

INTERVIEW 29

I : So what we are interested in is that you read the problem and then explain how you would solve it.

S : I would firstly get out the details that are given for the problem e.g. in this problem, at the moment when the traffic light turns green the car pulls away i.e.I'll take its initial velocity as 0 m s^{-1}; its acceleration as 2 m s^{-2} i.e. $a = 2 \text{ m s}^{-2}$. At the same time, we take the time to be 0 seconds, a lorry passes the car with a constant speed of 10 m s^{-1}. Now I'll write down the particulars for car A and then for car B.

(Writes) :

A	B
$u = 0 \text{ m s}^{-1}$	$u = 10 \text{ m s}^{-1}$
$a = 2 \text{ m s}^{-2}$	$v = ?$
$V = ?$	$a = 0 \text{ m s}^{-}$
$t = ?$	$t = ?$
$s = ?$	$s = ?$

Car B has initial velocity of 10 m s^{-1}. We do not know the final velocity for car A. We do not know the time. In any event, the time that A and B take to be next to each other will be the same; covering a certain distance. Then we can get the distance at that point.

Now after having written down all the particulars, I look at what is asked. They want to know how far from the starting point will car A overtake the lorry.

Now I look at suitable formulae that I know for motion in a straight line.

(Writes) $V = u + a\,t$

$$s = u\,t + \tfrac{1}{2}\,a\,t^2$$

$$v^2 = u^2 + 2\,a\,s$$

Now I look for particulars that will fit into a suitable formula. Since I know the time that the lorry and the car take to cover a certain distance, I know that the distance covered as well as the time is the same.

Hence, if I take the formula $s = u\,t + \frac{1}{2}\,a\,t^2$, then I know that the distance covered and the time is the same in this formula. Therefore I take the next formula $v = u + a\,t$ for which I know the acceleration and initial velocity. Let me first see if it is of any assistance.

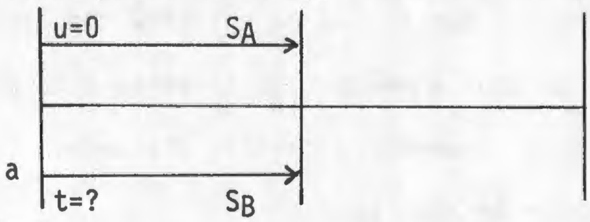
(Writes) $V = u + a\,t$

$$t = \frac{v - u}{a}$$

$$t = \frac{v - 0}{2} = \frac{v}{2}$$

Now I have a problem in that I don't know the initial velocity of the lorry. (Silence). Let me look again at what I want. (Reviews previous steps) Take the lorry as B.

(Draws) :



I : Is A the car?

S : Yes, A is the car and B the lorry. t is unknown and the final velocity is also unknown, but distance is the same.

Now writes down $S = u\,t + \frac{1}{2}\,a\,t^2$

$$S_A = \tfrac{1}{2}\,.\,2.\,t^2 \qquad S_A = t^2$$

Oh yes! The distance will be the same for car A and car B. Let's see if I can put it into this formula. No, that won't help at all.

I : Wait a little. Now you are busy with?

S : No, I got car A and that in this one formula $t = \frac{v}{2}$ for both.

I : What is V there?

S : The final velocity of car A is not given. Hence that formula won't help. Well, now I have a third formula. s is equal to ..... All I know is that in this formula time will be ..... the distance covered will be the square of the time that it is for car A. I know that the distance covered for car B is the same as that for car A, but the time that it takes, will also be the same. So it is the same for both. For car B I know that the initial velocity is equal to 10 m s^{-1} and from there it will proceed at a certain velocity. It moves at a constant speed i.e. a for car B will be zero. The car moves at a constant speed. Then s for this is vt; s = ut. The distance covered by B will be equal to t^2. $\therefore$ $t^2 = ut$

$\therefore$ t = u $\therefore$ t = 10 s.

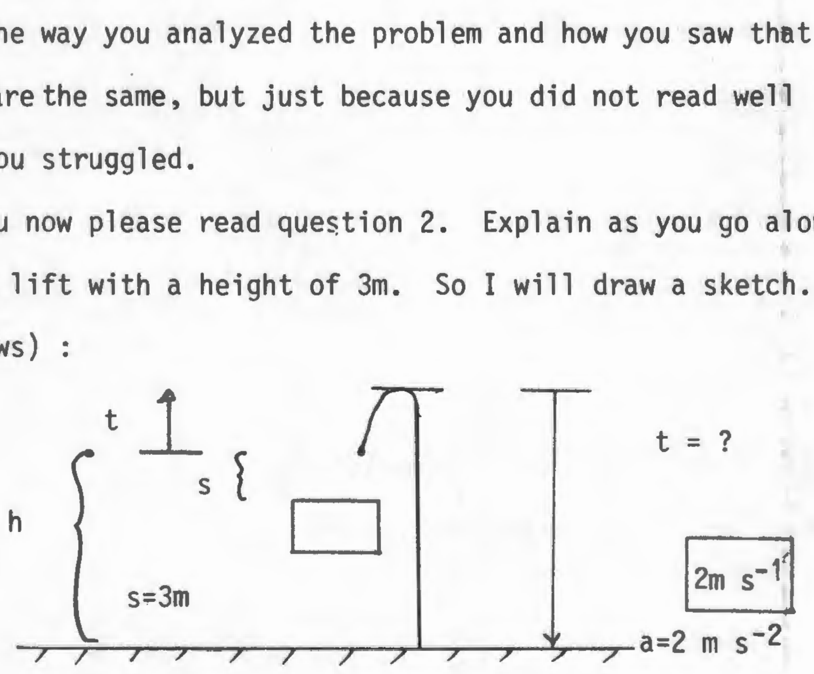
I : You look amazed.

S : No, I am not amazed (explains his last steps again as if to convince himself). I know that the acceleration of the lorry is equal to zero, but I forgot to write it here.

I : I like the way you analyzed the problem and how you saw that s and t are the same, but just because you did not read well enough you struggled.

Could you now please read question 2. Explain as you go along.

S : I have a lift with a height of 3m. So I will draw a sketch. (He draws) :



I have a lift 3m above the ground, so I have a distance there - a height of 3m. And it accelerates upwards at $2\ m\ s^{-2}$ i.e. $a = 2\ m\ s^{-2}$. At the moment that the velocity is $2\ m\ s^{-1}$ a screw falls out of the roof of the lift i.e.the lift moves upward and at a certain time t, at that moment the screw falls out of the roof of the lift; when $u = 2\ m\ s^{-1}$. Now, how long does the screw take to reach the floor of the roof, i.e. the screw will move upward with a certain velocity. At that point, the screw has an initial velocity of $2\ m\ s^{-1}$. I'll first consider the screw. Since it is now moving upward it has an acceleration of $g = 10\ m\ s^{-2}$ and it must stop at some point up there and $v = 0$. Then it will fall to the ground through a distance which we are to calculate. Good. Now I want to know how long does it take to reach the ground. Now I first want to see if I have all the particulars. I have the initial velocity that the lift has at a certain height ... at that specific height, the screw falls out i.e. the screw has a velocity of $2\ m\ s^{-1}$. The screw is now falling freely i.e. $g = 10\ m\ s^{-1}$.

(He writes):

<u>Screw</u>

<u>Upward</u>	<u>Downward</u>
$u = 2\ m\ s^{-1}$	$g = 10\ m\ s^{-2}$
$g = 10\ m\ s^{-2}$	$u = 0\ m\ s^{-1}$
$v = 0\ m\ s^{-1}$	$t = ?$
$S = ?$	

Now I must look at the formula.

I : What are these particulars?

S : Just for the screw .... ignore the lift.

(Writes down the three equations; adds circles and ticks variables as he tells whether they are known).

$s = u\ t + \frac{1}{2}\ g\ t^2$

$v^2 = u^2 + 2\ g\ s$

$v = u + g\ t$

I want to know what the height is that the screw will move from the instant that it leaves the lift till it reached the highest point. I have the initial velocity and the final velocity.

$$t = \frac{v - u}{g} = \frac{0 - 2\ m\ s^{-1}}{-10} = 1/5 \text{ or } 0{,}20$$

The screw will take 0,2s, from the moment it falls till it reaches the highest point, before it moves down. Now I want to know what amount of time the lift will take from the distance 3m till the screw falls out. I'll first look at what I have for the lift.

I : I just want to know what is the time that it takes from there to there. (Indicates on sketch). How long does the screw take to move down - oh no, it has got nothing to do with the lift. (Now discusses time of motion of screw, as on sketch, to floor).

I actually want to know the distance that it covers till there. I want to know what the distance is that the lift travelled before the screw fell. If I know the distance covered there, and I calculate the distance the screw covered from the moment it fell till it reached its highest point, I will have the total distance from the bottom to the highest point and can use the formula $s = ut + \frac{1}{2} gt^2$.

Then I can calculate the total time it took from up here to down here and add it to the time it took to move upward.

I : What is the s here? (Indicates on sketch).

S : The s is the distance that the lift covered before it reached a point where its velocity was 2 m s^{-1}.

Here is the screw, the lift is now there and the screw at that particular point. So it takes 2 m s^{-1} till there;its acceleration is the 2 m s^{-2} of the lift while it moves upward with a constant acceleration. So if it moves upward at 2 m s^{-2} .....

I : If you were to illustrate now, how does the problem look?

S : To obtain the total time I have to calculate the distance the screw falls from its highest point to the floor. I have now already calculated the time that it takes for the screw from up here at a certain point to its highest point. I have the time. Now I want to calculate the time that the screw takes from its highest point to the floor. Then I have to know the distance. If I do not want the distance I would at least have to know the velocity with which the screw hits the ground.

I : What distance do you want?

S : I want the distance from the floor to the highest point.

I : OK, how will you obtain this?

S : (Student not sure) Let me see ..... (Silence)

I : Here is the lift moving upwards. My hand is now the top of the lift. Right? And my other hand is the floor and it moves like this.

S : .... upwards together.

I : Right. Now at this moment when it gets here, the screw falls out. Now what happens to the screw?

S : It moves at the same velocity, 2 m s^{-1}.

I : It moves upward to its highest point and then it falls. What happens to the lift in the mean time?

S : The lift also moves upward at 2 m s^{-1}.

I : With an acceleration. So where does it hit the lift?

S : (Long silence. No answer).

I : Let us look here. Here is the lift. Here you have 3m. What is this 3m.

S : It is the height at which the lift is above the ground.

I : No, read again.

S : A lift ... it has a height of 3m. I presume it is 3m from the ground if I say the lift .....

I : What does that mean?

S : It means the lift has a height of 3m.

I : Yes it has a height of 3m. It is not 3m from the ground.

S : The lift has a height of 3m. It is not specified from where the screw falls, because it says : A lift with a height 3m accelerates at that much and it falls from the roof of the lift. So the screw will only fall 3m.

I : Will it fall 3m? Think a little.

S : Well I mean over a certain .... if it falls it will not fall further than 3m.

I : Think a little, here is the roof. Right?

S : Yes.

I : Let us look only at the screw. The screw goes like this. What happens to the lift in the mean time?

S : It moves upward.

I : So, there is the screw .... till there. Will it be 3m?

S : That is what must be calculated.

I : From the sketch you can see that it is (3-s) because the floor moves upward.

S : It moves upward together.

I : We have to read the problem well enough to analyze it. You are inclined to read the problem too quickly. This idea of 3m, where does it come from?

S : I have in fact confused the word, because I thought the lift was 3m from the ground. That is also the reason why I drew it like that on the sketch.

I : What are you thinking about now?

S : I am looking at the second part of the question; but first I have to calculate v^2.

I : What is v that you are using there?

S : The initial velocity.

I : Of?

S : Of the car because I have to look for a formula to calculate the displacement.

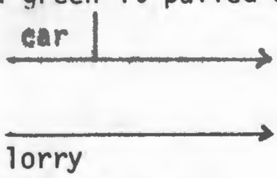
I : Let us do it in another way; if you were to describe the problem, how would you do this?

S : Basically it is just a car that stopped at a robot and pulled away when the light turned green; while at the same time a lorry came along at a constant speed and passed the car.

I : Correct. Make a sketch of it all.

S : Say this is the robot, here is the car, standing, but when the light turned green it pulled away.

(Draws)



S : But at the same time a lorry came along at a slow speed because it was already in constant motion.

I : OK. So where does the lorry overtake the car?

S : It is after the car started to move at a constant speed, after the initial acceleration, because the lorry was already at a constant speed when it approached the car.

I : But does the lorry overtake the car? That is the question.

S : Do you want to know the answer?

I : If you read the problem - you said that there is a stationary car; a lorry that passes. Now I just want to know where does the lorry pass the car.

S : At the robot.

I : Draw it there. (She writes : robot).
What is happening now? If you describe it, what happens next? Here is your car, here is your lorry and as you've said the lorry passes the car. What happens next?

S : Now the lorry will ..... i.e.the lorry's velocity is greater than that of the car.

I : And so ..... Say for example you are standing outside at the gate and there is a lorry here at the robot. Here comes the lorry - where does the lorry overtake the car - right here as you have said. What happens now? Now the light turns green. What is the car doing now?

S : The car must now accelerate.

I : It must accelerate. What happens to the lorry?

S : The lorry continues at a constant speed.

I : So where is the lorry compared to the car as they pull away?

S : It is more ahead, the car is more behind.

I : So what is the next important point in the problem? Look, here you have the robot, the car pulls away, the lorry passes and there it goes. What is the car going to do next?

S : The car will accelerate.

I : Now if you look at the problem, what is the next important point?

S : The fact that the car continues at a constant speed.

I : What continues at a constant speed?

I : The lorry.

ANALYSIS OF INTERVIEW 29

Semantic Difficulties

1) He interprets "height 3m" as meaning that the lift is 3m from the ground.

Cognitive Difficulties

1) Because of his initial haste to record numerical data and then to use equations, he seems to lose sight of some of the data recorded. For example, after saying and writing down the initial velocity of the lorry, he later says that it is unknown.
2) While he makes good qualitative evaluations of both problems, and in problem one at least relates the time and distance for the two vehicles, he seems to ignore more obvious implicit data such as the nature of the motions of the two bodies.
3) It is clear that with problem 1 and more especially problem 2, he has no planned or structured approach. In problem 1 he is visibly surprised when he gets the correct answer. In both his method consists of fitting available values into an equation.
4) He has difficulty in both problems in relating the motions of the two objects simultaneously. This is, however, especially relevant in problem 2 where the focus of his attention is almost entirely upon the screw.

INTERVIEW 30

I : Please try to do the first problem. How would you describe it?

S : Basically it is just one car that is stationary at the robot and pulls away while a lorry passes with a constant speed.

(Writes) : $s = ut + \frac{1}{2} at^2$

I : So what are you doing now?

S : Shall I first fill in all the symbols that they give us?

I : Do that.

S : The initial velocity is zero because it starts off from rest.

I : What object are you talking about?

S : The car that is standing at the robot.

I : OK.

S : This is the acceleration, 2 m s^{-2}.

(She writes) initial velocity = 0

acceleration = 2 m s^{-2}

displacement = s

I : Correct.

S : According to the first question they want to know at what distance will the car overtake the lorry. So they want displacement i.e. I must work out the displacement.

(She is silent for a while).

(Writes) : $v^2 = u^2 + 2\ a\ s$

$$2\ a\ s + u^2 = v^2$$

$$2\ a\ s = v^2 - u^2$$

$$s = \frac{v^2 - u^2}{2a}$$

$$= \frac{v^2 - 0}{2.2} = \frac{v^2}{4}$$

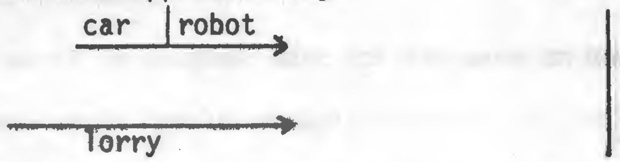
I : So here is the car, here is the lorry. The lorry passes the car at the robot and the car accelerates while the lorry continues at a constant speed. What will eventually happen?

S : I think the car will overtake the lorry.

I : Yes. Is that not what you were asked to find out? "How far from the starting point will the car overtake the lorry?" So where is this on your sketch?

S : It will be here approximately.

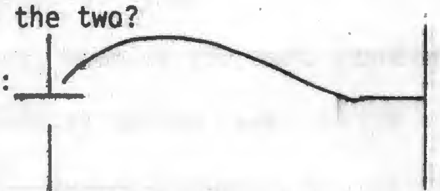
(Draws)



I : Say it is there ... Now as the car passes the lorry they will be next to each other won't they? So now you have two motions. You have the motion of the car and that of the lorry. What will be the same for the two?

S : It is this distance :

(She draws) :



I : But what is it? Indicate it on your sketch. What do you call it? What is the symbol that you use there?

S : s.

I : Put the s there. What about the lorry? If you were to describe the motion of the lorry - how does it move?

S : At a constant speed.

I : What is the distance that it covers?

S : It moves at 10 m s^{-1}. (Indicates on sketch).

I : What is it?

S : The distance.

I : Or the displacement. What is it according to your sketch?

S : (Not certain).

I : From where to where does the lorry move?

S : The lorry still moves from here to there at 10 m s^{-1}.

I : How long does it take?

S : (Does not answer).

I : That you don't know, hey. So the lorry moves from there to there. And the car?

S : It moves up to where it overtakes the lorry.

I : So what is the same for the car and the lorry?

S : The distance.

I : Yes. Here you have written down an equation for the car. Put in the values. (has $s = u\,t + \frac{1}{2}\,a\,t^2$)

S : s.

I : The distance for the car.

S : But I don't know what s is.

I : That is the problem. But use the symbol. What is u for the car?

S : 0

I : How did you write it here? What is a for the car?

S : a is the acceleration.

I : Yes. Put it in. And t? Do you know what t is?

S : No.

I : No, write down the t squared. (She writes hesitantly :

$s = 0 + \frac{1}{2} . 2 . t^2$

$= s = t^2$)

That is now for the car. What about the lorry? What is the initial velocity of the lorry?

S : I would say 0 m s^{-1} because it starts with an acceleration of

$2\ m\ s^{-2}$.

I : No, the lorry. That is the car - what about the lorry?

S : It does not give its initial velocity.

I : What is given for the lorry?

S : The constant speed.

I : What does constant speed mean?

S : (No answer).

I : If you say something is constant ....

S : Remains the same.

I : Remains the same. If the speed is constant what is the initial velocity?

S : It must be 10.

I : That's right. The initial velocity is 10. And the final velocity?

S : It is also 10.

I : And what is its acceleration?

S : It is zero.

I : So write s for the lorry, you don't know that;it is equal to the initial velocity times t. What is t actually?

S : It is the time.

I : What time?

S : That it takes to cover the distance.

I : What distance?

S : It is the distance that it covers over 10.

I : No, look we are talking about the distance from there to there. It is s. So t is the time it takes to cover s. And the acceleration?

S : It is 0.

I : It is 0. So this is 0. So there you have $s = 10t$ and here you have $s = \frac{1}{2} \times 2 \times t^2$. What can you do now?

S : I can simplify the equations and say I cancel $2 \times$ the $\frac{1}{2}$ ..... (With considerable difficulty she equates the two but is unable to solve the simultaneous equations).

ANALYSIS OF INTERVIEW 30

1) She has trouble understanding the implication of the word "constant" in "constant speed".

Cognitive Difficulties

1) She does not attempt to visualize the situation clearly. She makes a cursory initial analysis of the obvious situation at the robot and then looks for values to insert into an equation. When this leads nowhere she chooses a different equation and repeats the process. Her investigation of the data seems to be governed entirely by the choice of equation.
2) Her data analysis does not lead to a careful development of the problem. Rather, she seems to make an initial quick judgement of what principle is applicable and this entirely governs her further analysis.
3) She has difficulty relating the final point to what happens previously. She does not seem to see out of the problem formulation that the point where the vehicles draw level has significance.
4) She does not easily consider the two sources of information at once, focusing only on the car.

Mathematical Difficulties

1) She does not readily assign a symbol to an unknown quantity and then use it in an equation. She tends not to use an equation if there are not numerical values for certain data.

APPENDIX B : INSTRUMENTS DESIGNED TO COMPENSATE FOR DEFICIENT COGNITIVE FUNCTIONS

The following nine booklets were developed as instruments which were followed in the lecture presentation as well as being self-study units which were used by students.

MECH BL 1 : NEWTON'S THIRD LAW

Mech B1 1 : Newton's Third Law

Although Newton's third law is normally placed after the first, since it deals with forces acting on bodies due to other bodies around it, it has always been deemed necessary to discuss this before Newton's first law which requires the ability to draw forces on a specific body. We thus use this order of presentation.

Newton's Third Law states:

To every action there is an equal, but opposite reaction.

This booklet will attempt to answer the following questions:

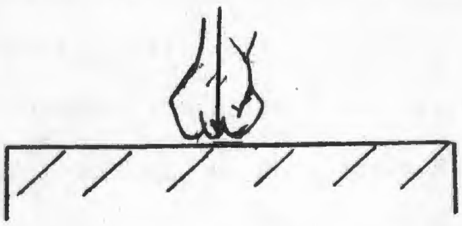
1. What do the terms "action" and "reaction" mean?
2. On what does the "action" and "reaction" operate?
3. How many bodies are involved in the law?
4. If all forces have an opposite, how is motion ever possible? Or, a related idea: what is the difference between Newton I and Newton III?
5. How must the term "unbalanced force" be understood?

After discussing the above, we will apply the idea to the donkey-and-cart problem.

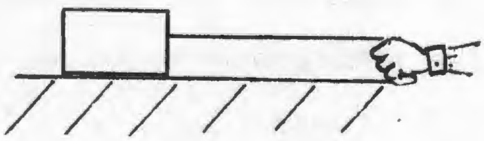
Finally, we will consider the algorithm which can be employed in determining "action-reaction" pairs.

1. Action: By this is meant a force exerted BY a particular body. Consider the following example:

   i) Clench your fist and strike a table. This represents a force exerted by your hand on the surface.



   ii) Tug at a box on the floor, using a rope. Your pull on the rope represents an action on the rope. The rope also pulls on the box. This represents an action by the rope on the box.



   The box pulls on the floor if it is rough. This is an action by the box on the floor.

   Clearly, what the action is, depends on the bodies under consideration.

   iii) If you stand on the Earth, the force of gravity exerts a pull on you, giving you weight. This force represents an action on you by the Earth.

   iv) The Earth around the Sun. The force of gravity of the Sun attracting the Earth is an action, i.e. a force exerted by the sun on the earth.

The action is always exerted ON another body. This body will in turn "react" to the force exerted on it and will provide a reaction force. Let us consider the above example:

i) The body on which the action (fist hitting table) is exerted is clearly the table. The table therefore provides the reaction force. It is this force which hurts your hand if you strike the table hard enough.

ii) In this example there are three actions. Let us consider the reactions to each.

a) Your pull (action) on the rope causes the rope to "react" by pulling on your hand. This force exerted on your hand by the rope can be quite painful at times.

b) The rope pulls on the box, causing the box to react by pulling on the rope.

c) If the surface is rough, the box will pull on the floor, which reacts by exerting a force to the left on the box.

iii) Since weight arises because of the action of the earth on the body, the body reacts to this force by exerting a pull on the earth, or put another way, any body attracts the earth as strongly as the earth attracts it.

iv) Here the earth reacts to the pull of the sun by exerting a force on the sun of the same magnitude, but, of course, in the opposite direction to that exerted by the sun on the earth.

Can you convince yourself from the above that it is immaterial which force is called the "action" and which the "reaction"?

2. To answer the question, let us take the example given above and write down the "action" and the "reaction" as it is set out in (1). (Before considering the answers below, see if you can formulate it by carefully going through (1) again).

i Action: Force exerted by the hand on the table.
Reaction: Force exerted by the table on the hand.

ii(a) Action: Force exerted by the hand on the rope
Reaction: Force exerted by the rope on the hand
The pull on the rope tends to stretch it. The reaction force to the pull (resisting the stretching) is called the TENSION in the rope. Note:

1. Tension is the force exerted by the rope on a body.
2. Consequently, it is always directed away from the body to which the rope is tied, e.g. hand, box, etc. but along the rope itself (see sketch later).

(b) Action: Force exerted by the rope on box.
Reaction: Force exerted by the box on rope.

(c) Action: Force exerted by box on floor.
Reaction: Force exerted by floor on box.
The reaction force here is directed away from the box and tends to prevent the box from moving. This force is called FRICTION.

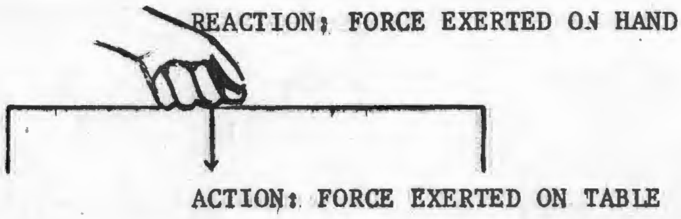
iii Action: Force exerted by earth on person.
Reaction: Force exerted by person on earth.

iv Action: Force exerted by sun on earth.
Reaction: Force exerted by earth on sun.
Gravitational forces such as in (iii) and (iv)

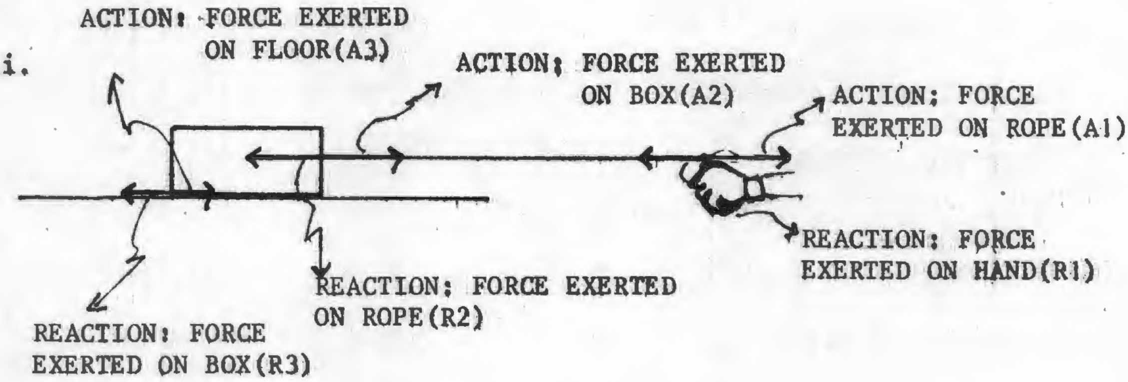
are always taken to act at the midpoint (or more correctly, centre-of-gravity) of the bodies concerned.

According to Newton's Third Law, the action and reaction are equal in magnitude, but opposite in direction. Please see if you are able to sketch out the above four situations showing the action - reaction pairs before turning the page to check your answers.

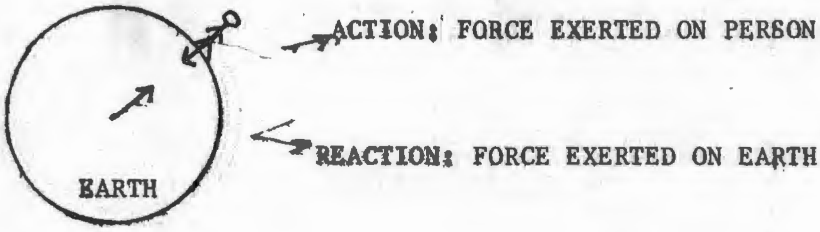
i.



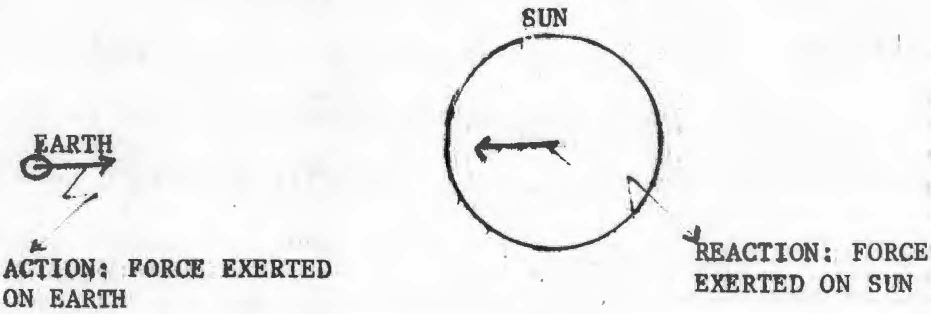
ii.



iii.



iv.

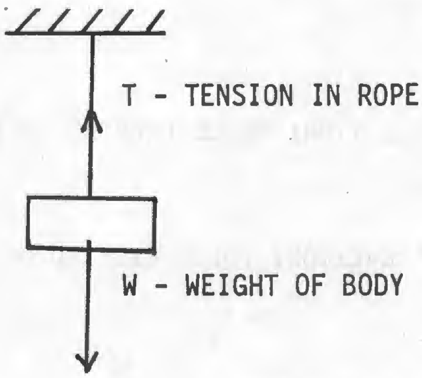


Note that the two bodies need not to be in contact for anaction - reaction pair.

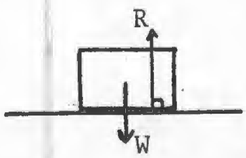
3. It should be clear by this time that there must be two bodies involved in the application of the law:
The body A which supplies the action on body B, or
The body B which "reacts" by exerting a reaction force on body A.
NOTE: THE ACTION AND REACTION FORCES ARE EXERTED ON DIFFERENT BODIES.

Please convince yourself of this point by looking at (1) & (2) above very carefully. If you find that the point is still unclear, please revise points (1) & (2).
It follows that since two bodies must be present in a system before an action - reaction pair can be defined, forces acting on the same body cannot be action - reaction pairs. Consider the following examples:

a)

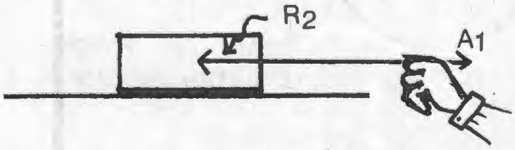


b)



What are the forces R & W?
Do they form an action - reaction pair?
If not, what is the other force in each case to make up the action - reaction pair?

c) Look again at example 1 (ii): (especially the sketch on P 3).



Are A1 and R" an action - reaction pair? Why?

If there is no movement, is A1=R2? Why?

Please answer the above before turning the page to check your answers.

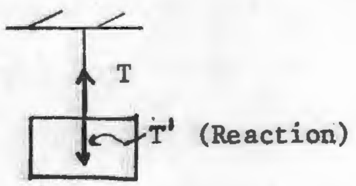
a) Since the tension in the rope and the weight of the body both act on the same body, they cannot represent an action - reaction pair.

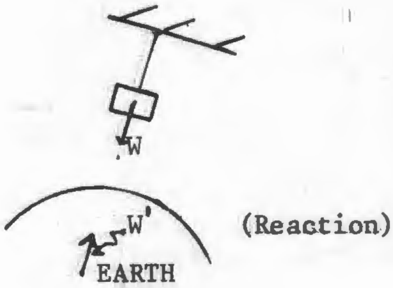
T : Force exerted by the rope on the body.

The reaction must then be the force exerted on the rope, clearly by the body i.e. the body pulls on the rope downward with a force equal in magnitude to the pull of the rope (tension) on the body.

W : Force exerted by the earth on the body.

The reaction must then be the force exerted <u>on</u> the earth by the body. (See 1(iii) above).





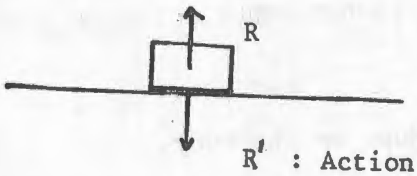
b) R is called the reaction on the body.

W is called the weight of the body.

Since R and W act on the same body, they do not form an action-reaction pair.

R is thus not the reaction to the weight W. The reaction to W is, as before, the force exerted by the body on the earth (see a) above). R is the force exerted by the surface on the body. Since it is called the reaction, then the action is the force exerted by the body on the surface (downward).

While this force is due to the fact that the body has weight, it is not correct to say that the weight is the action.



MASS, M : A2 - R3 = 0, equilibrium

If A2 - R3 > 0, movement (Newton II).

Here the forces R2 and A3, forces exerted by the box on the rope and the floor, play no part.

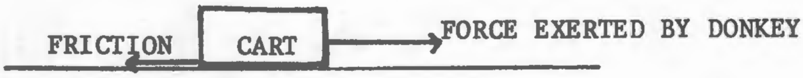
So, while Newton I & II deal with whether the body will move or not, Newton III deals only with forces exerted on and by a body.

5. In the light of the above, all forces have an opposite and in this sense, are balanced. The term "unbalanced force", applies only to the forces ON a particular body. If the vector sum of all forces on a body $\neq 0$, then the resultant is said to be an unbalanced force acting on the body.

A donkey refuses to pull a cart since he reasons that according to Newton's Laws, if he pulls on the cart, it pulls just as hard back on him and so he will not be able to move.

How will you reason with the wayward donkey!

1. Remember that movement is governed by Newton I and II.
2. Whether an object moves or not, depends on the forces exerted ON it.
3. So if the donkey exerts a force on the cart greater than that of the force of friction of the ground on the cart, the cart will move.



4. Of course, the cart also exerts a force on the donkey, equal and opposite to the force exerted by the donkey, but whether the cart will move or not, depends on how strongly the donkey can pull on the cart.

Algorithm To Employ In Solving Problems On Newton III

1. Establish whether it is indeed a problem involving N III i.e. whether action or reaction forces must be determined. Note that it may be part of a larger problem where you must determine the forces exerted on a body by other bodies.
2. Isolate the TWO bodies involved.
3. Determine what is the action.
   i) Size ii) Direction iii) Body on which it acts.
4. Remember that the reaction must have:
   i) Same size ii) Opposite direction iii) Must act on the body producing the action.
5. Test whether your answer can fit into the following:
   ACTION : Force exerted by body A on body B.
   REACTION : Force exerted by body B on body A.

MECH BL 2 : NEWTON'S FIRST LAW

Mech B1 2 : Newton's First Law

Newton 1 : A body will remain at rest or continue with uniform motion in a straight line, unless acted upon by an unbalanced force.

Would you please look at the above formulation of the law and answer the questions below? (without turning the page please!)

1. How many bodies are mentioned?

2. a) What is an "unbalanced force"?

   b) If a single force acts on a body, can the body remain at rest?

   c) Which forces are mentioned in the law?

3. The law is sometimes written as:

$$\Sigma \vec{F} = 0 \quad \text{i.e. } \Sigma F_x = 0 \ \& \quad \Sigma F_y = 0$$

   a) Can you prove this?

   b) What is implied in saying $\Sigma F_x = 0$, $\Sigma F_y = 0$?

4. How can you use the law to solve problems on equilibrium?

5. Can you use the law to give a definition of force?

6. What property of the body makes it act as described in the law?

7. What is implied by "at rest or continue with uniform motion in a straight line"?

The aim of this booklet is to help you to see what is implied by the usual statement of Newton's First Law, and also to solve problems which can be asked to test your understanding of the concept, in the Physics 1 course.

Let us discuss the points individually.

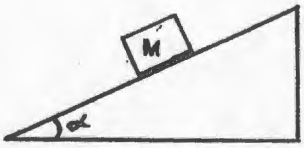
1. Only one body is mentioned. We are thus interested in the conditions for one body to be in equilibrium.
   Therefore in applying the law, you must first determine what the body is which is under consideration. If you are required to solve a problem involving various related parts, either consider the entire connected system as one, or isolate that part that you want to consider.

   Look at the following examples and isolate the "one body".

i.

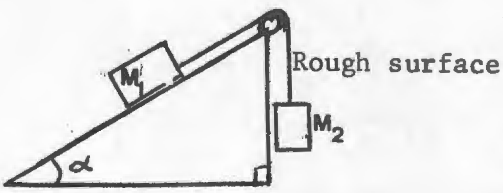
Rough surface, body weight W

ii.



Rough surface, body weight W

iii.



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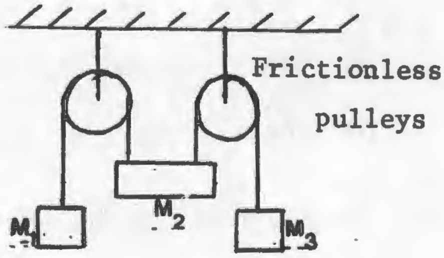

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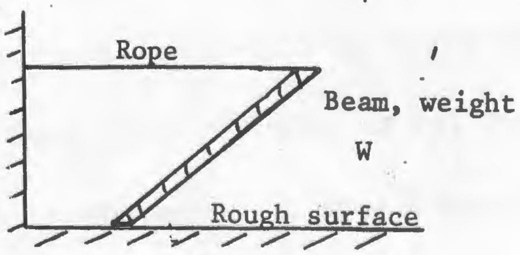

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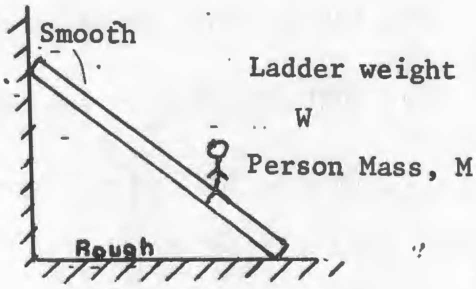
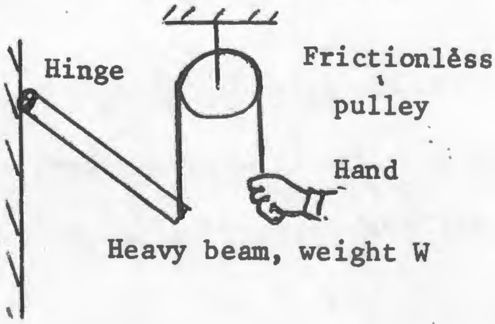


vii.



ANSWERS:

i) and ii) Obviously the mass M

iii) and iv) Where there are two or more masses, each mass is generally considered separately. The pulleys in iii) are usually used only as agencies to carry ropes etc.

v) The object of interest is of course, the beam. The rope is also a body but is used as in iii) and iv) to transmit a force.

vi) The body here is the ladder, not the person. We are interested in how the person influences the ladder.

vii) The heavy beam.

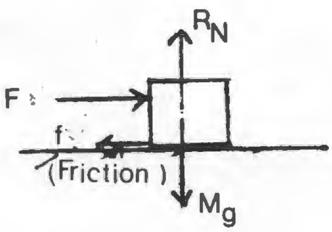
2. a) By "unbalanced force" is really meant, resultant force. This does not mean that there is necessarily only one force on the body, but the law requires that we determine whether the forces acting on the body are in equilibrium i.e. Resultant = 0, or not.

b) A single force acting alone on a body is always unbalanced, and therefore according to the law, the body must move. Thus if only one force is shown acting on a body and yet the body does not move or change its situation of uniform motion in a straight line, then there must be at least one other force acting on the body to balance the first.

c) We are interested only in the forces exerted <u>ON</u> the body. The body will in turn, also exert forces on other bodies (check Mech B1 1 - Newton III), but these forces do not influence the equilibrium of the body.

Now take the above examples (i) - (vii), isolate the "one body" as given above, and try to draw in all the forces acting ON that body (we assume that there is equilibrium), before turning the page to check the answers.

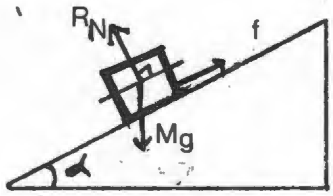
i.



R_N is exerted by surface ON M and is perpendicular to the surface.

f is exerted by surface ON M and is parallel to the surfaces.

ii.

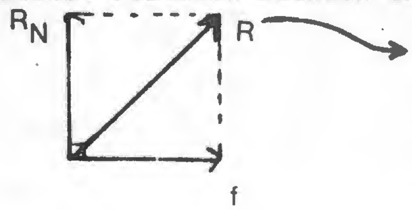


R_N is again perpendicular to surface

f is again parallel to surfaces and opposes the possible direction of motion.

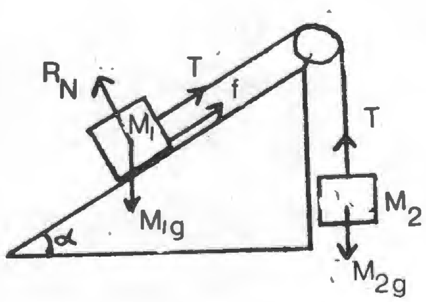
Note that in situations where there is contact as above between two surfaces, the effect of the one surface on the body is always a <u>normal</u> (perpendicular) reaction and a frictional force parallel to the two surfaces. If the surface is smooth, the reaction is still normal but f = 0.

In general, the actual reaction between the two surfaces is the resultant of R_N and f:



Force of reaction on the body exerted by the rough surface.

iii)

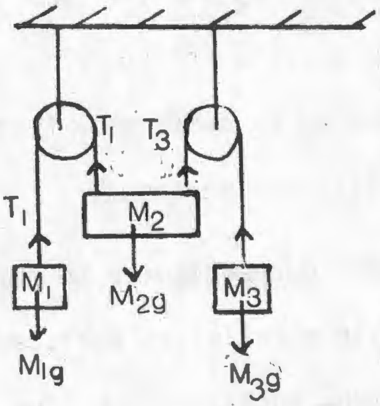


T: Tension in the rope is a force exerted by the rope ON the body. It is always exerted away from a body and is unaffected if the rope passes over a smooth pulley.

f: friction is only upward if the motion is down the plane.

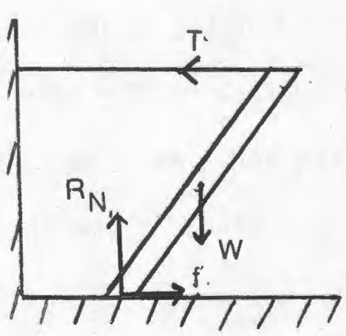
For purposes of equilibrium, we consider M_1 and M_2 separately and find the forces on each.

iv)



Here again, we are only interested in the tension in the ropes affecting the three masses separately.

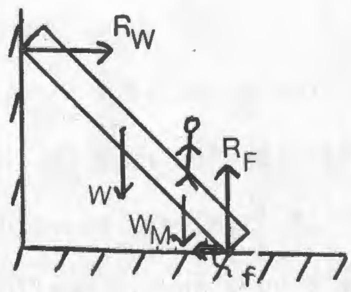
v)



W always acts through the midpoint of a uniform beam. R_N is again perpendicular even though the body meets the surface at an angle. (see point ii above)

f: friction will oppose possible direction of motion.

vi)



R_W & R_F: Reactions (i.e. forces exerted by) of wall and floor respectively, are again perpendicular.

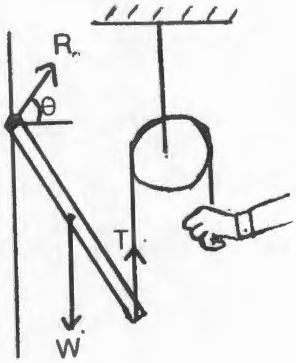
f: friction only at floor since wall is smooth

W_M: weight of the man is obviously a force exerted by man ON ladder.

(Why are we not concerned by the upward reaction on the man?)

W: weight of the ladder acting through its midpoint.

vii)



T: tension in the rope which always acts away from the point of application of the rope.

R: Please note that R is now not perpendicular to the wall. This is because the beam is not pressing against the wall, but is being supported by the hinge. Since the direction of the force which is now being exerted on the beam due to the interaction of the wall and the hinge is not immediately apparent, it is given an arbitrary direction.

It should now be clear from the above that in applying Newton's first law, it is crucial to draw in ALL the forces which act ON the body whose equilibrium is being investigated.

3. a) Since $\Sigma \vec{F} = \vec{0}$ means that the vector sum of the forces acting on the body must be zero, it follows that:

$$\sqrt{(\Sigma F_x)^2 + (F_y)^2} = 0$$

where ΣF_x and ΣF_y mean the sum of the x- components and the sum of the y - components of the force respectively. Since $(\Sigma F_x)^2$ and $(\Sigma F_y)^2$ are always positive, it follows that

$$\Sigma F_x = 0 \text{ and } \Sigma F_y = 0$$

since two positive quantities can only add up to zero if both are zero.

b) Since we are using F_x and F_y, it implies that we must have chosen a convenient x-and y-axis perpendicular to each other. All the forces acting on the body are then resolved into x and y components relative to these two axes.

4. To solve problems on equilibrium:

a) Isolate the body or bodies whose equilibrium is to be obtained.

b) Draw in all the forces acting ON the body or bodies.

c) Draw a convenient set of axes so that the forces acting on the body can easily be split up into components along these axes. If two or more bodies are involved in the problem, different sets of axes may be necessary. Remember that once you have a system of axes you have positive and negative directions. Therefore you must decide if any force or component along an axis is + or - .

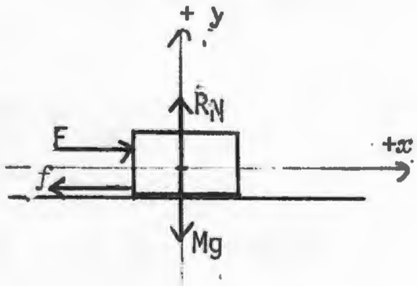
d) Now set up the equations $\Sigma F_x = 0$ and $\Sigma F_y = 0$ remembering:

   i) Some forces need to be split into components in direction of axes.

   ii) Each force or component must be assigned either a + or - sign.

Please look at the problems given above and choose what you feel is a convenient system of axes before turning the page.

i.



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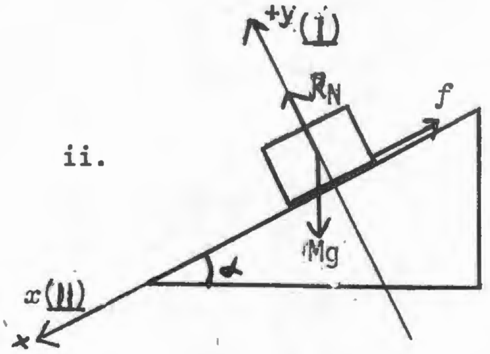

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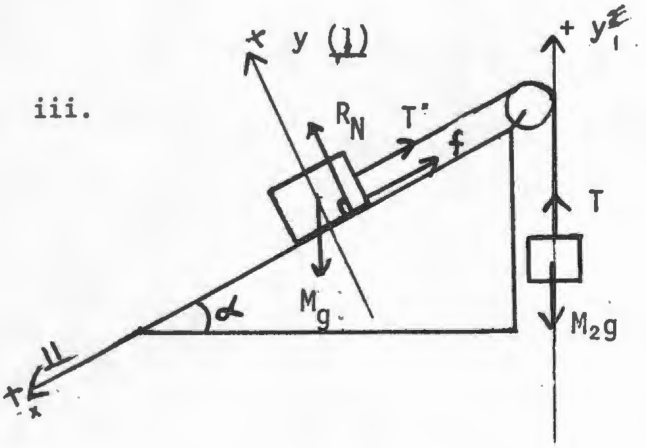

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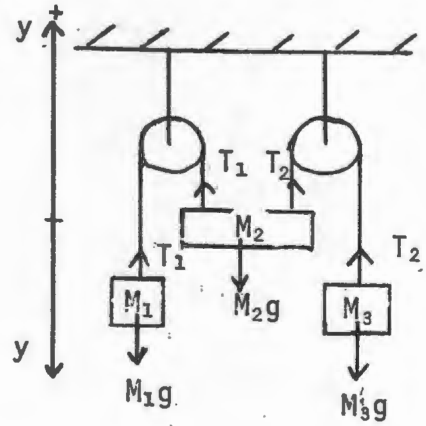

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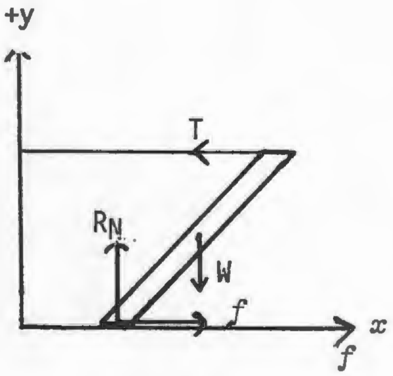

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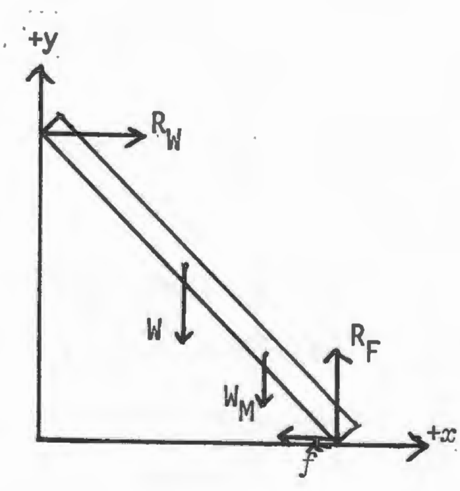

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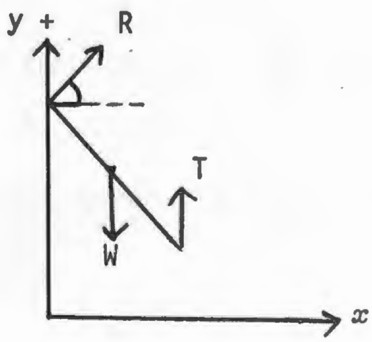


What now remains is to follow the procedure in 4 (d). Attempt it yourself please before consulting the answers given below.

i) $\Sigma F_x = F - f = 0$ (Note the - sign for some forces)

$\Sigma F_y = R_N - M_g = 0$

ii) $\Sigma F_x = Mg.\sin\alpha - f = 0$ (Note the components)

$\Sigma F_y = R_N - M g \cos\alpha = 0$

iii) M_2: $\Sigma F_{y'} = T - M_2 g = 0$

M_1: $\Sigma F_x = M_1 g \sin\alpha - T - f = 0$

$\Sigma F_y = R_N - M_1 g \cos\alpha = 0$

iv) M_1: $\Sigma F_y = T_1 - M_1 g = 0$

M_2: $\Sigma F_y = T_1 + T_2 - M_2 g = 0$

M_3: $\Sigma F_y = T_2 - M_3 g = 0$

v) $\Sigma F_x = f - T = 0$ (Why does f have a + sign?)

$\Sigma F_y = R_N - W = 0$

vi) $\Sigma F_x = R_W - f = 0$

$\Sigma F_y = R_F - W - W_M = 0$

vii) $\Sigma F_x = R\cos\theta = 0$ Thus $\theta = 90°$ since $R \neq 0$

$\Sigma F_y = R\sin\theta + T - W = 0$

In the above, can you clearly see why:

a) Some forces are negative?

b) Some forces are split into components, and how this is accomplished?

If not, please review the notes on components of forces.

Did you notice that for each body there are at most two equations? This means that in any problem at most two unknowns can be determined. The use of the concept of Moments (see Mech B1 4) allows another unknown to be added.

5. A force is that agency which changes or tends to change the position of rest or of uniform motion in a straight line, of a body.

6. That property of the body which opposes a change of its equilibrium state by a force, is called the inertia of a body. It is identified as the mass of a body.

7. Whether a body is at rest or whether it moves with constant <u>velocity</u>, is entirely equivalent. The resultant force on this body is zero.

MECH BL 3 : FRICTION

Mech Bl 3 : Friction

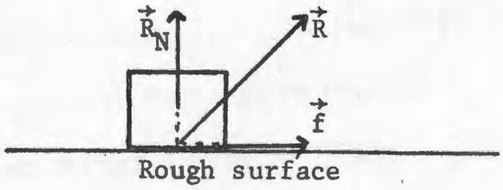
While an algorithm with Newton's first law is common, one dealing with friction appears to be non-existent in the usual first year university textbooks. Indeed it is not immediately apparent how one can be developed. Since frictional forces play a role in many problems both in statics and dynamics, it was decided to develop the booklet.

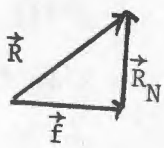
In this unit, we are not attempting to analyse in detail what causes friction, but rather to allow you to work with the force of friction quantitatively.

Very briefly, a frictional force arises when one surface moves over another because there is some interlocking between the surfaces. Although two surfaces may look very smooth, under a microscope they are seen to be very uneven. It is this which causes a retarding force which impedes or prevents the motion of the surfaces when they are pushed relative to one another.

Before we consider this force quantitatively, convince yourself of the following properties of the frictional force:

a) It always opposes the movement of a body (if moving) or the possible movement of a body (if stationary).
b) It is always parallel to the two surfaces in contact.
c) Generally, the frictional force $\vec{f}$, can be regarded as a component of the reaction force, $\vec{R}$, between the two surfaces, the other component being the normal reaction, $\vec{R}_N$.





(see Mech B1 2 and Mech B1 1 No 2 ii (c))

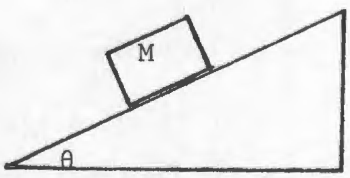
We now discuss different aspects which will help us to determine the magnitude and direction of the force of friction.

1. The <u>DIRECTION</u> of the force of friction, $\vec{f}$:

   Remembering that $\vec{f}$ will always be opposite in direction to the direction of motion and also parallel to the surfaces of contact, consider the following situations, and draw in the frictional force (it is assumed, of course, that the surfaces are rough):

i.

ii.



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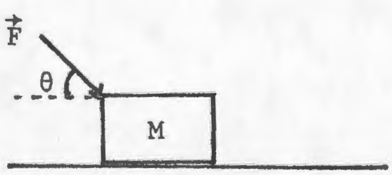

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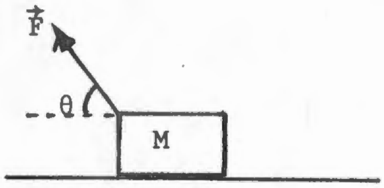

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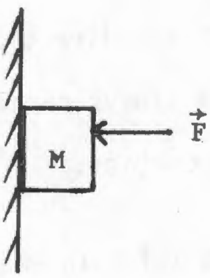

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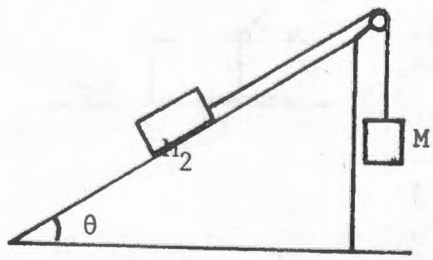

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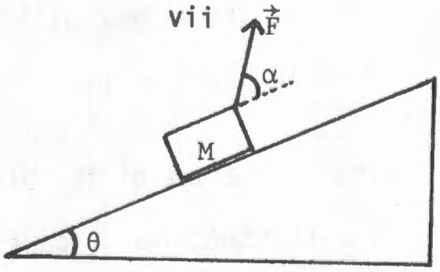
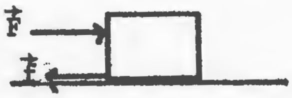
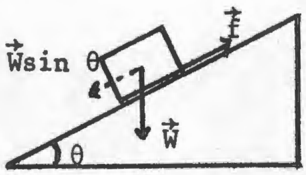
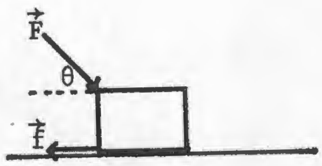


In each case the friction is parallel to the surfaces in contact and opposite in direction to the possible or actual movement of the mass:

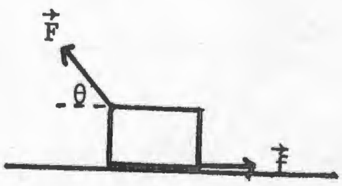
i) 

ii) 

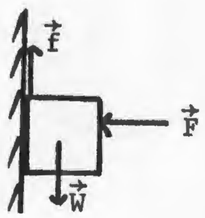
The component of the weight, $\vec{W}$, parallel to the plane viz. Wsin θ, will cause or tend to cause movement, downward.

iii) 

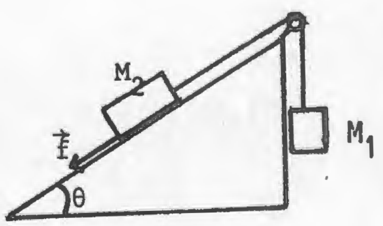
Note that regardless of the direction of $\vec{F}$, the friction $\vec{f}$ is always parallel to the surfaces in contact.

iv) 

Please look at iii) and iv) : In which of the two do you think $\vec{f}$ will be larger? The answer will be furnished later in the booklet.

v) 

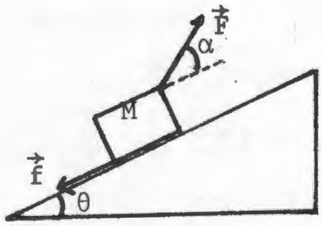
Here the weight of the block will tend to pull it downward, causing the frictional force to be upward.

vi) 

In this type of problem it is not always easy to see which way the system will move unless of course this is stated explicitly. M_2 will tend to move upward if:

$M_1g - M_2g \sin\theta > 0$, depending on the maximum value of $\vec{f}$. $\vec{f}$ will of course, be downward and vice-versa.

vii)



In this case, M will move upward if $\vec{F}\cos\alpha - Mg\sin\theta > 0$, depending on the maximum value of $\vec{f}$ which is downward in this case.

2. Consider now if the system is STATIONARY:

   Here we distinguish between 2 possibilities which we can illustrate as follows: take a reasonably heavy object and place it on a rough surface. Now gently push on the object, parallel to the surface, gradually increasing the force you apply. What do you observe? (PLEASE TRY IT!)

   You will find that as you increase the force slowly the box will not move until you reach a certain maximum value, when it suddenly starts to move. This is explained as follows: the frictional force exerted by the surface on the object will be sufficient to balance the force you exert, but only up to a certain maximum value. The maximum value is reached just before the object starts to move. So, if the object is stationary, there are two possibilities for f:

   a) The force of friction is just sufficient to prevent movement taking place, but is not a maximum. The value of f is calculated by applying Newton 1 to the problem.

      NOTE : UNDER THESE CONDITIONS THERE IS NO EXPRESSION FOR f ITSELF.

   b) The object is on the point of moving. The frictional force is now a maximum, and can be expressed quantitatively as

$$f = \mu_S R_N$$

μ_S = Constant = Coefficient of friction

R_N = Reaction force (exerted by surface on the object) normal to the surface.

NOTE: THIS RELATION ONLY APPLIES IF THE OBJECT IS ON THE POINT OF MOVING.

It does NOT apply to case a) above.

3. If the system is moving (sliding): Repeat the experiment in 2 above, but continue pushing until the object slides for a while. What do you observe about the force required to get the object moving as opposed to the force required to keep it moving? PLEASE TRY IT!

One finds that once the body moves, a smaller force is required to keep it moving, than to move it in the first place. Quantitatively, the frictional force, f, can be expressed as:

$$f = \mu_K R_N$$

where μ_K = Coefficient of kinetic friction

R_N = Reaction force on the body perpendicular to the surface.

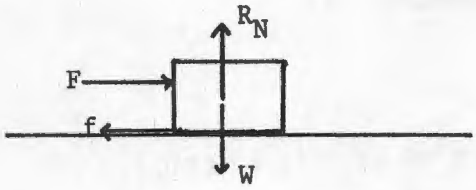
<u>PROBLEM</u>: On the basis of the above which do you expect to be larger μ_S or μ_K? Why?

4. You will note that if 2b) or 3) applies i.e.maximum static friction, or kinetic friction, then R_N needs to be determined. Before applying the steps below, be sure that the problems deal with the so-called critical value of friction i.e. either the system is on the point of moving or it is moving (in which case $f = \mu_{K(s)} R_N$ applies).

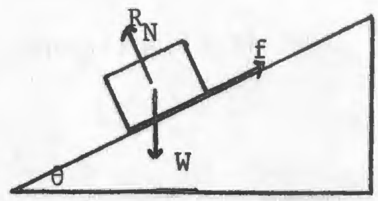
To do this, three steps are needed.

(A) All the forces acting <u>ON</u> the body (system), including R_N, must now be inserted. Try to do this in the examples given, before checking your answers with the sketches on the next page.

i.



ii.



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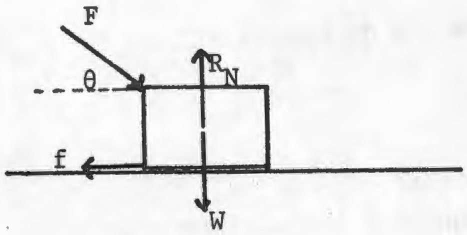

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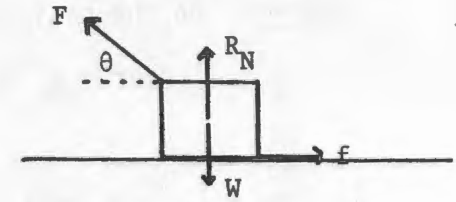

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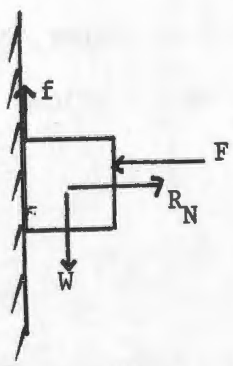
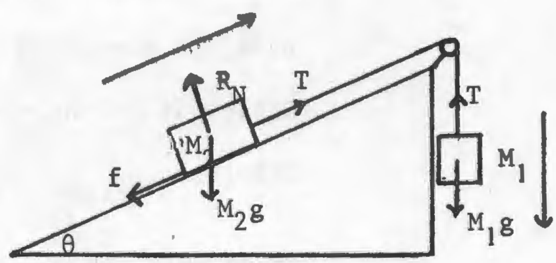
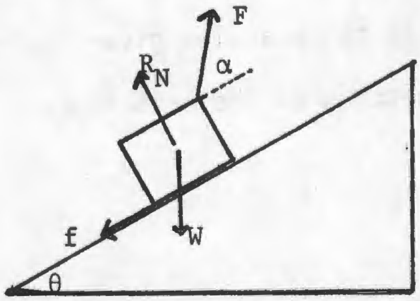


vi.



Assume possible or actual movement in the direction of arrows. If opposite, of course, f will be upward.

vii.

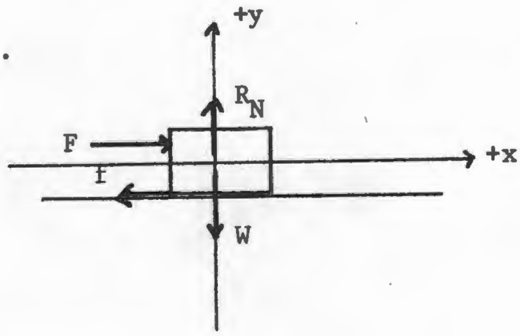


(B) A suitable system of axes must now be chosen (See Mech B1.2 and Mech B1.5, Point 3b).

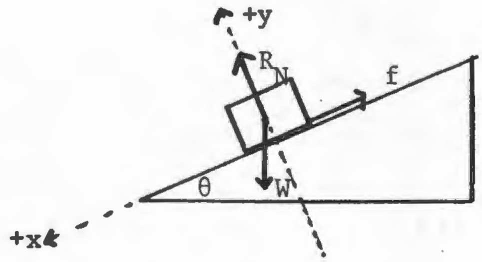
Since f and R_N are perpendicular to each other, the axes are conveniently chosen in these directions.

If you check how this corresponds to the choice of axes in Mech B1s 1 and 5, you will note that it is identical. In Mech B1 5, the +x (or +y) axis is drawn in the direction of movement of the system, thus always making f negative.

i.



ii.



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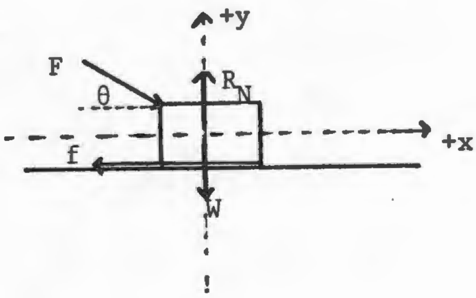

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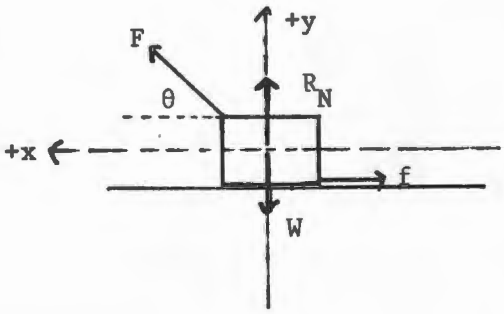

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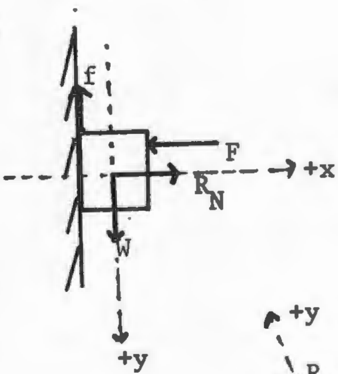

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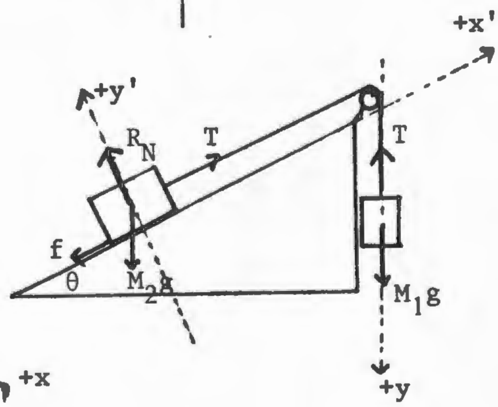
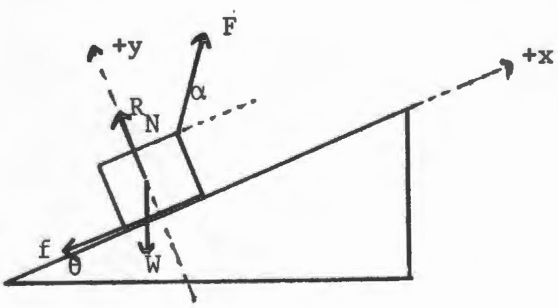


vii.



(C) To determine R_N, the sum of the components of the forces in the direction of the axis parallel to R_N must now be determined. This sum will always equal zero, whether the entire system is in equilibrium or not. (Check Mech B1. 5.).

Write down in each of the above examples, the sum of components along axis parallel to R_N, before checking the answers below:

(i)	Σ y-COMPS. = $R_N - W = 0$	$R_N = W$
(ii)	Σ y-COMPS. = $R_N - W \text{ Cos } \Theta = 0$	$R_N = W \text{ Cos } \Theta$
(iii)	Σ y-COMPS. = $R_N - W - F \sin \Theta = 0$	$R_N = W + F \sin \Theta$
(iv)	Σ y-COMPS. = $R_N - W + F \sin \Theta = 0$	$R_N = W - F \sin \Theta$
(v)	Σ y-COMPS. = $R_N - F = 0$	$R_N = F$
(vi)	Σ y-COMPS. = $R_N - M_2 g \text{ Cos } \Theta = 0$	$R_N = M_2 g \text{ Cos } \Theta$
(vii)	Σ y-COMPS. = $R_N + F \sin \alpha - W \text{Cos } \Theta = 0$	$R_N = W \text{ Cos } \Theta - F \sin \alpha$.

Did you find the above answers in your calculation? If not, check again the procedure to follow in finding components of forces. Note: There is no standard answer for R_N. It differs from situation to situation and depends upon the forces acting on the system.

The value of f in each of the above examples is now simply obtained:

1. Object stationary, but on the point of moving.

$$f = \mu_S R_N$$

2. Object moving:

$$f = \mu_K R_N$$

Are you now in a position to answer the questions posed on p.28 concerning examples (iii) and (iv): in which case is the friction larger?

Commonsense of course, furnishes the answer without any recourse to the equations above. If an object is pushed down hard onto a surface, there will obviously be greater interlocking of the surfacesand thus the friction is larger than normal. On the other hand, if a body is pulled upward, the friction between it and the surface is decreased. Thus obviously f in (iii) is larger than in (iv), assuming the same mass and external conditions.

This can also be seen from the equations above

(iii) $\quad R_N = W + F \sin\theta \qquad f_{(iii)} = \mu\,(W + F\sin\theta).$

(iv) $\quad R_N = W - F \sin\theta \qquad f_{(iv)} = \mu\,(W - F\sin\theta).$

Are you convinced from the equations that

$$f_{(iii)} > f_{(iv)}?$$

ALGORITHM FOR FRICTION

1. Determine the direction of the force of friction i.e. find the direction in which the system moves, or tends to move.

2. If stationary, two possibilities:
   (a) The friction is just sufficient to stop movement taking place.
   (b) The system is on the point of moving.
   Then friction is a maximum,

$$f = \mu_S R_N$$

3. If <u>moving</u> (sliding), then

$$f = \mu_K \; R_N$$

4. If (2b) or (3) is the case, determine R_N.

   (a) Draw in all forces <u>ON</u> body.

   (b) Choose suitable axes.

   (c) To find R_N, perpendicular components.

MECH BL 4 : MOMENTS AND THE SECOND EQUILIBRIUM CONDITION

Mech B1 4 : Moments and the Second Equilibrium Condition

The booklet developed for this section is longer than the others and really consists of two parts:

a) Devising a plausible algorithm for the determination of the moment of a force.

b) Combining the algorithm for Newton I and that for the second equilibrium condition into one coherent algorithm.

In the three previous booklets consideration has been given to forces and how their magnitude and direction influence the equilibrium of a body.

However, in many instances, another factor is brought into play. Consider the simple action of opening a door. It is the common experience of all of us that it is easiest to open a door by pushing on the end furthest from the hinge. If the same pushing force is applied at the middle of the door, it swings open with greater difficulty, this difficulty increasing the closer the force is applied to the hinge. If you push on the hinge itself, the door will not open at all! Thus, from this little experiment we deduce that it is not simply a question of applying a force, but also where the force is applied relative to the axis of rotation of the body (the hinge in the case of the door).

The physical quantity which measures the ability of a force to cause rotation of a body, is called the Turning Moment (or just Moment) or the Torque of the force about the particular axis. The magnitude of the Turning Moment is defined as: The product of the force applied to the body and the distance from the axis to the line of action of the force, which is perpendicular to both axis and line of action.

Note that the Moment consists of the product of:

1. The force
2. Line which is simultaneously perpendicular to both the axis of rotation and the line of action of the force.

In the 1st year, we consider only the Moment of coplanar forces. Please convince yourself of the fact that if the forces are coplanar, the axis of rotation will always be perpendicular to the plane of the forces and will pass through some point on the plane.

Any line drawn from the axis to the force will always be perpendicular to the axis as represented by the point,O, and hence we need only determine the way in which it is perpendicular to the line of action of the force.

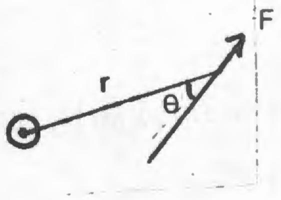
In calculating the Moments of coplanar forces, therefore, we use the following steps:

1. Choose the point, O, (axis) about which moments are to be taken.

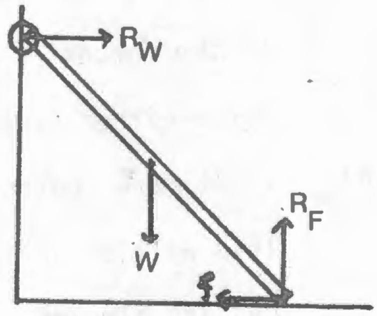
2. Determine which forces will have a moment about O.

Look at the following examples and see if you can determine this:

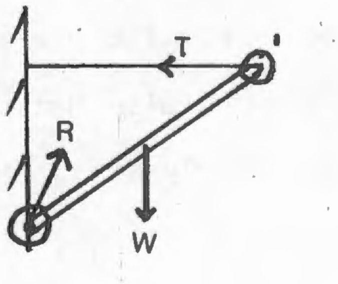
(i)



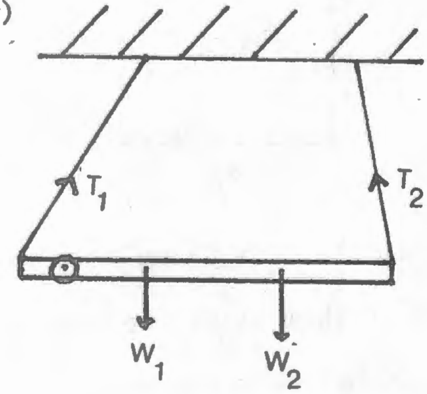
(ii)



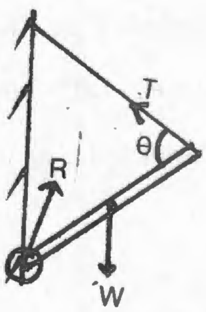
(iii)



(iv)



(v)



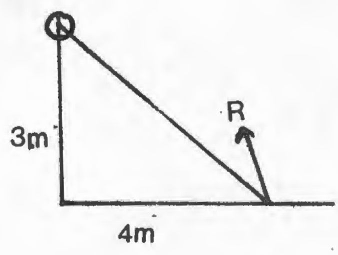
<u>Answers</u>: The only forces which do not have a moment about a point are those forces whose lines of action pass through the point.

Thus the forces R_W in ii), R in iii) and R in v) will have no moment about 0 since they act through 0.

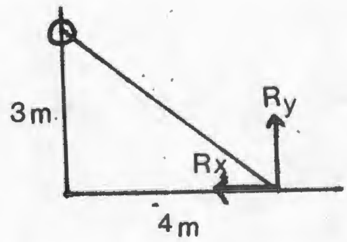
3. a) Is it possible to draw a perpendicular line from 0 onto each of the forces? If not, extend the line of action of the force either backward or forward.

   b) Is it more convenient to break the force into components? This will be so especially if certain perpendicular distances are already given in the problem.

<u>Example</u>:



If you are required to find the reaction R: The perpendicular distances about 0 can immediately be written down if R is split into R_x and R_y:



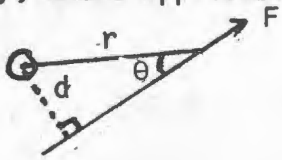
It is then necessary to extend the line of action of the components to be able to draw in the perpendicular lines.

Look at the five examples given on the previous page and decide in which of them it may be convenient to split certain forces into components before taking moments. (You can check your answers later when the problems are worked out)

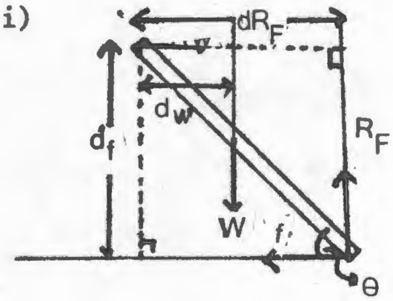
4. Draw the perpendicular from 0 to the force or line of action of

the force (or the component if the force is split up into such).

Please attempt it before checking your answers below: (we draw only the body, where applicable, and the forces acting on it).

i) 

Clearly $d = r \sin\theta$.

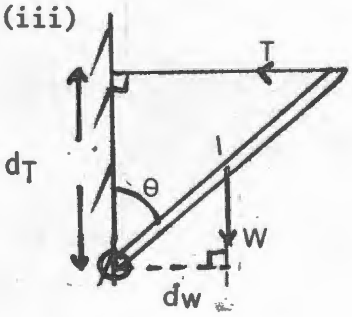
(ii) 

Please convince yourself that:

i) dR_F is $\perp$ to line of action of R_F

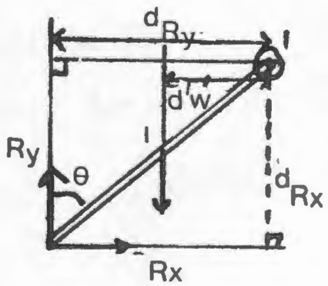
ii) d_W is $\perp$ to line of action of W

iii) d_f is $\perp$ to line of action of f

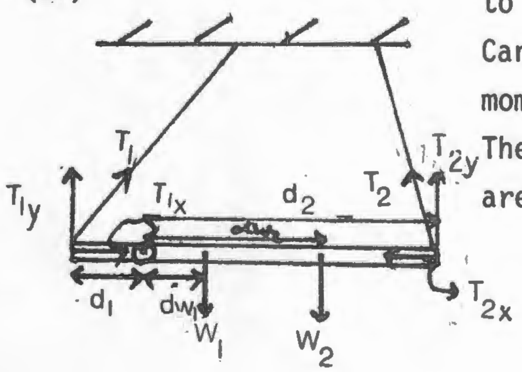
(iii) 

Perpendicular to W & T are d_W and d_T respectively for moments about 0.

If moments are taken about 0', then it is convenient to divide R into components R_x and R_y as shown.

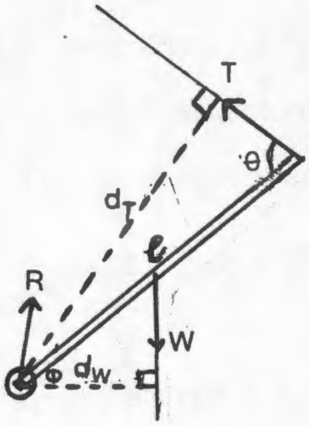


The perpendiculars to R_x, R_y and W are d_{Rx}, d_{Ry}, and d'_W as indicated.

(iv) 

In this case also, it is more convenient to break up T_1 & T_2 into components.
Can you see that both T_{1x} and T_{2x} have no moment about 0?
The perpendiculars to T_{1y}, W_1, W_2 & T_{2y} are d_1, dw_1, dw_2, d_2 respectively.

v)



In this case it is simpler to draw the perpendicular d_T onto the line of action of T, rather than splitting T into components.

The perpendicular onto the line of action of W, is as before, d_W.

5. It is now necessary to determine the Moment by:

   i) calculating the product: Force x Perpendicular distance

   ii) inserting the correct sign.

If you are in any doubt about the sign of the Moment, place a ruler along the line perpendicular to the line of action of the particular force. Let the force act on the ruler while keeping the part of the ruler at the axis of rotation, 0, fixed.

If the ruler is turned clockwise by the force, the Moment is negative and vice-versa.

Look at the above five examples, write down the force and the Moment of the force, remembering, of course, the correct sign.

Try it before turning the page to check your answers.

		FORCE	MOMENT
(i)		F	$+ Fd = F\ r \sin\Theta.$
(ii)		R_F	$+ R_F.d_{R_F} = R_F.\ell.\cos\Theta.$
		f	$- f.d_f = -f.\ell \sin\Theta.$
		W	$-W.d_w = -W. \frac{\ell}{2} \cos\Theta.$
(iii)	About 0:		
		T	$+T.d_T = -T.\ell \cos\Theta.$
		W	$-W.d_W = -W . \frac{\ell}{2} \sin\Theta.$
	About 0':	R_y	$- R_y.d_{R_y} = - R_y . \ell \sin\Theta.$
		R_x	$+ R_x.d_{R_x} = + R_x .\ell\cos\Theta.$
		W	$+ W . d'_W = + W. \frac{\ell}{2} \sin\Theta.$
(iv)		T_{1y}	$- T_{1y}.d_1$
		W_1	$- W_1 d_{W_1}$
		W_2	$- W_2 d_{W_2}$
		T_{2y}	$+ T_{2y}.d_2$
(v)		T	$+ T.d_T = + T.\ell \sin\Theta.$
		W	$- W.d_W = - W.\frac{\ell}{2} \cos\phi.$

ALGORITHM TO DETERMINE MOMENTS

1. Choose the point 0 (axis of rotation) about which moments are to be taken.
2. Determine which forces have a moment about 0.
3. a) Is it possible to draw a perpendicular from 0 onto each of the forces? If not, extend the line of action of the force either forward or backward.

   b) Is it more convenient to use components of a particular force? Extend the line of action of these if necessary.
4. Draw the perpendicular from 0 to the force or the line of action of the force (or the components if the force is split up into such).
5. Determine the moment:

   i) Calculate the product : Force x Perpendicular distance.

   ii) Insert the sign of the moment.

The reason for wanting to calculate the moment of a force, is that we are now in a position to apply the so-called Second Equilibrium condition:

If a body is in equilibrium under the action of a number of coplanar forces, the algebraic sum of the torques about any arbitrary axis is zero.

As in the case of Newton's First Law (see Mech B1 2) let us analyze the principle carefully to see what is meant in the above formulation.

1. How many bodies are involved.

   Since only one body is mentioned, we are interested in the conditions for one body to be in equilibrium. In a system containing a number of connected bodies, each one is treated separately.

2. Which moments are of concern?

   The sum of the moments on the one body must be found. Before the moments can be found, all the forces acting on the body must therefore be obtained.

Please compare the above two points with 4a & 4b in Mech B1 2 which gives the algorithm for applying Newton I. You will note that they are identical.

3. Which point must be taken to determine the moments of the forces acting on the body?

   The principle states that "any point" can be taken as the axis about which moments are to be taken. Since you are at liberty to choose any point, it is normally convenient to take moments about the point through which most of the unknown forces pass, in this way eliminating some of the unknowns from the equation.

4. Why is the algebraic sum of the moments taken?

   Since all the forces are coplanar, the axis of rotation is always perpendicular to the plane of the forces, through the point O and the moment is either clockwise or anticlockwise. Since the moment is a vector either up or down along the axis of rotation, all the moments will lie along the same line and can therefore be added algebraically.
   (The same as adding forces which have the same line of action).

In a problem on equilibrium therefore, two principles are available:

a) Newton I

b) 2nd equilibrium condition

The application of these two principles results in three equations:

Newton I : i) $\Sigma F_x = 0$

ii) $\Sigma F_y = 0$

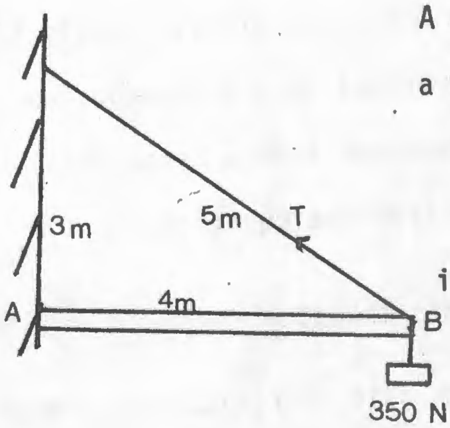
see Mech. B1 2

2nd equilibrium condition iii) Σ Moments About Any Point = 0,

Thus the problem can have at most, three unknowns.

To illustrate the overall strategy to employ in solving problems on equilibrium, using Newton I & the 2nd equilibrium condition, we will consider two actual examples.

1.



A uniform beam has a weight of 500 N with a weight of 350 N attached at B. Calculate:

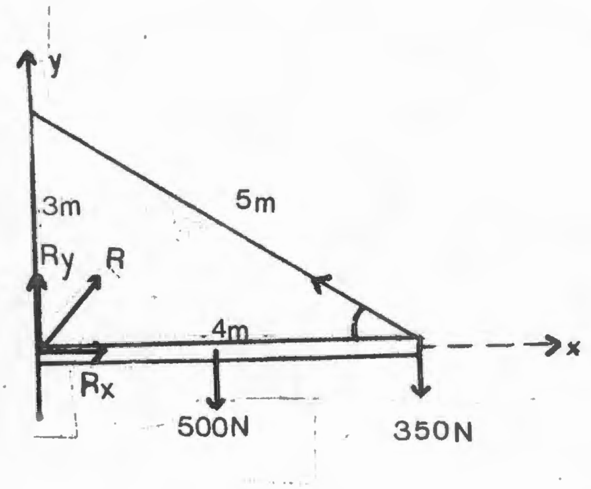
i) the tension T in the cable holding the beam horizontally;

ii) the magnitude and direction of the reaction force exerted by the wall on the beam at A.

This is clearly a problem on equilibrium and thus the strategies for Newton I and Moments, have to be applied.

Step 1: Isolate the body or bodies involved.
This is obviously, the beam.

Step 2: Draw in all the forces on the body.



Step 3: Choose a convenient system of axes.

This is obviously x & y as shown above.

It is convenient to split R into R_x & R_y as shown.

Step 4: $\Sigma F_x = 0 \qquad \Sigma F_y = 0$:

$\Sigma F_x = R_x - T\cos\theta = 0$

$\Sigma F_y = R_y + T\sin\theta - 500 - 350 = 0$

If you have difficulty with any of the above four steps, please check Mech B1 2 again.

Step 5: Apply 2nd equilibrium condition, viz.

Σ Moments about any point = 0

To carry out this step, the algorithm on Moments on P 45 must now be applied.

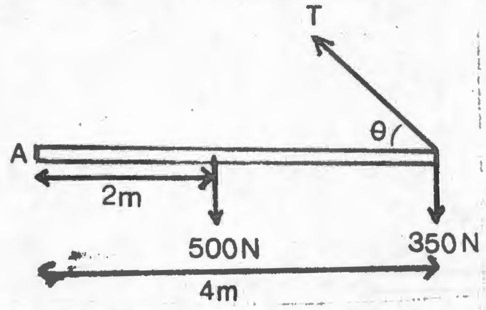
i) Choose point about which moments are to be taken.
Since the unknowns are R_x, R_y, T, if Moments are taken about A only T will have a moment.

ii) Determine which forces have a moment about A:
Clearly 500 N, 350 N and T.

iii) a) Determine perpendiculars

&

iv)



Clearly 500N & 350N are 2m & 4m from A. (perpendicular)

b) Is it more convenient to use components of a particular force?

If T is split into T_x: $-T\cos\theta$

T_y: $T\sin\theta$

Then only T_y has a moment and is perpendicular distance of 4m from A.

v) Determine the moment

500 N: -500×2

350 N: -350×4

T_y : $+ T\sin\theta \times 4$

$\therefore$ Σ Moments about A $= -500 \times 2 - 350 \times 4 + T\sin\theta \times 4 = 0$

Step 6: Solve the 3 equations: ($\sin\theta = \frac{3}{5}$; $\cos\theta = \frac{4}{5}$ from sketch).

$$R_x - T\cos\theta = 0$$

$$R_y + T\sin\theta - 850 = 0$$

$$-2400 + 4\,T\sin\theta = 0$$

$\therefore$ $T = 1000$ N

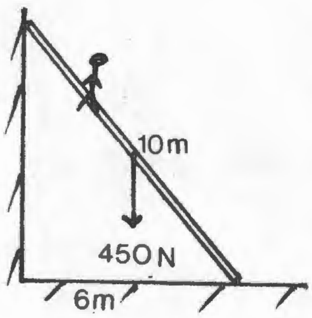
$\therefore$ $R_x = 800$ N

$R_y = 250$ N

$\therefore$ $R = \sqrt{R_x^2 + R_y^2} = 838{,}2$ N

$$\tan \theta = \frac{R_y}{R_x} = \frac{250}{800}$$

$$\therefore \qquad \theta = 17{,}4^{\circ} \rightarrow$$

2. 

A uniform ladder 10m long, rests against a vertical frictionless wall with lower end 6m from the wall. Weight of ladder = 450 N. Coefficient of friction between the foot of ladder and the ground = 0,40. A man, weight 800 N stands on the ladder. Calculate:

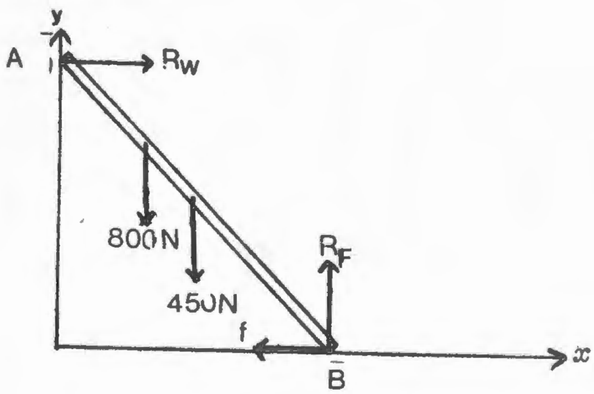
i) Maximum frictional force exerted by the ground on the ladder.

ii) Where the man stands if the ladder is on the point of slipping.

From the questions it would appear that this is a problem on friction. However, since to calculate the force of friction, the normal reaction is required it is clearly a problem requiring forces to be determined. It is thus generally a problem of equilibrium.

Step 1: Isolate the body or bodies involved.
This is obviously the ladder.

Step 2: Draw in all the forces.



Step 3: Choose convenient axes.

These are as shown.

Step 4: $\Sigma F_x = 0 \qquad \Sigma F_y = 0$

$\Sigma F_x = R_W - f = 0$

$\Sigma F_y = R_F - 450 - 800 = 0$

Always keep in mind what you are required to determine:

i) Maximum frictional force.

This is $f = \mu_s \cdot R_F$ in this case.

But $R_F = 1250$ N

$\therefore$ $f = 0{,}4 \times 1250$ N $= 500$ N.

$\therefore$ From above $R_W = 500$ N.

The second question involves the position of the man on the ladder. Please remember that distances cannot be obtained from ΣF_x and ΣF_y. For that, moments are required.

Thus:

Step 5: Apply the 2nd equilibrium condition.

Σ Moments about any point = 0

Strategy for moments:

i) Choose convenient point.

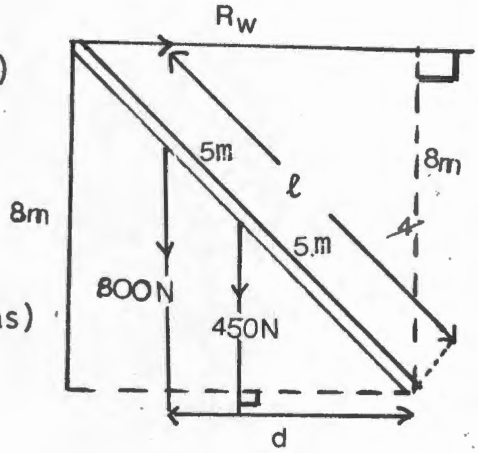
Point B eliminates two forces and is therefore convenient.

ii) Determine which forces have moments.

R_W, 800 N, 450 N.

iii) Determine perpendicular

& (iv)



(Pythagoras)

R_W : 8 m

450 : $5 \cos\theta = 5 \times \frac{6}{10}$

$= 3\text{m}$

800 : $d = \ell \cos\theta = \ell \times 0{,}6$

Step 6: $\Sigma \text{ Moments} = -R_W \times 8 + 450 \times 3 + 800 \times \ell \times 0{,}6 = 0$

$$\therefore \ \ell = \frac{8 \times 500 - 3 \times 450}{800 \times 0{,}6} = 5{,}52 \text{ m} \rightarrow$$

OVERALL ALGORITHM FOR DEALING WITH PROBLEMS ON EQUILIBRIUM

1. Establish that is is a problem on equilibrium.
2. Isolate the body or bodies involved.
3. Draw in all the forces on the body.
4. Choose a convenient system of axes.
5. Write down the equations.

$$\Sigma F_x = 0$$

$$\Sigma F_y = 0$$

6. Carry out the strategy on Moments
7. Write down the equation:

$$\Sigma \text{ Moments about convenient point} = 0$$

8. To the three equations thus obtained add the equation $f = \mu R_N$ if and where applicable.
9. Solve the equations for the unknowns.

MECH BL 5 : NEWTON'S SECOND LAW

Mech B1 5 : Newton's Second Law

Before considering this unit, please review Mech B1 1 on Newton I. The material discussed below is an extension of that unit.

Newton II: The acceleration of a body is proportional to, and in the same direction as the resultant force acting on it, and inverseley proportional to the mass of the body. This normally reduces to: $\vec{F} = m.\vec{a}$

Please look at the following questions and see if you are able to answer them by looking at the above formulation of the law:

1. Since only one body is mentioned, how will you deal with a system involving related masses?
2. a) Which forces are needed to apply the law?
   b) Do all the forces on the body influence the acceleration?
3. a) Which two directions are implied in the law?
   b) How does this influence your choice of axes?
   c) What would be the equations in these directions?
4. How can you use the law to solve problems?
5. Is the resultant force causing the acceleration necessarily in the direction of motion?

We will now attempt to answer these questions individually and in so doing demonstrate what is required to use the law.

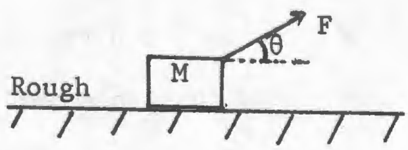
1. In seeking to apply the law, we must divide the system into its constituent parts i.e.each mass must be considered individually and the law applied to each one. (Please look again at point 1 of Mech B1 2).

   Since we are dealing with movement, in the case of connected masses it is necessary to check how the acceleration of the masses are related. So while the law is applied to each mass on its own, the equations which result may have common variables.

   Look at the following problems, identify the "body" on which the law is to be applied and at the same time see if the acceleration of that body will in any way influence that of any other body.

i.

ii.



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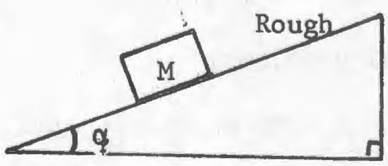

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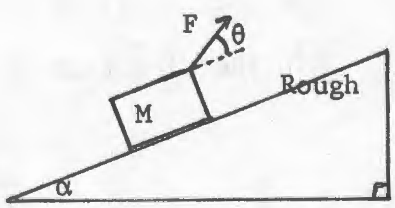

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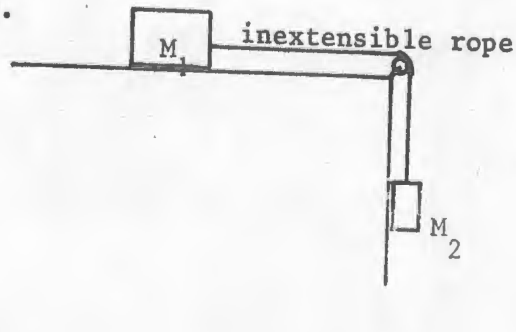

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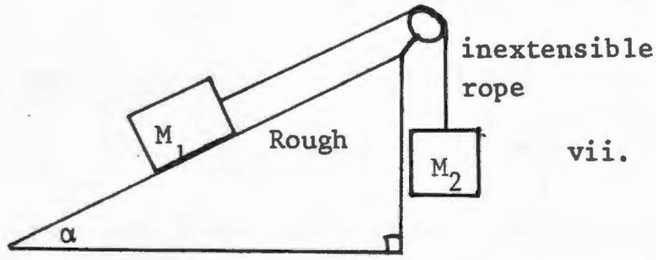

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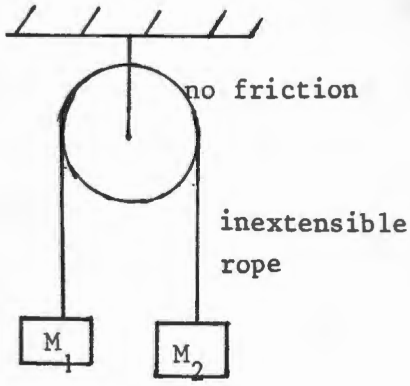
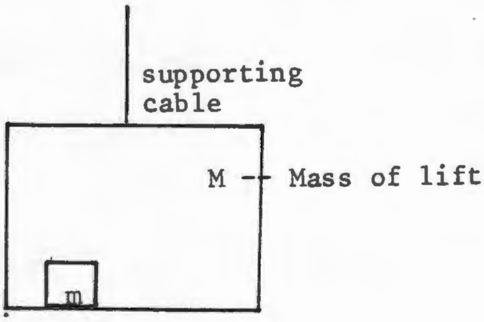


viii.



<u>DISCUSSION</u>:

i) - iv) Here the body on which the law is to be applied to predict its movement, is clearly the mass M.

v) - vii) M_1 and M_2 are usually regarded as separate masses on which the law is to be applied individually. Since they are connected by an inextensible rope, the <u>magnitude</u> of the acceleration of M_1 is the same as that of M_2. Obviously the direction of the acceleration differs. This is considered in point 3 below.

viii) The mass under consideration depends on the particular problem. If the problem has to do with the apparent weight of m, then obviously, this mass is considered. If the movement of the lift is in question, then please note that the mass to be used in calculations is (M + m).

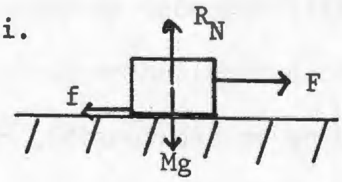
2. a) It is the RESULTANT force ON the body which causes the acceleration. The direction of the acceleration is that of the resultant force. As a first step it is thus necessary to draw in all the forces acting on the body.

(Check again point 2 in Mech B1 1).

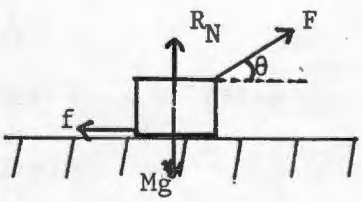
Please draw in all the forces acting on the various masses in the above examples.

b) After you have drawn in all the forces and checked your answers by looking at the next page, can you specify which forces directly effect the acceleration, which effect it only indirectly, and which have no effect on the acceleration of the mass? Please remember that it is the resultant force on the body which causes the acceleration.

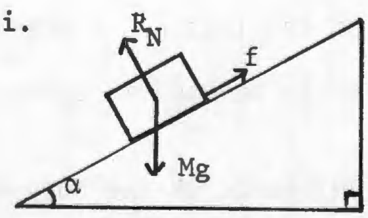
i.



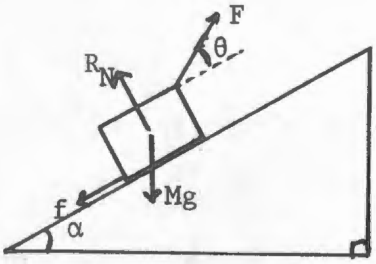
ii.



iii.

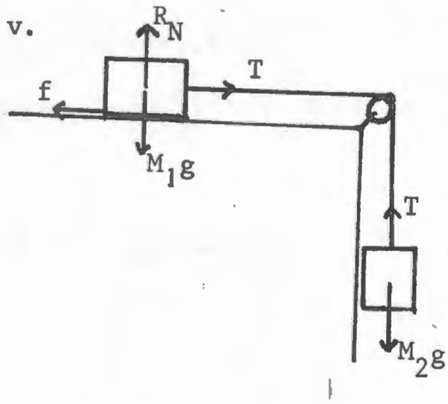


iv.

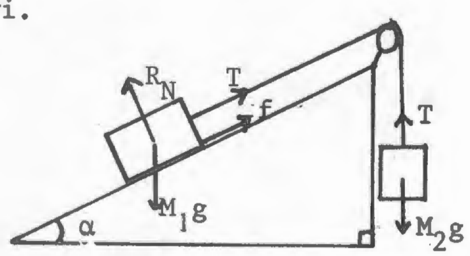


Note: In this problem, it is also possible for the mass to move downward. Then f, friction, will be upward. The direction of motion must be specified in this particular problem.

v.

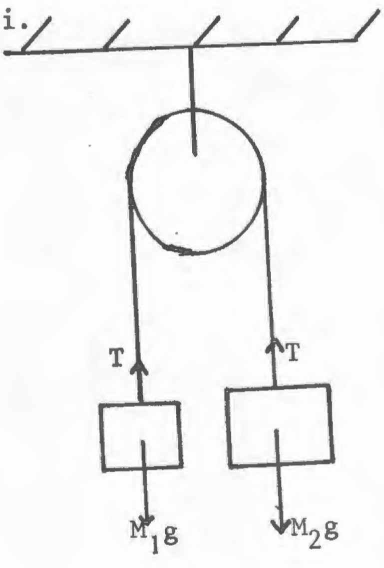


vi.

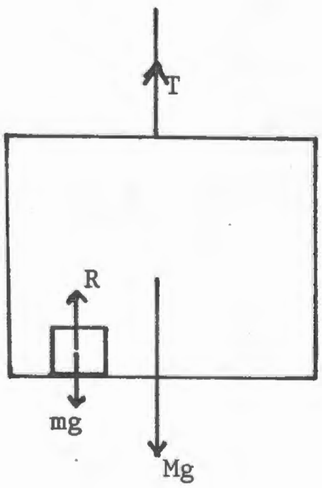


Here again, f will be upward only if the movement of M_1 is down the plane. The direction of motion is usually specified in problems of this nature. Thus if M_2 moves downward (i.e. M_1 upward), then f is down the plane.

vii.



viii.



Discussion of 2b

The forces which are perpendicular to the resultant force, obviously do not contribute directly to the resultant force. (Check notes on components of forces). However, if friction is present, since $f = \mu R_N$, all the perpendicular forces (which of course have a bearing on the value of R_N) contribute indirectly to the resultant force via f, which is parallel to the direction of motion.

Please see if you agree with the following chart:

No.	Forces directly affecting acc.	Forces indirectly affecting acc.
i)	F, f	R_N, Mg
ii)	Fcos θ, f	Fsin θ, R_N, Mg
iii)	Mg sin α, f	R_N, Mg cos α
iv)	F cos θ, Mg sin α, f	F sin θ, R_N, Mg cos α
v)	M_1: T, f	R_N, M_1g
	M_2: T, M_2g	
vi)	M_1: T, f, M_1g sin α	R_N, M_1g cos α
	M_2: T, M_2g	
vii)	M_1: T, M_1g	
	M_2: T, M_2g	
viii)	M : T, (M + m)g	
	m : R, mg	

3 a) It is clear that two directions are implied:

i) The direction of acceleration in which Newton II is applicable.

ii) The direction perpendicular to the direction of acceleration, in which Newton I is applicable i.e. in which there is equilibrium. (compare point 3 in Mech Bl 2).

b) Thus in choosing a system of axes, the one axis is chosen in the direction of the acceleration, the other perpendicular. Since the + direction of acceleration is generally in the direction of motion, this direction is chosen for the one axis. (see point 5 below for exceptions).

c) In the direction of acceleration, say x:

$$\Sigma F_x = M \cdot a$$

Perpendicular to the direction of acceleration, say y:

$$\Sigma F_y = 0$$

This means that in some cases, the components of the forces in these directions must be determined.

NOTE: In determining the direction of motion, example vii) provides the method for establishing this:

if $M_2 > M_1$ i.e. $M_2g - M_1g > 0$, M_2 will move down i.e. find the direction of the resultant of the external forces on the system. This will be the direction of motion if the system moves on its own. (friction is excluded)

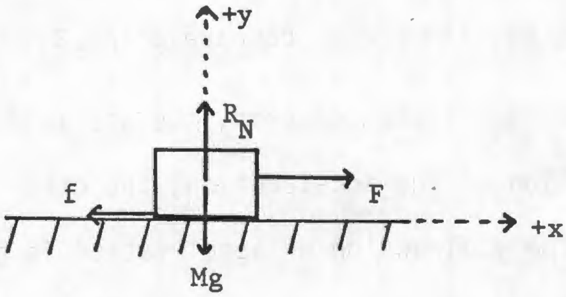
iv) $F \cos \theta - Mg \sin \alpha > 0$, movement up and vice-versa.

v) $M_2g > 0$ (which is obviously so!), M_2 thus moves downward.

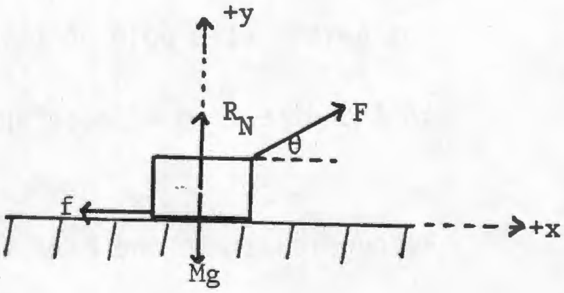
vi) $M_1g \sin \alpha - M_2g > 0$ then M_1 moves down plane and vice-versa.

The above examples all presuppose that motion takes place. The frictional force, though external to the system, always reacts to the direction of motion but does not influence it. At most, it can prevent motion.

i.



ii.



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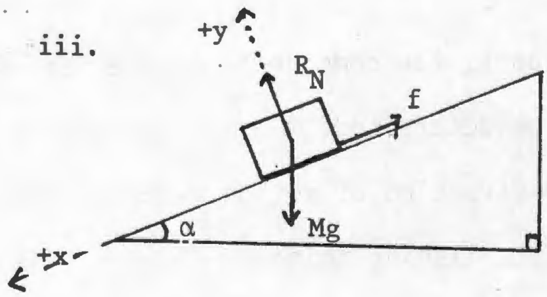

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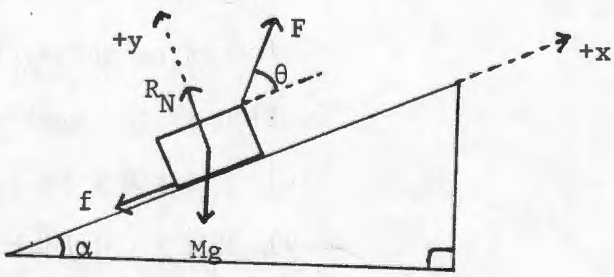
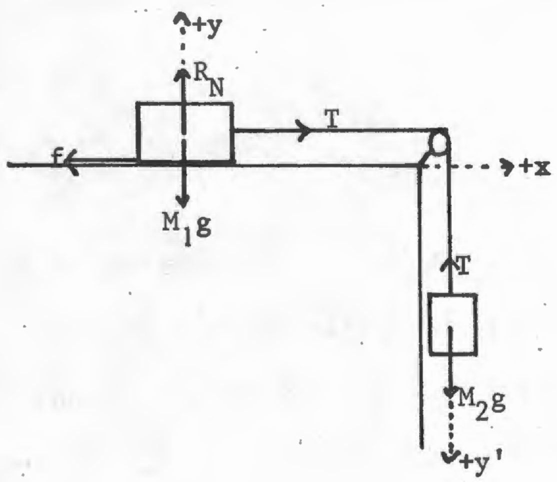
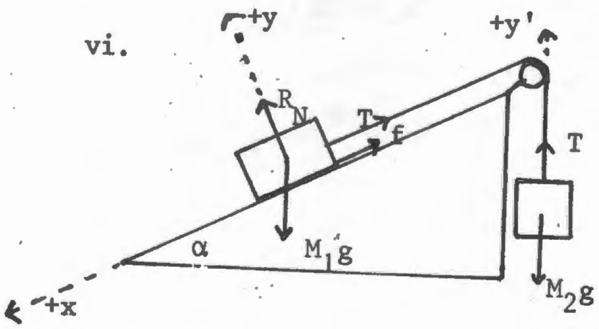


v.

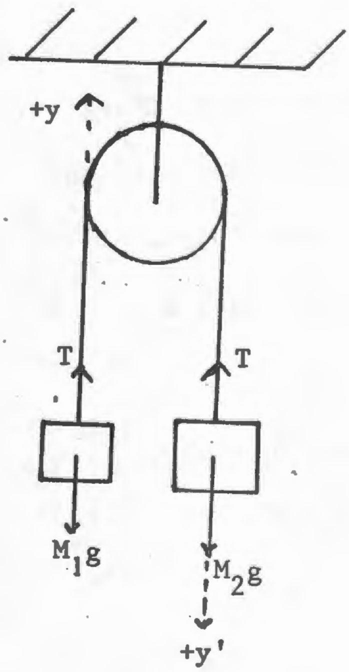


Note that although y and y' are + and - in the same line, the use of the (') indicates that we are dealing with a different mass.

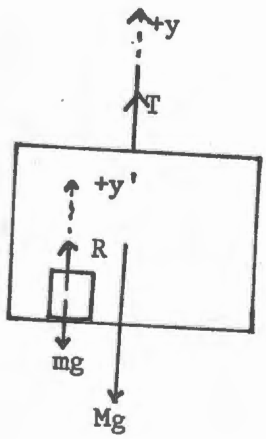
vi.



vii.



viii.



4. i) Determine the mass that is being accelerated. (point 1 above)

ii) Draw in all the forces acting on the mass. (point 2 above).

iii) Determine the direction of motion, if this is not given, by using the procedure at the bottom of page 60.

iv) Choose one axis in this direction, the other perpendicular.

v) In the case of connected masses, see how the accelerations are related.

vi) Determine if forces in the perpendicular direction, influence forces parallel to the direction of motion. Thus

$$\Sigma F_{perpendicular} = 0 \text{ and } f = \mu R_N$$

vii) $$\Sigma F_{parallel} = M . a$$

viii) Repeat the procedure for each mass individually.

ix) Solve the equations.

We have already carried out the procedure up to point iv) above, having predetermined arbitrary directions of motion in examples iv), vi), and vii). Try to carry out points v) - ix) above in the examples given, before consulting the answers below.

i) $\Sigma F_y = R_N - Mg = 0 \quad \therefore \quad f = \mu R_N = \mu M_g$

$\Sigma F_x = F - f = Ma$ or $F - \mu Mg = Ma$

ii) $\Sigma F_y = R_N + F \sin \theta - Mg = 0$

$\therefore \quad R_N = M_g - F \sin \theta$ and $f = \mu R_N = \mu(Mg - F \sin \theta)$

$\Sigma F_x = \cos \theta - f = Ma$

$\therefore \quad F \cos \theta - \mu(Mg - F \sin \theta) = Ma$

iii) $\Sigma F_y = R_N - Mg \cos \alpha = 0$

$\therefore \quad f = \mu R_N = \mu R_N = \mu Mg \cos \alpha$

$\Sigma F_x = Mg \sin \alpha - f = Ma$

$\therefore \quad a = g \sin \alpha - \mu g \cos \alpha$

iv) $\Sigma F_y = R_N - Mg \cos \alpha + F \sin \Theta = 0$

$\therefore \quad f = \mu R_N = \mu(Mg \cos \alpha - F \sin \Theta)$

$\Sigma F_x = F \cos \theta - f - Mg \sin \alpha = Ma$

$\therefore \quad F \cos \theta - \mu (Mg \cos \alpha - F \sin\theta) - Mg \sin\alpha = Ma$

v) If M_2 has acceleration "a" in direction +y', then M_1 will have acceleration "a" in direction +x.

$M_1 : \Sigma F_y = R_N - M_1 g = 0 \quad \therefore \quad f = \mu R_N = \mu M_1 g$

$\Sigma F_x = T - f = M_1 a$

$\therefore \quad T - \mu M_1 g = M_1 a \qquad (1)$

$M_2 : \Sigma F_{y'} = M_2 g - T = M_2 a \qquad (2)$

Two unknowns can now be obtained by solving (1) and (2)

vi) If M_1 has acceleration "a" down the plane in direction +x, then M_2 has acceleration "a" upward in direction +y'.

$M_1 : \Sigma F_y = R_N - M_1 g \cos \alpha = 0 \therefore f = \mu R_N = \mu M_1 g \cos \alpha$

$\Sigma F_x = M_1 g \sin \alpha - T - f = M_1 a$

$\therefore \quad M_1 g \sin \alpha - T - \mu M_1 g \cos\alpha = M_1 a \qquad (1)$

$M_2 : \Sigma F_{y'} = T - M_2 g = M_2 a \qquad (2)$

By putting values into equation (1) and (2), unknowns such as "T" and "a" can be found.

vii) $M_1 : \Sigma F_y = T - M_1 g = M_1 a$

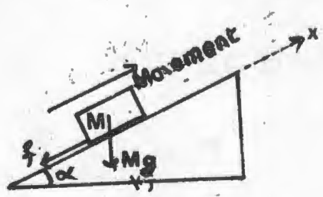
$M_2 : \Sigma F_y' = M_2 g - T = M_2 a$

Note: "a" is the same for both masses although the directions differ.

viii) Lift: $T - (M + m)g = (M + m)a$

$R - mg = ma$

5. Please distinguish between direction of <u>motion</u> and direction of <u>acceleration</u>. In the above examples it was assumed that the direction of motion and of acceleration are the same. Consider example iii) above, but now assume that the mass M is projected up the plane. If the upward direction parallel to the plane is taken as positive, then



$\Sigma F_x = -Mg \sin\alpha - f = Ma$

$\therefore \; a = -(g \sin\alpha + \mu g \cos\alpha)$

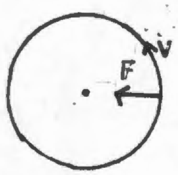
which is negative. The (-) shows that the direction of motion and the direction of acceleration are opposite.

NOTE: With problems iii), iv), and vi), either:

a) the direction of motion is given as part of the problem, in which case the direction of friction must be inserted accordingly.

or b) no direction of motion is given. Here it does NOT help to guess a direction and then to say that if it works out negative it simply means that the acceleration is in the opposite direction. The reason for this is that the frictional force, f, must always oppose the motion. Thus the direction of motion of the system must first be established using the procedure on p 60. Of course, if friction is absent, then an arbitrary direction of motion can be assumed. A negative acceleration will then simply indicate that the acceleration is opposite to the chosen direction.

In the case of circular motion, the force causing the motion in the



circle is directed inward radially, and the acceleration (centripetal) is also in this direction.

$$F = \frac{mv^2}{r}$$

(see Mech. B1.8)

MECH BL 6 : RELATIVE VELOCITY

Mech. B1 6 : Relative Velocity

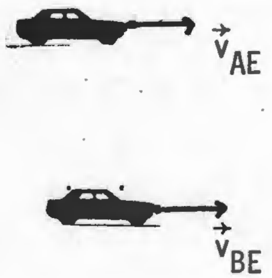
Consider the following example and see if you can determine the procedure which you follow to arrive at an answer:

Two cars A & B are moving at 60 Km/h.

a) in the same direction. Clearly both can be said to be moving relative to the stationary earth.

$\vec{v}_{AE}$: velocity of A relative to the earth.

What is the velocity of A relative to B?

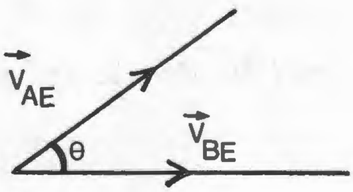


(Clearly the answer is 0)

b) in opposite directions

Can you see that the velocity of A relative to B is 120 km/h?

c) at angle θ w.r.t. each other.



What is the velocity of A relative to B?

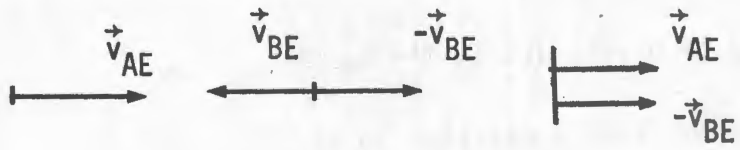
It is more difficult to write down the answer to (c) than is the case with (a) & (b). Let us look at the two problems again.

a) $\xrightarrow{\hspace{2em}} \vec{v}_{AE}$ It is easy to give the velocity of each w.r.t. the earth, since the earth can be regarded as

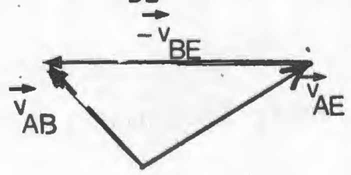
$\longrightarrow \vec{v}_{BE}$ stationary. Therefore to establish the velocity of A relative to B, stop B by giving it a suitable back velocity. This must them be added also to velocity of A:

$$-\vec{v}_{BE} \longleftarrow\!\!+\!\!\longrightarrow \vec{v}_{BE} \quad -\vec{v}_{BE} \longleftarrow\!\!+\!\!\longrightarrow \vec{v}_{AE} \quad \therefore \quad \vec{v}_{AB} = 0$$

Notice that the addition must be done vectorially

b) 

c) In dealing with this problem therefore, stop B by giving it a back velocity i.e. $-\vec{v}_{BE}$. Add this same back velocity vectorially to $\vec{v}_{AE}$:



The vector equation is thus:

$$\vec{v}_{AB} = \vec{v}_{AE} + (-\vec{v}_{BE})$$

$$\boxed{\therefore \quad \vec{v}_{AE} = \vec{v}_{AB} + \vec{v}_{BE}} \qquad \text{.......(6.1)}$$

Note carefully how the subscripts are ordered.

Ensure that you understand clearly how this vector equation is obtained. The rest of the booklet is designed to demonstrate the procedure you need to use this equation in solving problems on relative velocity.

Consider the following four problems:

1) A cyclist is travelling due north at 6 m s^{-1} on a day when the wind blows from the east at 2,5 m s^{-1}. Calculate the

velocity of the wind relative to the cyclist.

2) a) The water in a river flows at an average velocity of 1,5 $m\ s^{-1}$, and a boat moves at 2 ,5 $m\ s^{-1}$ on the water. Calculate:

i) the direction in which the boat is to be steered for it to reach B, a position on the opposite bank directly across from A the starting point.

ii) the velocity of the boat relative to the earth.

b) If the boat is steered in direction AB, calculate its velocity relative to the earth.

3) A helicopter flies at 120 $km\ h^{-1}$. At a particular instant a motor car is 10 km away exactly due South of the helicopter and travelling at 80 $km\ h^{-1}$ in a South-Easterly direction. Assuming that the helicopter flies just above ground level and there is no wind, determine

i) the direction in which the helicopter must fly to intercept the car ;

ii) the time taken to make the interception.

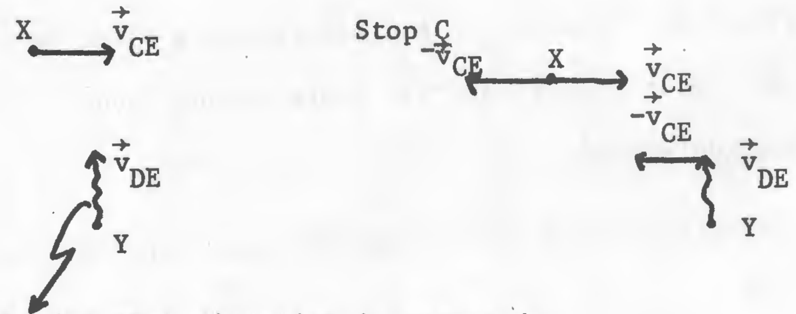
4) A bus travels at a velocity of 5 $m\ s^{-1}$. Raindrops falling vertically with respect to the earth, strike the window of the bus at an angle inclined at 30° to the vertical. Calculate the velocity of the raindrops (a) with respect to the earth; (b) with respect to the bus.

Before establishing the strategy for dealing with this type of problem, it is perhaps necessary to clarify two points:

A. Remember that $\vec{v}_{MN}$ the velocity of body M relative to the body N does not deal with actual velocity e.g. Car M $\rightarrow$ 40 $km\ h^{-1}$
Car N $\rightarrow$ 80 $km\ h^{-1}$ $\therefore$ $\vec{v}_{MN}$ $\leftarrow$ 40 $km\ h^{-1}$. Obviously neither M nor N is moving backward! But relative to N, M appears to be.

If you need to determine that body M is moving in direction , then the velocity of M relative to the earth will give this i.e. the actual direction in which a body M will move is determined by $\vec{v}_{ME}$.

B. If the problem is one of interception i.e. the paths of two bodies cross or they approach each other to within a certain distance, then the relative velocity must be along the line joining the two bodies. To illustrate, let us say that C is moving as shown with <u>velocity</u> $\vec{v}_{CE}$, at pt X at a particular instant. At the same instant D, at point Y, moving with speed v_{DE} changes direction so as to intercept C:



Indicating that direction is uncertain.

C is now stationary. How must D now move relative to C? Clearly vel. of D rel. to C i.e. $\vec{v}_{DC}$ must be along the line YX for interception to take place since relatively speaking C is now stationary. Thus for an interception: RELATIVE VELOCITY of D to C must be along the line joining the two bodies.

It is essential that you think carefully about points A & B and have them clear in mind before proceeding.

<u>STEP 1</u>

Read through the four problems above and try to determine the number of bodies involved in each and which is the reference body. See if you can write them down before checking on the next page for the answers.

PROBLEM 1 There are clearly 3 bodies involved: cyclist (C), the wind (W) and the earth (E).

The wind is, of course, not a body strictly speaking, but for purposes of relative velocity we will regard it as such.

While the earth is not explicitly stated obviously the velocities of C & W are given relative to the stationary (to C & W) earth.

PROBLEM 2 a & b : Here there are again 3 bodies: water (W), boat (B) and the earth (E).

Again in the first part of the problem the earth is not stipulated, but the velocity of the water is obviously relative to the earth.

PROBLEM 3: Here the velocity of both the helicopter (H) and the car (C) is stated relative to the earth (E). So again there are 3 bodies: H, C, E.

PROBLEM 4: The bus, (B), the raindrops, (R), are moving relative to the earth (E).

NOTE:

i) In each of the above problems there are 3 bodies. While any amount is possible, the equation (6.1) on page 69 only allows three to be handled at a time.

ii) In each of the cases, the earth is the reference body. Again others are possible in different problems, but it is normally so.

STEPS 2 & 3

Steps 2 & 3 are executed together. These involve determining the magnitude and direction of the three velocities in equation (6.1). Remember two velocities are relative to the reference body and the third relative to each other. Assume bodies A, B, E, construct the following matrix:

	MAGNITUDE	DIRECTION
$\vec{v}_{AE}$:		
$\vec{v}_{BE}$:		
$\vec{v}_{AB}$:		
or BA		

Read the problems and fill the given values in the table, remembering the following:

a) Four of the six possible items must be given if you are to use equation (6.1). (Why?)

b) Points A & B of p.70 & 71 will enable you to determine some directions.

Please try it before checking your answers over the page.

PROBLEM 1:

	MAGNITUDE	DIRECTION
$\vec{v}_{CE}$:	6 m s^{-1}	$\uparrow$
$\vec{v}_{WE}$:	$2{,}5 \text{ m s}^{-1}$	$\leftarrow$
$\vec{v}_{CW}$	?	?

or WC

$\vec{v}_{WC}$ is required according to the problem

PROBLEM 2a:

$\vec{v}_{WE}$:	$1{,}5 \text{ m s}^{-1}$	$\xrightarrow{}$ a
$\vec{v}_{BE}$:	?	$\uparrow$ b
$\vec{v}_{BW}$:	$2{,}5 \text{ m s}^{-1}$	?

a The direction of the river can obviously be arbitrarily chosen.

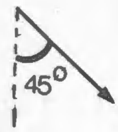
b Note that the problem stipulates that the boat must reach the point exactly opposite its starting point. This means that the velocity of the boat relative to the earth must be in this direction. (Please check point A on page 70 .).

PROBLEM 2b:

$\vec{v}_{WE}$:	$1{,}5 \text{ m s}^{-1}$	$\rightarrow$
$\vec{v}_{BE}$:	?	?
$\vec{v}_{BW}$:	$2{,}5 \text{ m s}^{-1}$	$\uparrow$ a

a. Note that the problem states that the boat is steered in direction AB i.e.its velocity relative to the water is in this direction. It will of course in practice be swept downstream by the river i.e.its velocity relative to the earth will certainly not be straight across.

Please think carefully about these 3 points in problems 2a & b. They are troublesome:

PROBLEM 3:	MAGNITUDE	DIRECTION
$\vec{v}_{HE}$:	120 km h^{-1}	?
$\vec{v}_{CE}$:	80 km h^{-1}	
$\vec{v}_{HC}$:	?	 a

a. To understand the direction of $\vec{v}_{HC}$, please read carefully again point B, p.71. Note that at the instant in question C, is due South of H. For interception, the relative velocity must be along this line.

PROBLEM 4:		
$\vec{v}_{BE}$:	5 m s^{-1}	 a
$\vec{v}_{RE}$:	?	
$\vec{v}_{RB}$:	?	 b

a. Arbitrarily chosen direction.

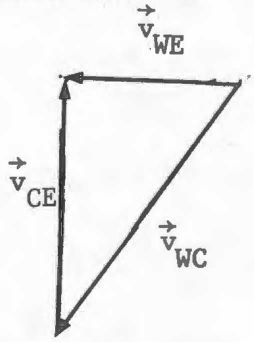
b. One needs to exercise a bit of commonsense here. 30° to the vertical could be  or  . Can you see why it is chosen as it is?

STEP 4:

Now all that remains is to construct the vector diagram in such a way that the correct triangle results from the given values. Try it before checking your answers over the page. Take your time - this is the crucial step.

PROBLEM 1: $\vec{v}_{WE} = \vec{v}_{WC} + \vec{v}_{CE}$

The resultant must be $\vec{v}_{WE}$ while $\vec{v}_{WC}$ is unknown.



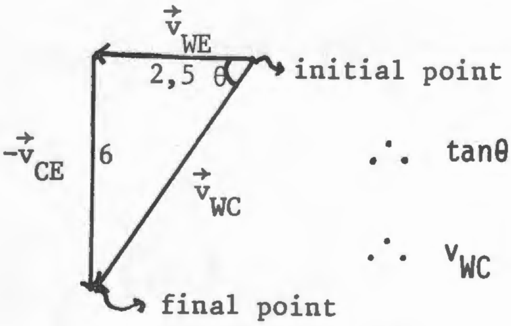
Because $\vec{v}_{WE}$ is resultant, the head of this vector must coincide with the head of $\vec{v}_{CE}$.

The tail of $\vec{v}_{WE}$ must coincide with the initial point of $\vec{v}_{WC}$.

Thus $\vec{v}_{WC}$ must have that direction.

If two directions are known ($\vec{v}_{WE}$ & $\vec{v}_{CE}$) in this example, group these two on one side and add vectorially:

$$\therefore \quad \vec{v}_{WC} = \vec{v}_{WE} - \vec{v}_{CE} = \vec{v}_{WE} + (-\vec{v}_{CE})$$



$$\therefore \quad \tan\theta = \frac{6}{2,5} \quad \therefore \theta = 67° \ 24'$$

$$\therefore \quad v_{WC} = \frac{6}{\sin\theta} = 6,5 \text{ m s}^{-1}$$

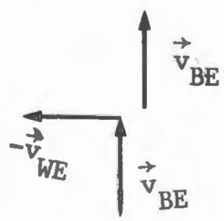
$\therefore$ $\vec{v}_{WC}$ i.e. velocity of wind relative to the cyclist appears to be from 22° 36' E of N.

PROBLEM 2a:

Here $\vec{v}_{BE} = \vec{v}_{BW} + \vec{v}_{WE}$

Since the direction of $\vec{v}_{BE}$ & $\vec{v}_{WE}$ are known:

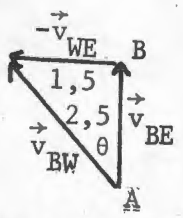
$$\vec{v}_{BW} = \vec{v}_{BE} + (-\vec{v}_{WE})$$



The diagram is thus started with $\vec{v}_{BE}$.

$(-\vec{v}_{WE})$ is added vectorially:

$\vec{v}_{BW}$ as the resultant is then obtained as the vector from the initial point of $\vec{v}_{BE}$ to the final point of $(-\vec{v}_{WE})$:

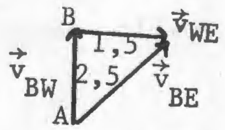


$$\sin\theta = \frac{1,5}{2,5} \quad \therefore \quad \theta = 360\ 52'$$

$$v_{BE} = 2,5 \cos\theta = 2 \text{ m s}^{-1}$$

<u>PROBLEM 2b</u>: Again $\vec{v}_{BE} = \vec{v}_{BW} + \vec{v}_{WE}$

Since the directions of $\vec{v}_{BW}$ & $\vec{v}_{WE}$ are known, the vector addition can be executed as it stands:

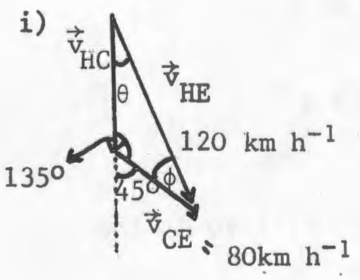


$$v_{BE} = \sqrt{2,5^2 + 1,5^2} = \underline{2,95 \text{ m s}^{-1}}$$

<u>PROBLEM 3</u>: In this case: $\vec{v}_{HE} = \vec{v}_{HC} + \vec{v}_{CE}$

Since the directions of $\vec{v}_{CE}$ & $\vec{v}_{HC}$ are known, the vector addition is done as it is:

i)



Now $\dfrac{\sin 135°}{v_{HE}} = \dfrac{\sin\theta}{80}$

$\therefore \sin\theta = \dfrac{8}{120} \times \sin 45° \ \therefore \ \underline{\theta = 28,1°}$

So helicopter flies at angle <u>28,1°</u> East of South.

ii) To determine the time taken to make the interception, 10 km due South must be covered at the relative velocity $\vec{v}_{HC}$. (See point B, p.71.)

$$\frac{v_{HC}}{\sin\phi} = \frac{v_{HE}}{\sin 135°} \quad \therefore \quad v_{HC} = \frac{\sin 16,9°}{\sin 135°} \times 120 = 49,3 \text{ km h}^{-1}$$

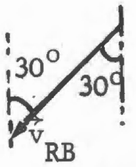
$$\therefore \text{ Time taken} = \frac{10}{49,3} \text{ h} = 0,2 \text{ h} = 12 \text{ min.}$$

PROBLEM 4:

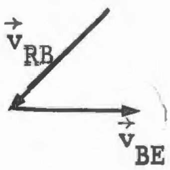
Here: $\vec{v}_{RE} = \vec{v}_{RB} + \vec{v}_{BE}$.

In this case all three directions are given, so the addition can take place as the equation is set up with due regard for the various directions.

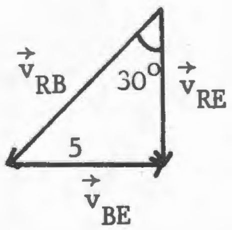
Start with $\vec{v}_{RB}$:



To this is added $\vec{v}_{BE}$



Since the resultant $\vec{v}_{RE}$ must be vertically down, the endpoint of $\vec{v}_{BE}$ must be directly below the initial point of $\vec{v}_{RB}$.



a) $\dfrac{5}{v_{RE}} = \tan 30°$

$\therefore \; v_{RE} = \dfrac{5}{\tan 30°} = \underline{8{,}66} \quad m\ s^{-1}$

b) $\dfrac{5}{v_{RB}} = \sin 30° \quad \therefore \; v_{RB} = \dfrac{5}{\sin 30°} = 10\ m\ s^{-1}$

From the above the algorithm for dealing with problems on relative velocity is the following:

1) Determine the three bodies involved, A,B,E and establish which is the reference body, E.
2) Establish which of the magnitudes $|\vec{v}_{AE}|, |\vec{v}_{BE}|, |\vec{v}_{AB}|$ are known.
3) Establish which of the directions of $\vec{v}_{AB}$, $\vec{v}_{AE}$, $\vec{v}_{BE}$ are known.

Remember

a) The actual direction in which a body travels is given by the direction of its velocity relative to the earth.

b) For an interception of two bodies A&B, the relative velocity $\vec{v}_{AB}$ must be along the line joining them.

4) Draw a vector diagram of the velocities according to:

$$\vec{v}_{AE} = \vec{v}_{AB} + \vec{v}_{BE}$$

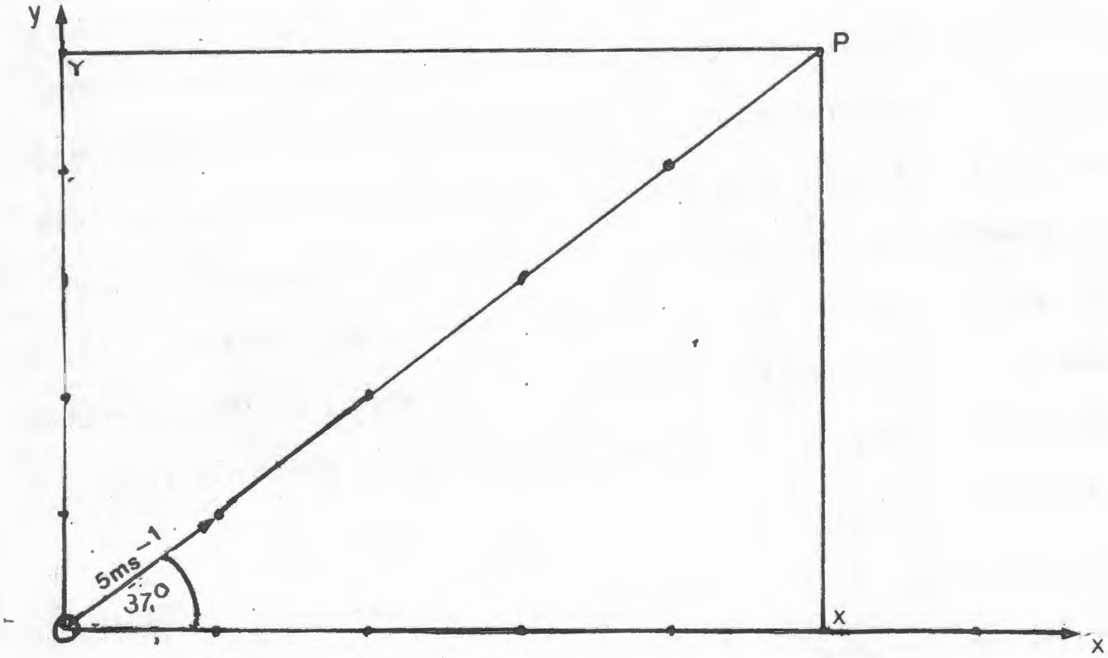
5) Determine the unknowns.

MECH BL 7 : PROJECTILE MOTION

Mech B1 7 : Projectile Motion

One of the principal difficulties in dealing with the motion of an object in 2-Dimensions under gravity, is to separate the movement into two components, horizontal and vertical, and then to combine these to the extent needed.

Let us first illustrate what is involved, using a simple example.



Assume that an object travels at 5 m s^{-1} at an angle of 37° to a direction designated as Ox. Assume that each of the intervals along the line OP represents 1 s. Clearly after travelling for 5 s it will reach P and will have covered 25 m. However, there is an alternative manner of expressing this situation:

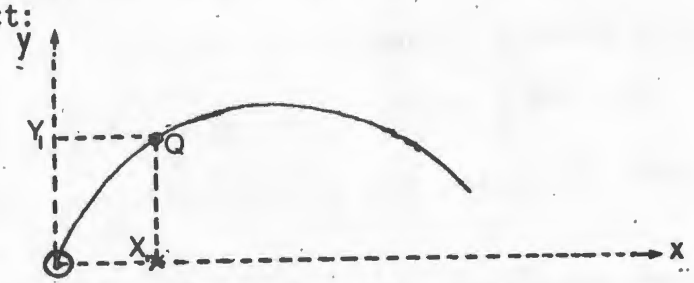
The movement can be regarded as the compounding of two simultaneous movements in directions Ox and Oy with speeds of

$$v_x = 5 \cos 37° = 4 \text{ m s}^{-1} \text{ and } v_y = 5 \sin 37° = 3 \text{ m s}^{-1}$$

respectively. Please note that the actual direction in which the body moves is obviously OP but in the x-direction it will cover distance OX while in the y-direction it covers distance OY in 5 s.

The actual displacement OP is obtained by combining the projected motions OX and OY.

In the case of projectile motion under gravity the actual parabolic motion of the object:



can more easily be considered by dividing the movement into a horizontal (x) and vertical (y) component. Thus instead of considering the actual path of motion from O to Q we consider that the particle has covered the distances OX and OY, in the x- and y-directions respectively. Clearly the time t taken to reach the point Q is the same as that to reach X_1 in the x-direction and Y_1 in the y-direction. The important reason for separating the x and y movements is the following:

The only force acting on the particle as it moves along the parabolic path is its own weight. Since this acts vertically downward then, by Newton's second law, the only acceleration which the body experiences will be in this direction. Thus in the x-direction there is no acceleration i.e. the body moves with constant speed.

So the two components making up the parabolic movement, are:

1) x-direction. Constant speed.
2) y-direction. Accelerated motion with acceleration g always directed downward.

Please remember:

a) Since acceleration is a vector it must have a direction, in this case downward. If you choose your axis y positive upward, then g is $-9{,}8\ m\ s^{-2}$ (or $-10\ m\ s^{-2}$ for convenience) and vice-versa.

b) The actual time t to reach point Q along the path is the same for the corresponding component movements OX_1 and OY_1.

Please make sure that you understand the above points clearly. We will now develop a procedure for dealing with problems on projectiles by considering the solutions to the following four problems:

1) A body, projected upward at an angle of 37° with the horizontal direction, has an initial speed of $50\ m\ s^{-1}$. Calculate:

   a) Maximum height it reaches above the ground and the time taken.

   b) How far from the point of projection does it reach the ground and how long does it take?

   c) At what angle does it strike the ground?

2) A body is projected at a speed of $50\ m\ s^{-1}$ and an angle of 37° to the horizontal and lands on a 25 m high plateau.

   i) How long does it take to reach there?

   ii) What is the horizontal distance covered?

   iii) With what velocity does it strike the plateau?

3) A ball rolls off the edge of a horizontal table top, 1,8 m high, and strikes the floor at a distance of 2,5 horizontally from the edge of the table. What was the velocity of the ball at the instant of leaving the table?

4) An object is thrown upward from the roof of a building 10 m high with velocity $10\ m\ s^{-1}$ at an angle of 37° with the horizontal. Calculate:

i) Highest distance above the ground the object will reach.

ii) Distance from the foot of the building the object will land.

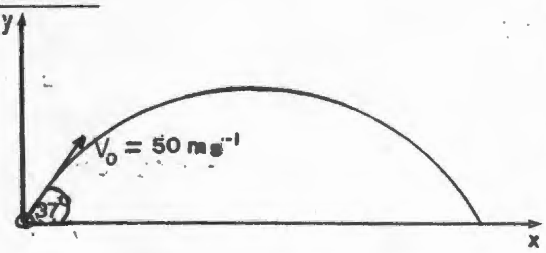
You will find it useful, before actually working through the solutions, to read through the problems carefully, note the facts given and try to determine what exactly is required to solve the problem. Please do this before going into the procedure developed on the following pages.

STEP 1:

Choose suitable axes x (horizontal) and y (vertical) and the origin at the point of projection of the object.

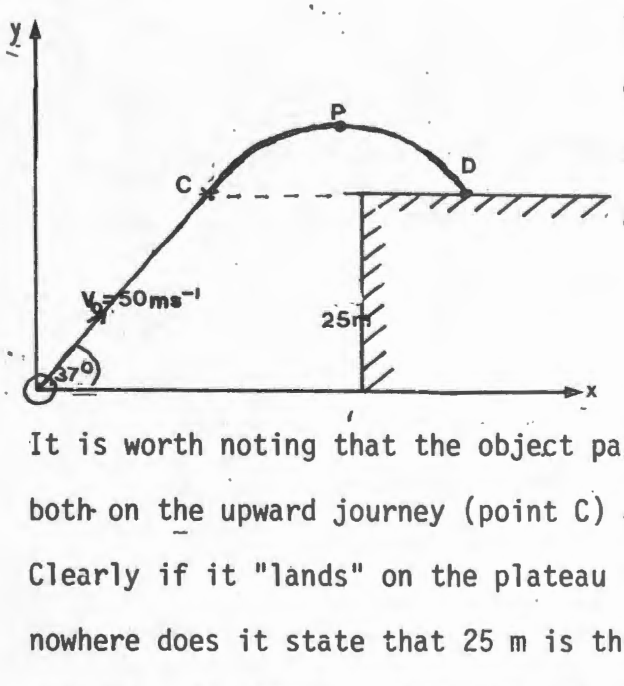
This is obviously necessary in view of the discussion before. Please attempt this before checking your answers below.

PROBLEM 1



It is clear from the problem that the x-axis must be at ground level since the problem says nothing to the contrary.

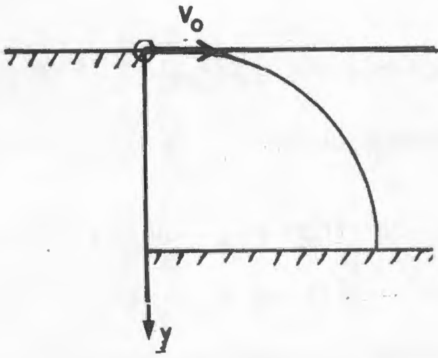
PROBLEM 2



In this case the object is thrown from ground level, thus determining the origin of the system of axes. It lands 25 m above ground level.

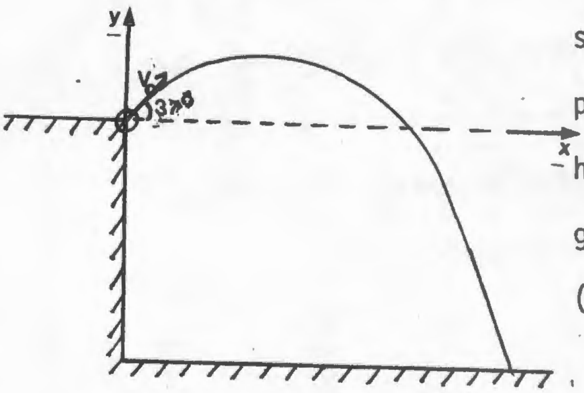
It is worth noting that the object passes through the 25 m mark both on the upward journey (point C) and the downward (point D). Clearly if it "lands" on the plateau it cannot do so at C. Also, nowhere does it state that 25 m is the highest point the object reaches. Be careful not to place interpretations on the data given which are not stated or apparent from the physical situation.

PROBLEM 3:



In this case the movement of the object is only downward and hence there is no point to having the positive direction of the y-axis upward since there is no movement in that direction. Note also that although it is not explicitly stated in the problem the initial velocity, V_o, of the ball (as it leaves the table) must be horizontal.

PROBLEM 4



Note again that the origin of the system of axes is at the point of projection, thus at the roof. The horizontal (x) axis is thus not the ground which now has a y-coordinate (-10).

STEP 2

Determine the x and y components of the initial velocity, V_o.

PROBLEM 1

$$v_{ox} = V_o \cos\theta = 50 \cos 37° = 40 \text{ m s}^{-1}$$

$$v_{oy} = V_o \sin\theta = 50 \sin 37° = 30 \text{ m s}^{-1}$$

PROBLEM 2

$$v_{ox} = V_o \cos\theta = 50 \cos 37° = 40 \text{ m s}^{-1}$$

$$v_{oy} = V_o \sin\theta = 50 \sin 37° = 30 \text{ m s}^{-1}$$

PROBLEM 3:

In this case the problem is to determine V_o. Please remember that in dealing with the horizontal motion use

$$v_{ox} = V_o \cos\theta$$

while with the vertical motion

$$v_{oy} = V_o \sin\theta$$

Since the solution of the problem will yield v_{ox} and v_{oy} independently, it will be necessary to recombine the components to yield the initial velocity, V_o:

$$V_o = \sqrt{v_{ox}^2 + v_{oy}^2}$$

In this particular problems the initial velocity is horizontal. Hence

$$V_{ox} = V_o \cos\theta = V_o \text{ (since } \theta = 0)$$
$$v_{oy} = V_o \sin\theta = 0$$

PROBLEM 4:

$$v_{ox} = V_o \cos\theta = 10 \cos 37° = 8 \text{ m s}^{-1}$$
$$v_{oy} = V_o \sin\theta = 10 \sin 37° = 6 \text{ m s}^{-1}$$

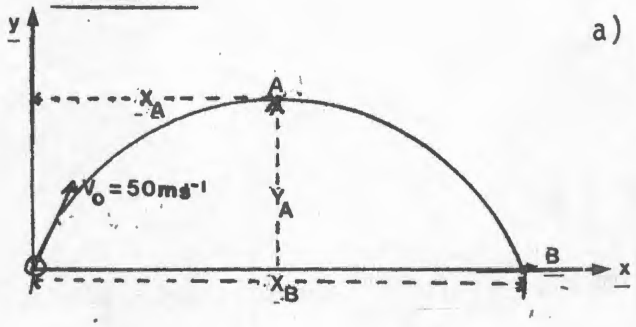
When writing down the components please realize that the angle θ used above, is the angle between the x-axis and the initial velocity.

STEP 3

Read the problem and write down explicitly the coordinates (x,y) of the point or points in question.

This is the crucial step since these coordinates give you the displacements from the origin in the x and y-directions at any time t. Please read the problems carefully and study the graphs on pages 85 and 86 before comparing your answers with those given below.

PROBLEM 1



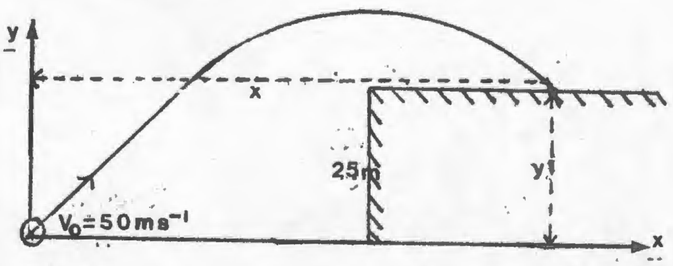
a) Maximum height reached is at point A. The problem gives no coordinates for this point, so assign coordinates

x_A, y_A

b) This is obviously the point B. While the x-coordinate is not given and can thus be arbitrarily named, the body has returned to the level of the origin i.e.its vertical or y displacement is zero. Thus the coordinates are

$(x_B; 0)$

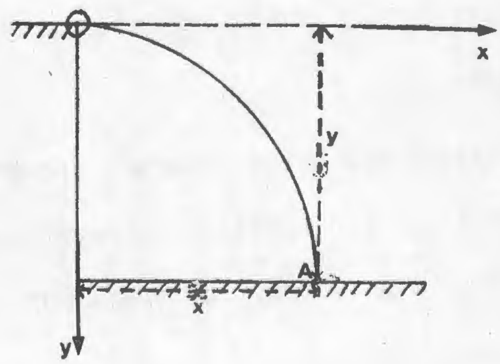
PROBLEM 2



The point in question is of course D. The x-coordinate of D is not specified, while the y coordinate is + 25 i.e. vertically displaced 25 m from Ox.

Thus the coordinates are: (x, +25)

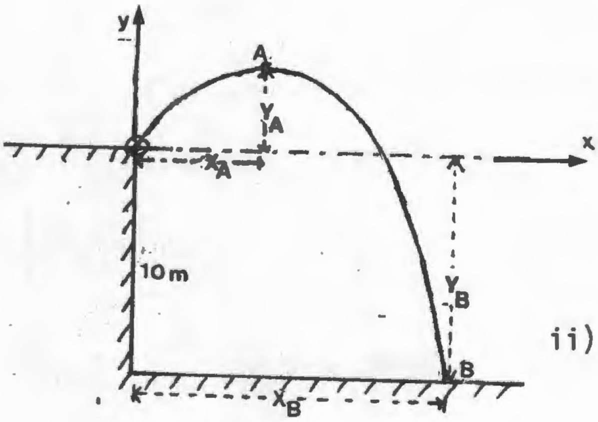
PROBLEM 3



Here the point A is the point in question. y would then represent the vertical distance from the top of the table to the ground while x is the horizontal distance from the foot of the table to the point where the ball lands. Clearly, reading the problem the coordinates are

$x = +2,5; \quad y = +1,8.$

PROBLEM 4:



i) The highest point the object will reach is obviously the point A. Neither coordinate is given so can be arbitrarily designated as (x_A, y_A).

ii) The point where the object lands is B. The x coordinate, x_B, is not known - indeed that is the problem.

The y-coordinate is y_B - it is the vertical distance of the ground below the x-axis, but since we are dealing with a coordinate system with an origin, the coordinate must have a sign.

Thus $y_B = -10$

The coordinates of B are thus: $(x_B, -10)$.

NOTE

1. The coordinate y is a displacement relative to the origin, not necessarily the actual vertical distance covered.

   e.g.PROBLEM 1: while y_A is the actual vertical distance covered, y_B is not.

   $y_B = 0$ can be regarded as made up of two displacements

The resultant displacement $= y_B = 0$

2. The coordinates x and y of a particular point will of course both be reached in same time, t.

STEP 4

VERTICAL MOTION under constant acceleration $\pm g$:

a). Fill in from the problem the values of the 5 variables:

$a = \pm g$ $\left\{ \text{sign depends on direction of y-axis: } +y \text{ upward: } -g \;\; +y \text{ downward: } +g \right\}$

$s = y$ (y-coordinate Step 3)

$t = ?$ (same as for the horizontal coordinate, x)

$v_o = v_{oy}$ (Step 2)

$v = ?$ (at highest point, $v = 0$).

b) Use the appropriate kinematic equations to determine the unknown(s).

$$\left\{ \begin{array}{l} v = v_o + at \\ s = v_o t + \frac{1}{2}at^2 \\ v^2 = v_o^2 + 2as \end{array} \right\}$$

Please remember that $v_o = v_{oy}$ since you are dealing only with the vertical motion.

PROBLEM 1

a) $a = -g = -10 \text{ m s}^{-2}$; $s = y_A$; $v_o = v_{oy} = 30 \text{ m s}^{-1}$.

$v = v_y = 0$ (highest point). $t = t_A$.

The problem requires maximum height, y_A, and t_A to be calculated. Since 3 out of 5 variables are known and there are three equations, this can be calculated:

$v = v_o + at$: $0 = 30 - 10 \times t_A$ $\therefore$ $\underline{t_A = 3s}$ $\rightarrow$

$s = v_ot + \frac{1}{2}at^2$: $y_A = 30 \times 3 - \frac{1}{2} \times 10 \times 3^2 = \underline{45m}$ $\rightarrow$

b) We now deal with point B with coordinates: $(x_B,0)$ (Step 3)

$a = -g = -10\ m\ s^{-2}$ $s = y_B = 0$; $v_o = v_{oy} = 30\ m\ s^{-1}$, $v = v_y$, $t = t_B$.

To calculate t : since v_y is not known the only equation that can be used is:

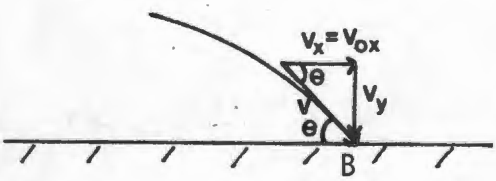
$s = v_{ot} + \frac{1}{2}at^2$.

$\therefore$ $0 = 30 \times t - \frac{1}{2} \times 10 \times t^2$ $\therefore$ $t(30 - 5t) = 0$

$\therefore$ $t = 0$ or $\underline{t_B = 6s}$. $\rightarrow$

To calculate the horizontal distance we must consider the horizontal motion. This will be done in STEP 5.

c) To calculate the angle it strikes the ground, it is necessary to calculate the y-component of the velocity v_y at the particular point. (The x-component of velocity is, of course, constant).



It should be clear from the sketch that the angle at which the body strikes the ground will be given by the direction of its velocity, V. This will again be determined by the components, v_x and v_y.

The point in question is B: $a = -g = -10\ m\ s^{-2}$; $s = y_B = 0$, $v_o = v_{oy} = 30\ m\ s^{-1}$, $t = t_B = 6s$; $v = v_y$.

$v = v_o + at$: $v_y = 30 - 10 \times 6 = -30\ m\ s^{-1}$.

The - sign indicates that the velocity is downward which is,of course, the case since it is striking the ground.

$\therefore \quad \tan\theta = \frac{(-)30}{40} \quad (v_x = v_{ox} = 40 \text{ m s}^{-1}$ - STEP 2)

$\therefore \quad \theta = 37°$ (as expected?)

PROBLEM 2:

i) $a = -g = -10 \text{ m s}^{-2}$; $s = y_D = +25$ m; $t = t_D$; $v_o = v_{oy} = 30 \text{ m s}^{-1}$; $v = v_y$.

The problem is to calculate $t = t_D$. Since v_y is not known, use eqn.

$s = v_o t + \frac{1}{2}at^2$; $25 = 30 \times t - \frac{1}{2} \times 10 \times t^2$

$\therefore \quad t^2 - 6t + 5 = 0. \quad \therefore \quad (t - 5)(t - 1) = 0.$

$\therefore \quad t = 5\text{s} \quad t = 1$ s.

Since the body "lands" on the plateau $t = t_D = 5$s.

(See p.85 ; $t = 1$s would be for point C).

ii) Horizontal distance will be calculated in STEP 5.

iii) The problem is to calculate v_y. Using values in i):

eqn $v = v_o + at$: $v_y = 30 - 10 \times 5 = -20 \text{ m s}^{-1}$.

$v_x = v_{ox} = 40 \text{ m s}^{-1}$.

$\tan\theta = \frac{20}{40} = 0{,}5$

$\therefore \quad \theta = 26{,}6° \quad \therefore \quad V = \sqrt{40^2 + 20^2} = 44{,}7 \text{ m s}^{-1}$.

The -20 m s^{-1} again indicates that the "striking" velocity is downward, which is obvious.

PROBLEM 3:

$a = +g = 10 \text{ m s}^{-2}$; $s = y = 1{,}8$ m; $v_{oy} = 0$; $v = v_y = ?$; $t = ?$.

(Check again p.89 to convince yourself that $v_{oy} = 0$).

Since the problem is to determine the initial (horizontal) velocity, we may reason that we do not need the vertical motion. As will be seen in Step 5, the only equation for the horizontal motion is $x = v_{ox} . t$. While we have x (p,39), to calculate v_{ox}, we need t. This can be obtained from the vertical motion.

Using $s = v_o t + \frac{1}{2}at^2$: $1,8 = 0 + \frac{1}{2}.10 \times t^2$ $\therefore$ $\underline{t = 6s}$ $\rightarrow$

Horizontal motion in STEP 5.

PROBLEM 4

i) $a = -g = -10 \text{ m s}^{-2}$; $s = y_A$; $v_o = v_{oy} = 6 \text{ m s}^{-1}$.

$v = v_y = 0$ (highest point) $t = t_A$.

Using $v^2 = v_o^2 + 2as$: $0 = 6^2 - 2x\ 10 \times y_A$. $\therefore$ $y_A = 1,8$ m.

$\therefore$ Highest distance above the ground = 1,8 + 10 = 11,8 m.

ii) The distance required in the problem is x_B. This is calculated from the horizontal motion using $x_B = v_{ox} . t$. t must thus be calculated from the vertical motion.

$a = -g = -10 \text{ m s}^{-2}$; $s = y_B = -10$ m (do not forget the sign!)

$v_o = v_{oy} = 6 \text{ m s}^{-1}$; $v = v_y$; $t = t_B$.

Using $s = v_o t + \frac{1}{2}at^2$; $-10 = 6t - \frac{1}{2} \times 10\ t^2$

$\therefore$ $5t^2 - 6t - 10 = 0$ $\therefore$ $t = \frac{6 \pm \sqrt{6^2 + 200}}{2x5} = 2,1s$ or $-0,94s$.

Clearly only the +2,1s is acceptable.

The horizontal distance will be calculated in Step 5.

REMEMBER

In executing STEP 4 you are dealing only with the vertical motion. Use only the y-coordinate of the point in question for displacement s and the y-component, v_{oy}, of the initial velocity in the three equations.

STEP 5

HORIZONTAL MOTION: $x = v_{ox} . t$

As seen on the previous page, this equation, the only one for the horizontal motion, is not always necessary to solve the problem. We use it now only to complete some of the problems handled in STEP 4.

PROBLEM 1b

$x = v_{ox} \cdot t; \quad v_{ox} = 40 \text{ m s}^{-1}$. (p.86)

Since the point in question is B, $x = x_B$ and the time taken is $t_B = 6$s as determined for the vertical motion.

$$\therefore \quad x_B = 40 \times 6 = \underline{240 \text{ m} \rightarrow}.$$

PROBLEM 2(ii)

The point in question is D and the x-coordinate is thus x_D. The time, $t_D = 5$s as for the vertical motion.

$$v_{ox} = 40 \text{ m s}^{-1}. \quad \text{(p. 86)}$$

$$\therefore \quad x_D = v_{ox} \cdot t_D = 40 \times 5 = \underline{200 \text{ m s}^{-1} \rightarrow}$$

PROBLEM 3

Time to land is 0,6s while x = 2,5 m.

$\therefore \; x = v_{ox} \cdot t \; \therefore \; 2{,}5 = v_{ox} \cdot 0{,}6$

$\therefore \; v_{ox} = V_o = 4{,}17 \text{ m s}^{-1} \rightarrow$.

PROBLEM 4(ii)

We are dealing with point B, x-coordinate, x_B.

$t = 2{,}1$s determined for the vertical motion.

$v_{ox} = 8 \text{ m s}^{-1}$. (p.87)

$\therefore \; x_B = v_{ox} \cdot t = 8 \times 2{,}1 = 16{,}8 \text{ m} \rightarrow$

PROCEDURE FOR PROBLEMS ON PROJECTILE MOTION

1. Choose suitable axes x (horizontal) and y (vertical) and the origin at the point of projection of the object.
2. Determine the x- and y-components of the initial velocity, V_o.
3. Read the problem and write down explicitly the coordinates of the point or points in question.
4. Analyse the vertical motion taking place under constant acceleration $\pm g$:

   a) Fill in from the problem the values of the 5 variables:

   $a = \pm g$ (sign depends on direction of y-axis +y upward: -g)
   +y downward: +g)

   $s = y$ (y-coordinate of point in question; STEP 3)

   $t = ?$ (same as for horizontal coordinate, x)

   $v_o = v_{oy}$ (STEP 2)

   $v = v_y$ (at highest point, $v_y = 0$)

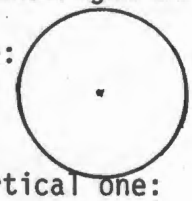
   b) Use the appropriate kinematic equation to determine the unknown(s).

5. Use horizontal motion $x = v_{ox}.t$, where necessary.

MECH BL 8 : CIRCULAR MOTION

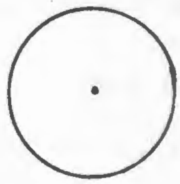
Mech B1 8 : Circular Motion

Students sometimes find it difficult to understand all that is involved in describing the dynamics of circular motion. One of the possible reasons is that it is necessary to think in three-dimensions. Try doing this right at the outset: if a problem calls for a horizontal circle:



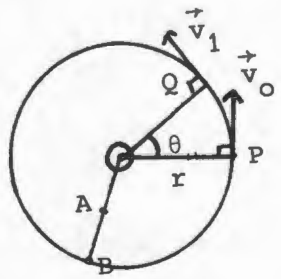
,

as opposed to a vertical one:



both of which are of course identical in two-dimensions, use your imagination to visualize the physical situation exactly.

It is perhaps instructive to review the kinematics of motion in a circle.



The expression "linear velocity" in this context, is used as opposed to angular velocity. Then $\vec{v}_0$ and $\vec{v}_1$ are the instantaneous velocities of the particle tangential to the circle. The angular velocity ω gives the angle (in radians) swept out by the radius per unit time. The relationship is, of course,

$$\boxed{v = \omega r} \qquad \vec{v} = \vec{\omega} \times \vec{r}$$

Say that the circle is a solid rotating disc with a mass at the circumference, an other at half the radius from the centre, O. (The points B and A respectively). Please make sure that you understand the following clearly: if the disc is rotating, the mass at B will have twice the linear velocity of the mass at A, but the angular velocity of both masses will be the same. (Remember, the radius OAB will move through angle θ in time t i.e. both masses will move through this angle in time t).

The velocities $\vec{v}_0$ and $\vec{v}_1$ have directions tangential to the circle at P and Q and magnitudes v_0 and v_1 respectively. Because velocity has both magnitude and direction, two situations arise:

1. The magnitude can vary i.e. $v_0 \neq v_1$

   An acceleration thus arises:

$$a_T = \frac{v_1 - v_0}{t}$$

   a_T is called the tangential acceleration (taken here to be constant).

NOTE: The expression above for a_T is only valid if the force causing the acceleration tangentially, is of constant magnitude.

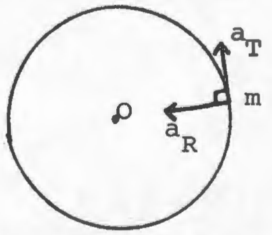
2. The direction of $\vec{v}_0$ is different to $\vec{v}_1$

   In this case the acceleration is

$$a_R = \frac{v^2}{r} = \omega^2 r$$

   (Please check the derivation in your notes or textbook)

   a_R is called the radial acceleration. The directions of the respective accelerations are as indicated on the sketch:



Are the two situations specified above compulsory for circular motion? Obviously, the particle in moving around the circumference of the circle, need not increase or decrease its <u>speed</u> e.g.a car travelling at 60 km^{-1} around a curve can maintain this speed. Thus a_T can possibly be zero. It could also quite possibly not be zero. But if it is to move in a circle, it must change its direction. Thus while the speed (the magnitude of the velocity) may be the same, the direction of the velocity must change i.e. there must be acceleration for any circular motion, where the direction of the acceleration points from the mass to the centre of the circle of rotation i.e. is radial. So while two accelerations are possible, a_T and a_R, one, a_R, is compulsory.

While acceleration takes place in the plane of the circle, perpendicular to this plane there is equilibrium.

Combining the above ideas with Newton's Second and First Laws, the following emerge:

1. For circular motion there must be a radial acceleration. Therefore there must be a resultant radial force causing it. Hence the basic problem in working with a particle moving in a circle, is to determine the resultant force directed toward the centre of the circle.

2. If there is also a tangential acceleration then there must be a resultant force also in the direction at a tangent to the circle. The radial and tangential directions are of course, perpendicular. (Why?)

3. Perpendicular to the plane of the circle there is no acceleration and thus the resultant force in this direction is zero i.e. Newton I applies.

NOTE: There is not necessarily only one force in these directions. Rather, the resultant would be made up of the sum of the components of the forces acting on the body.

We will now discuss the various steps required to deal with problems on circular motion doing so as we solve the following typical problems:

1. A stone mass 2 kg, is attached at the end of a light string and rotated in a circle of radius 0,8 at a linear speed of 4 m s^{-1} in a vertical plane. Calculate the maximum and minimum tensions in the rope.

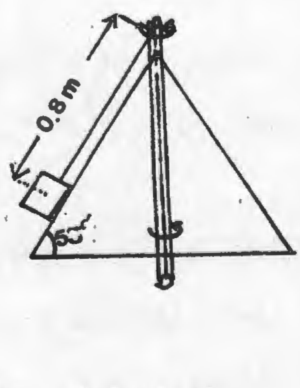
2. A 10 kg mass is fixed at the end of a rope 2 m long, attached to the ceiling. The mass is swung in a horizontal circle with constant speed so that the rope makes an angle of 37° with the vertical. Calculate:

   i) the angular velocity of the mass ;

   ii) the tension in the rope.

3. a) At what maximum speed can a motorcar travel on a horizontal circular road, radius 125 m and μ = 0,5 without skidding sideways?

   b) If the road is perfectly smooth, calculate at what angle the road should be banked so that the car is on the point of skidding at a speed of 25 m s^{-1}.

4. 

The figure shows a mass of 10 kg resting on a smooth cone, base angle 53°, and held in place by a light rope, length 0,8 m, parallel to the conical surface. The cone rotates about the central axis. Calculate:

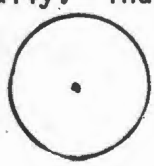
a) Tension in the rope and the reaction of the conical surface when the system rotates at 20 r.p.m.

b) The r.p.m. when the body is on the point of lifting off the cone.

In demonstrating how these problems are solved, we will develop the procedure which can be applied generally to solve problems on circular motion.

STEP 1

Visualize the geometry of the problem in three-dimensions.

In most cases, circular motion is a 3-D movement in spite of the fact that the circle of motion is obviously only 2-D. It is essential to make sure that you have a clear picture of the physical situation in 3-D. In drawing out the essential parts of the problem, do so realistically. Thus while a vertical circle can easily be represented by 

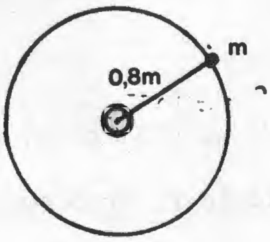
a horizontal circle will more fittingly be 

You will find that to draw in forces acting on the body in the correct directions, clear visualization is essential.
While this is not a written step you will find it vital to your comprehension of the problem.

STEP 2

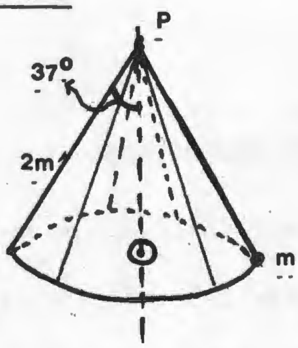
Identify the plane of the circle of rotation and the centre of the circle. Please read through the problems given and sketch the essential parts, clearly identifying the circle of rotation and its centre. Try it before checking your answers below.

Problem 1



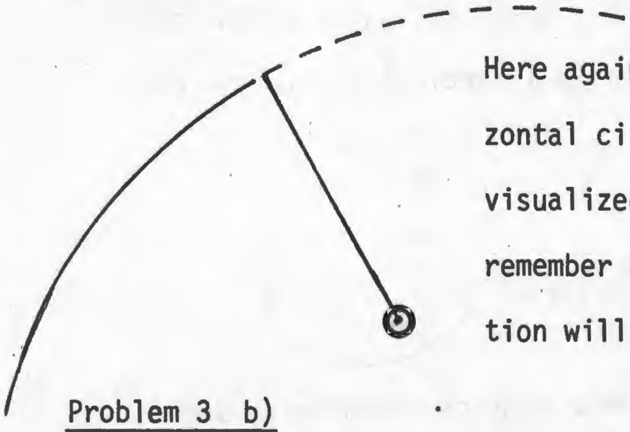
This is obviously the simplest type of example to visualize and sketch. The plane of the circle is that of the paper.

Problem 2



Here we are dealing with a horizontal circle i.e.the base of a cone with apex P. The centre of the circle of rotation is obviously O and not P.

Problem 3 a)

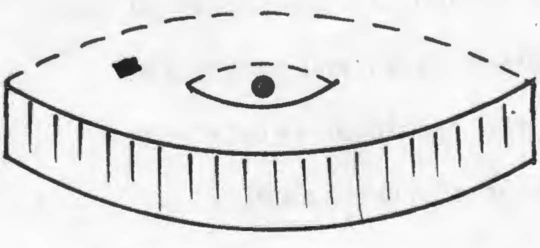


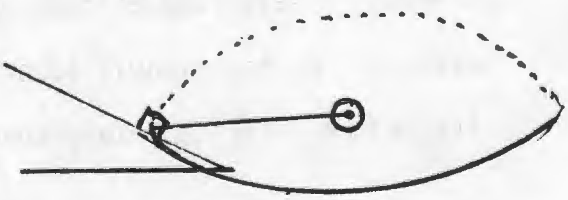
Here again the problem specifies a horizontal circle. While it can easily be visualized in the plane of the paper, remember that the perpendicular direction will then be out of the paper.

Problem 3 b)

This is difficult to draw. Hold your hand at an angle to a horizontal surface e.g.table top, and move it around in a circle, always keeping the thumb against the surface. This should give you a good idea of the geometry of the problem.

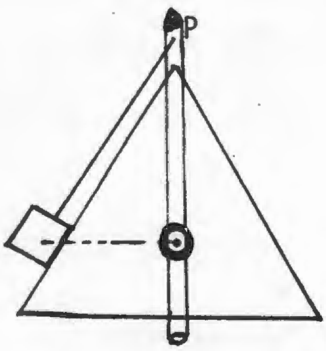
The essential part is again:

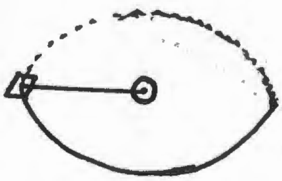




Here again we have a horizontal circle with centre as shown.

Problem 4





As the cone rotates about the vertical axis, the mass describes a horizontal circle with the centre of the circle of rotation being located in the horizontal plane opposite the mass, therefore at O, not P.

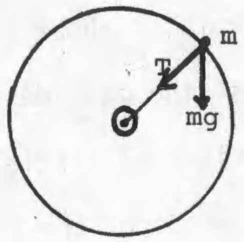
Please note carefully that in all the problems, the plane of the circle of rotation is either vertical (Problem 1) or horizontal (all the other). You will seldom come across any other possibilities.

STEP 3

It is now necessary to draw in all the forces acting on the body, keeping in mind in problem 3 a), that there are two kinds of friction operating on a car when it moves in a circle. Remember to identify all the bodies which exert forces on the mass in question. Finally remember that you may be dealing with a problem in 3-D and so need to draw in force directions realistically.

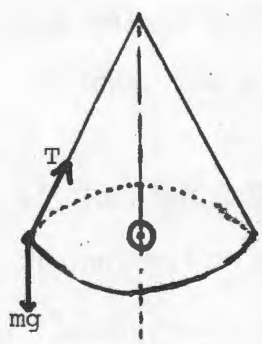
Please try it before checking over the page for the answers.

Problem 1



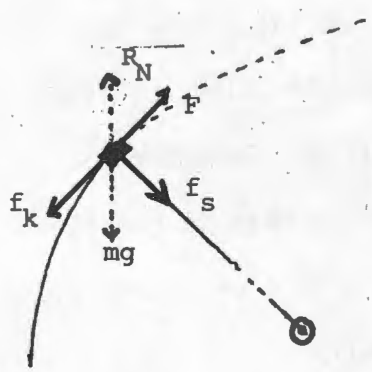
In this case it is clear that the tension in the rope and the weight both lie in the plane of the vertical circle.

Problem 2



Note carefully that the weight, mg, is perpendicular to the plane of the circle. The tension, T, is obviously along the rope and thus at an angle to the plane of the circle.

Problem 3 a)



It is necessary to think carefully about the forces acting on the car:

F: This is the force exerted by the engine to enable the car to move forward.

f_k: The usual force of kinetic friction opposing the movement of the car.

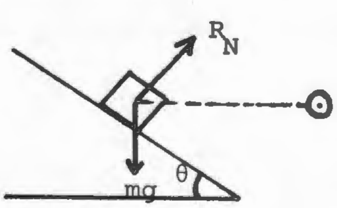
R_N: Reaction force exerted by the ground on the car. It is perpendicular to the road and hence to the plane of the circle of rotation and thus to all the forces lying in this plane. (F, f_k, f_s)

mg: The weight, perpendicular to the plane of the circle of rotation.

f_s: It is clear that none of the four forces above can supply the radial force necessary for the car to move in a circle (see p.98). This is supplied by the force of friction between the tyres and the road, but not the usual (f_k) force, but rather perpendicular to the direction in which the tyre is moving (i.e.radially). This is a static friction which holds the car on the circumference of the circle. The car does not move along the radius. Think carefully about this point.

If, for example, a book is lying on the seat of the car, it might slide outward as the car rounds a bend. The reason is that while the tyres supply a sufficient radial frictional force (f_s) to allow the car as a whole to turn, for the book also to round the bend the seat must exert a sufficient radial frictional force on it. If it does not, the book will slide until it comes in contact with the door which now pushes on it and hence supplies the radial force for the book also to round the bend. If there were no door the book would simply slide out of the car since nothing can supply the radial force necessary to enable the book also to move in a circle.

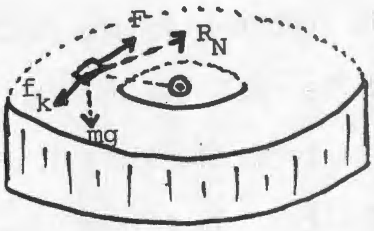
<u>Problem 3 b)</u>



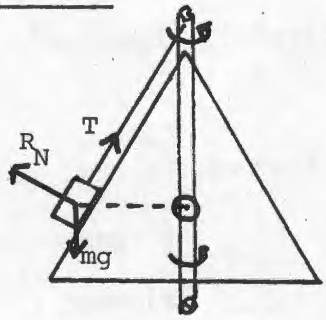
Two of the forces acting, the weight and the normal reaction are shown. If f_s in problem 3a were present, it would act down the plane.

Remember, however, that the forces F and f_k in 3a will also be acting. They will operate in the plane perpendicular to the page and tangential to the circle of rotation. The centre of the circle will be at 0.

If the situation is viewed from above:



Problem 4



All the forces can be regarded as falling in the plane of the page while the plane of the circle of rotation is of course perpendicular to the page.

STEP 4

It is now necessary to specify three directions:

a) RADIAL(Ra): The radial axis is chosen with origin at the body and direction positive to the centre of the circle.

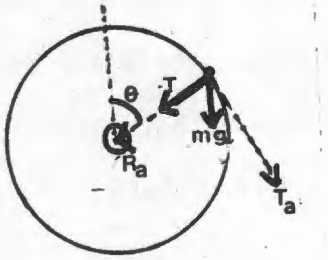
b) TANGENTIAL (Ta): This direction is perpendicular to the radial direction but still in the plane of the circle of rotation.

c) PERPENDICULAR ($\perp$): This is the direction perpendicular to the plane of the circle, generally chosen positive upward.

The origin for all these directions is chosen to be at the particular mass in question.

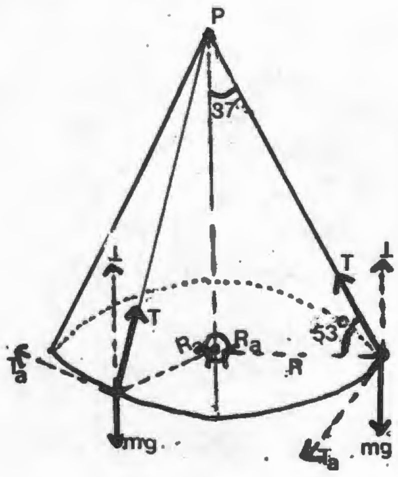
Try to indicate (where possible) these directions on the sketches drawn of the above problems. Do this before checking your answers.

Problem 1



Perpendicular to the plane of the paper will of course be perpendicular to the page. In practice the perpendicular axis will be horizontal since the circle is vertical.

Problem 2

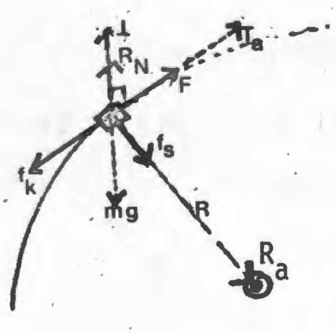


Note that the set of axes moves with the mass as origin.

They are merely instituted to indicate the primary directions in dealing with circular motion.

Check carefully on the angles between the forces and the axis Ra.

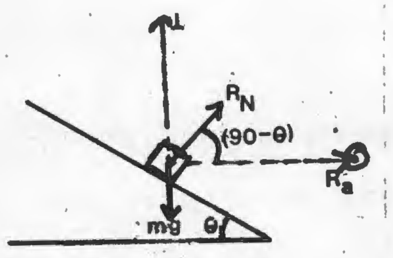
Problem 3 a)



The perpendicular axis is perpendicular to the plane.

Make sure that you are clear on the angles between the forces and the axis Ra.

Problem 3 b)

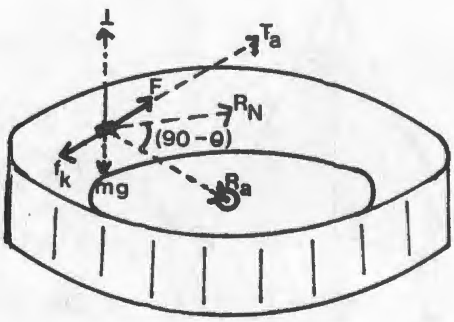


Note that the perpendicular axis is not the usual perpendicular we are accustomed to with the inclined plane. In this case "perpendicular", means perpendicular to the plane of the circle of rotation, not perpendicular to the inclined plane.

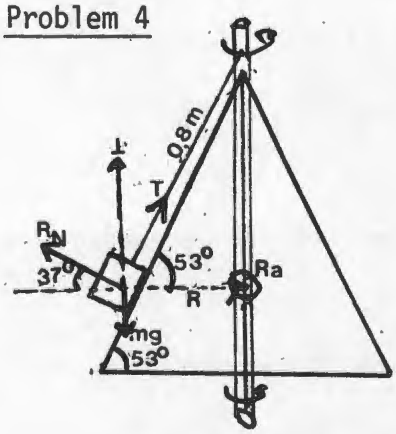
The tangential axis will be into the paper. Seen from the top:

Remember that the three axes, Ra, Ta, ⊥ are mutually perpendicular.

Check that you are clear on the angles between the forces and the axis Ra.



Problem 4



Note again that the perpendicular axis is perpendicular to the plane of the circle of rotation, not to the inclined plane.

Check carefully on the angles between the forces and the axis Ra.

STEP 5

Since for circular motion the radial acceleration is compulsory, it is essential now to write down Newton's 2nd Law in the radial direction. Determine all the forces which have a component in the radial direction and set up the equation:

$$\Sigma F_{RAD} = m \omega^2 R = m \frac{v^2}{R}$$

If there are forces with components in the perpendicular direction, $\perp$, these will be in equilibrium. Thus

$$\Sigma F_{\perp} = 0$$

You will find frequently that in many problems on circular motion, these two equations are sufficient to solve the problem. Especially when friction is present, the equation $f = \mu R_N$ depends on the above.

Please try to set up the equations before checking your answers below.

<u>Problem 1</u>

$$\Sigma F_{RAD} = T + m g \cos \theta = \frac{mv^2}{R}$$

$$\Sigma F_{\perp} = 0$$

Please make sure of the components. Note that the angle θ is zero when the particle is at the top of the circle. The particle is assumed to move clockwise.

<u>Problem 2</u>

$$\Sigma F_{RAD} = T \cos 53° = \frac{mv^2}{R}$$

Note that R = radius of the circle of rotation = 2 cos 53° = 1,2 m

$$\Sigma F_{\perp} = T \sin 53° - m g = 0$$

Are you clear about the angle 53° and the components indicated on the previous page?

Problem 3 a)

$$\Sigma F_{RAD} = f_s = \frac{mv^2}{R}$$

$$\Sigma F_{\perp} = R_N - m g = 0 \; ; \quad f_s = \mu R_N$$

Note that R is given as 125 m.

Problem 3 b)

$$\Sigma F_{RAD} = R_N \cos (90 - \theta) = R_N \sin \theta = \frac{mv^2}{R}$$

Note that the radius R is given as 125 m.

$$\Sigma F_{\perp} = R_N \sin (90 - \theta) - m g = 0$$

$$\therefore \; R_N \cos \theta - m g = 0$$

Problem 4

$$\Sigma F_{RAD} = T \cos 53° - R_N \cos 37° = \frac{mv^2}{R}$$

Here R = 0,8 cos 53° = 0,48 m

$$\Sigma F_{\perp} = T \sin 53° + R_N \sin 37° - m g = 0$$

STEP 6

It is now necessary to find the sum of the components in the tangential direction. In most cases which you meet in this course the speed of the particle is constant and thus the resultant tangential force is zero. In most cases the step is therefore unnecessary.

Convince yourself in reading through the problems given that this is indeed the case.

STEP 7

Solve the problem by solving the equations established in STEP 5. We leave this as an exercise but supply the answers below.

Problem 1 (Note that the maximum and minimum tensions are obtained when the stone is at the bottom ($\theta = 180°$) and the top ($\theta = 0$) respectively, of the revolution)

$T_{max} = 60$ N; $T_{min} = 20$ N

Problem 2 $T = 125$ N; $\omega = 2,5$ rad s^{-1}

Problem 3 a) $v = 25$ m s^{-1}

Problem 3 b) $\theta = 26° 34'$

Problem 4 a) $\omega = 20$ r p m $= \frac{20 \times 2\pi}{60}$ rad s^{-1} $= 2,09$ rad s^{-1}

$T = 92,6$ N $R_N = 259,2$ N

Problem 4 b) Here $R_N = 0$ $\omega = 3,95$ rad s^{-1}

The algorithm for solving problems on circular motion is the following:

1. Visualize the geometry of the problem in three-dimensions.
2. Identify the plane of the circle of rotation and the centre of the circle.
3. Draw in all the forces acting on the body.
4. Draw in the three axes Ra, Ta and $\perp$.
5. Set up the equations:

$$\Sigma F_{RAD} = m \omega^2 R = \frac{m v^2}{R}$$

$$\Sigma F_{\perp} = 0$$

6. Set up the equation tangentially if necessary.
7. Solve the problem.

STEP 7

Solve the problem by applying the equations derived in the article. As we did with every exercise we supply the answers below.

Problem [illegible]

[illegible]

[illegible] value of the [illegible]

$$F_{max} = [illegible] \sin [illegible]$$

Answer 2: [illegible]

Problem 3: [illegible]

Problem 4: [illegible]

Problem 5: [illegible]

Problem 6: [illegible]

[illegible] the solving procedure [illegible] circular motion [illegible]

1. Visualize the geometry of the problem in three dimensions.
2. Identify the plane of the circle of rotation and the center of the circle.
3. Draw in all the forces acting on the body.
4. Choose the [illegible] axes, [illegible]
5. Set up the equations:

$$\Sigma F_r = m a_r = m\frac{v^2}{r}$$

$$\Sigma F_z = 0$$

6. Solve the equation (mathematically if necessary).
7. [illegible] the problem.

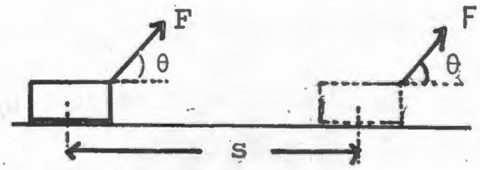
MECH BL 9 : WORK AND ENERGY

Mech B1.9 : Work and Energy

If a force F acts on a body, mass M, so that the body is displaced a distance s, then the force does work on the body. The definition is:

Work done by F on body

$= (F \cos\theta).s.$ i.e. the definition requires the component of F in the direction of the displacement.



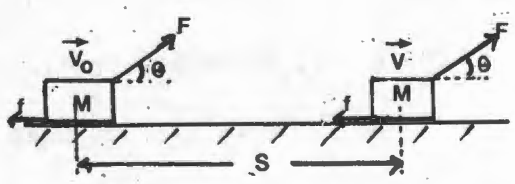
In calculating the W.D. by any F:

1) Determine the direction of displacement of the body.
2) Determine the component of the force in this direction.

Note that all forces or components in the direction perpendicular to the direction of the displacement do no work during the displacement, s.

Consider the following two situations:

1.



Let $\vec{F}$ be the constant resultant applied force, $\vec{f}$ the force of friction, $\vec{v}_0$ the initial velocity and $\vec{v}$ the velocity after the mass M has covered a distance, s.

By Newton II:

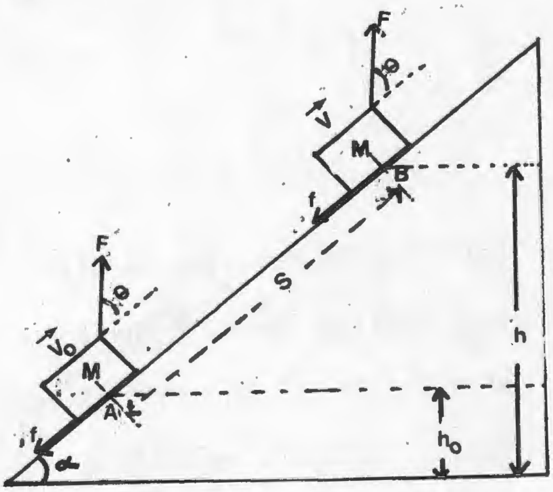
$$(F\cos\theta - f) = M.a = M \times \frac{v^2 - v_0^2}{2s}$$

$$\therefore \quad (F\cos\theta - f)s. = \left(\frac{Mv^2}{2} - \frac{Mv_0^2}{2}\right)$$

Clearly ($F\cos\theta - f$) is the resultant force in the direction of displacement; therefore the term on the left is the total work done on the mass M. (the normal reaction force and the weight are both perpendicular to the direction of motion, and hence do no work).

The work done on the body produces a change in the quantity ($\frac{1}{2}mv^2$) of the body. The work done by the resultant force is thus the same as the change in this quantity, called, of course, the kinetic energy, E_K. One is thus at liberty to speak either of the work done on M or the change in kinetic energy of M. The descriptions are equivalent except that in the former, forces and their directions are important, whereas in the latter only the initial and final values of E_K are relevant.

2.



Consider the mass M moved from position A to position B, a distance s along an incline, angle α. A is a vertical height h_0, while B is a vertical height h, above an arbitrary reference level. The speed of M up the incline changes from $\vec{v}_0$ to $\vec{v}$.

Resultant force in the direction of motion = $(F\cos\theta - f - Mg\sin\alpha)$.

$\therefore$ By Newton II: $(F\cos\theta - f - Mg\sin\alpha) = Ma = M\left(\frac{v^2 - v_0^2}{2s}\right)$

$\therefore$ $(F\cos\theta - f)s = \left(\frac{Mv^2}{2} - \frac{Mv_0^2}{2}\right) + Mg\,s\sin\alpha.$

$\therefore$ $(F\cos\theta - f)s = \left(\frac{Mv^2}{2} - \frac{Mv_0^2}{2}\right) + (Mgh - Mgh_0)\{s\sin\alpha = (h-h_0)\}.$

The term on the left is the work done by all the external forces except the weight (the normal reaction and the perpendicular component of F do no work in moving the object parallel to the plane). The work done on the body not only changes that quantity $(\frac{Mv^2}{2})$ of the body, but also the quantity Mgh of the body, from its initial value Mgh_o to Mgh. Notice that the change in the quantity Mgh, called, of course, the Potential Energy, E_p, also depends only on the initial and final values of E_p.

Rewrite the equation as follows:

$$(F \cos\theta - f)s = (\frac{Mv^2}{2} + Mgh) - (\frac{Mv_o^2}{2} + Mgh_o)$$

$$\therefore \quad (F \cos\theta - f)s = (E_K + E_P)_{FINAL} - (E_K + E_P)_{INITIAL} \quad \ldots(1)$$

In using this equation to solve problems on work and energy, please note the following:

i) $(F \cos\theta)$ is the component of the resultant force applied to the body (excluding friction, normal reaction and the weight).

ii) The work done by the force of friction on the body, is always (-f.s). Do not neglect the negative sign.

iii) The work done by the weight, Mg, is always included on the right of the equation as the potential energy.

iv) In writing down the value of the potential energy, an arbitrary reference level must first be chosen.

The following procedure now suggests itself for solving problems on work and energy:

1. Determine whether any external forces, other than the weight and the normal reaction, act on the body.

Determine the Total Work done by these forces (see p. 113).

2. Identify the first situation of the system and choose an appropriate reference level.

   a) Determine E_K (c) $(E_K + E_P)_{\text{INITIAL}}$ (Use symbols if values are not given).

   b) Determine E_P

3. Identify the final situation of the system.

   a) Determine E_K (c) $(E_K+E_P)_{\text{FINAL}}$ (Use symbols if values are not given).

   b) Determine E_P

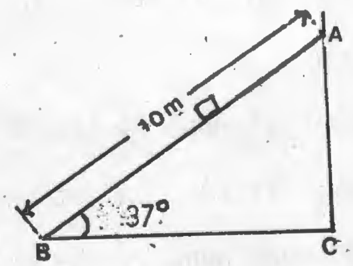
4. $\therefore$ $(F\cos\theta - f)s = (E_K+E_P)_{\text{FINAL}} - (E_K + E_P)_{\text{INITIAL}}$

5. Solve for unknowns.

Let us apply this procedure to solve some typical problems.

PROBLEM 1

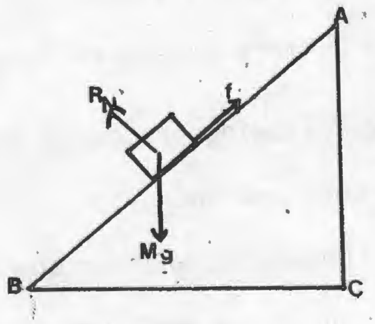
A block, mass M, slides from rest down a 37° incline and reaches a speed of 6 m s^{-1} after covering a distance of 10 m. Calculate the coefficient of kinetic friction of the surfaces, using energy considerations.



STEP 1:

a) Determine whether external forces, other than the weight and the normal reaction, act on the system:

   In drawing in the forces acting on the mass M, it is clear that only the force of friction, f, is



acting (apart from R_N and Mg).

b) Determine total work done by these forces:

Remember that we are determining the term $(F \cos\theta - f)s$ (in eqn 1):

$$(F \cos\theta - f)s = -f.s = -f.10$$

since F = 0 (i.e.there are no external applied forces apart from f), and s = actual distance the body moves.

STEP 2

Identify the first situation of the system and the reference level.

The body (system) moves 10 m, thus from A to B. A therefore represents the first situation of the body.

It is convenient (usually!) to choose the lowest point that the body reaches as the reference level (i.e. the level where $E_p = 0$). In this case it is BC.

At A: $E_K = 0$ since the body starts from rest.

$E_p = Mgh$, where h is the vertical distance of A from BC.

$$\therefore \; E_p = Mg.s \sin 37° = Mg \times 6$$

$$\therefore \; (E_K + E_P)_{INITIAL} = 6.Mg$$

STEP 3:

Identify the final situation of the system.

This is obviously at B.

At B: $E_K = \frac{1}{2}M \,.\, v^2 = \frac{1}{2}\, M.6^2$ since $v_B = 6 \text{ m s}^{-1}$

$E_p = 0$ since B is on the reference level.

$$\therefore \; (E_K + E_P)_{FINAL} = \tfrac{1}{2}M.6^2$$

STEP 4:

$$(F \cos\theta - f)s = (E_K + E_P)_{FINAL} - (E_K + E_P)_{INITIAL}$$

Using the values determined in Steps 1 - 3:

$$-f.10 = \tfrac{1}{2}.M.6^2 - 6\ Mg.$$

STEP 5:

Solve for unknown.

While this step is placed at the end, it is obvious that before the previous steps can be executed one must have determined what the problem requires. In this case the coefficient of kinetic friction, μ_K, is required. Since

$\mu_K = \frac{f}{R_N}$, clearly both f and R_N must be determined.

f is determined by energy considerations, while R_N is obtained from Newton I.

From the above: ($g = 10\ m\ s^{-2}$)

$$f = 4,2M$$

Σ Forces perpendicular to plane $= R_N - Mg \cos 37° = 0$

$$\therefore\ R_N = Mg \cos 37° = 8M$$

$$\therefore\ \mu_K = \frac{4,3\cancel{M}}{8\cancel{M}} = \underline{0,525} \rightarrow$$

Normally, in cases where the mass M is not given, it will cancel.

PROBLEM 2

A skier takes off from a ski jump with a velocity of 50 m s^{-1} at an unknown angle and lands at a point where vertical distance below the point of take-off is 100 m. Calculate his speed just before landing, ignoring air friction.

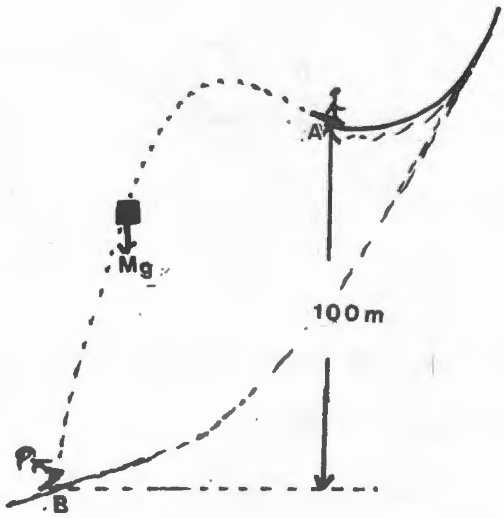
ANALYSIS OF PROBLEM:

Since the angle at which the skier leaves the ski jump is unknown, projectile motion cannot be used to solve this problem. Energy considerations which depend only on the initial and final points and not the path in between, must therefore be considered.

STEP 1:

a) Determine whether external forces, other than the weight and the normal reaction, act on the system.

It is necessary to draw a force diagram which is facilitated by using a sketch. The forces at A and B are not of consequence; we are interested only in the forces along the path between A and B.



Clearly, if friction is neglected, the only force acting is the weight. Since this is excluded from ($F \cos\theta - f$) - equation 1- it follows

$$(F \cos\theta - f) = 0$$

b) i.e. Total Work Done = 0.

It is perhaps relevant to note that under these conditions equation 1 becomes:

$$0 = (E_K + E_P)_{FINAL} - (E_K + E_P)_{INITIAL}$$

i.e. $(E_K + E_P)_{FINAL} = (E_K + E_P)_{INITIAL}$

Energy is thus conserved.

STEP 2:

Identify the first situation of the system and the reference level.

The initial position of the system is at A.

It is convenient to choose the reference level as the horizontal line through B.

At A: $E_K = \frac{1}{2}M.\ v_A^2 = \frac{1}{2}.M.\ (50)^2$

$E_P = Mgh = M.\ 10 \times 100$

$\therefore\ (E_K + E_P)_{\text{INITIAL}} = \frac{1}{2}M(50)^2 + M.1000$

STEP 3

Identify the final situation of the system.

This is clearly at B.

At B: $E_K = \frac{1}{2}M.v_B^2$

$E_P = 0$, since B is on the reference level.

$\therefore\ (E_K + E_P)_{\text{FINAL}} = \frac{1}{2}\ M.v_B^2$

STEP 4:

$$(F \cos\theta - f)s = (E_K + E_P)_{\text{FINAL}} - (E_K + E_P)_{\text{INITIAL}}$$

Using the values from Steps 1 - 3:

$$0 = \frac{1}{2}\ M.v_B^2\ -\ (\frac{1}{2}\ M.50^2 + M.1000).$$

As before, M will cancel.

STEP 5:

Solve for unknowns.

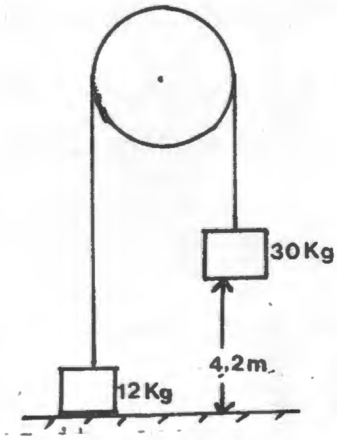
The unknown is the value of the velocity at B, i.e. v_B.

$$v_B^2 = 4500$$

$$\therefore \quad v_B = \underline{67,1 \text{ m s}^{-1}} \rightarrow$$

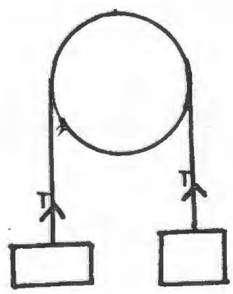
PROBLEM 3

The system in the sketch is released from rest with the block of mass 12 kg touching the floor. Use energy considerations to calculate the velocity with which the 30 kg block strikes the floor. Neglect friction and inertia in the pulley.



ANALYSIS OF PROBLEM:

Since the problem stipulates it, energy considerations, and thus equation 1, must be used rather than Newton's 2nd law although it too can be used to solve the problem. It is worthwhile noting that the tension is directed upward away from the 12 kg and 30 kg masses. Thus the work done if the 12 kg mass moved upward a distance s will be (+T.s) while if the 30 kg mass moved downward the corresponding distance s, the work done by the force of tension on the right will be (-T.s) i.e.the work done by T is zero. Thus internal forces such as the tension can always be neglected in using equation 1.



STEP 1 a):

Determine whether external forces, other than the weight and the normal reaction act on the system.

In this case, the only force apart from the weight, which acts on the system, is the tension. As shown above, the tension does no work on the system as a whole.

(Please distinguish between this case and that in which a single body is pulled by a rope. Obviously in the latter the tension in the rope will do work).

STEP 1 b)

Total work done = $(F \cos\theta - f)s = 0$.

STEP 2

Identify the first situation of the system and the reference level.

The first situation of the system is as shown in the sketch, with the 30 kg above and the 12 kg on the surface. The surface obviously is a suitable reference level.

INITIAL:

$E_k = 0$. Both the 12 kg and 30 kg masses are stationary.

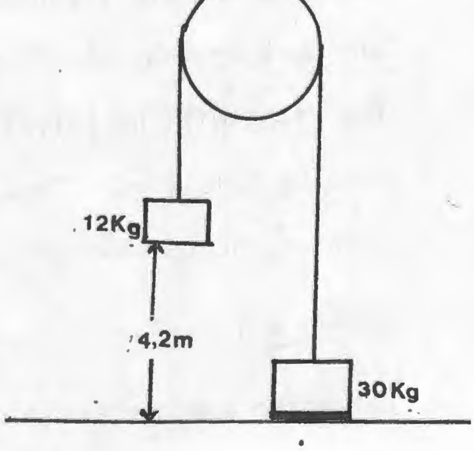
$$E_p = 30 \times g \times h = 30 \times g \times 4{,}2.$$

The 12 kg mass is on the reference level and thus has potential energy = 0.

STEP 3

Identify the final situation of the system.

This is clearly as is shown in the sketch.



FINAL: $E_p = 12 \times g \times 4{,}2$

The 30 kg mass is now on the reference level and thus has potential energy = 0. In determining the kinetic energy of the system, please think carefully about it. The two masses are connected - thus the speed of each is the same.

$$\therefore \quad E_k = \tfrac{1}{2}\, 12 \times v^2 + \tfrac{1}{2}\, 30 \times v^2.$$

STEP 5:

Solve for the unknowns

The unknown is the velocity, v.

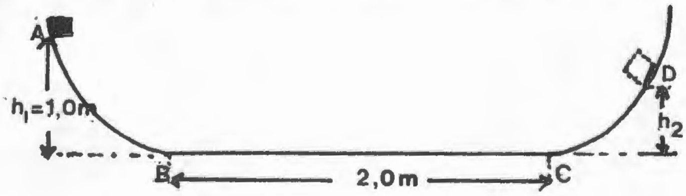
$$\therefore \quad 0 = (E_k + E_p)_{FINAL} + (E_k + E_P)_{INITIAL}$$

$$0 = (\tfrac{1}{2} \times 12 \times v^2 + \tfrac{1}{2} \times 30 \times v^2 + 12 \times g \times 4{,}2) - 30 x g x 4{,}2$$

$$\therefore \quad \underline{v = 6{,}0 \text{ m s}^{-1}}.$$

PROBLEM 4

A particle slides from rest along a plane with the sides bent up and smooth. The central part BC, length 2,0 m, is horizontal and has a kinetic coefficient of friction of 0,2. If the particle slides from A, height h_1 = 1,0 m, calculate the height h_2 the particle will reach. Use energy considerations.



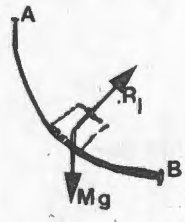
ANALYSIS OF PROBLEM

The problem obviously consists of three distinct parts: A → B moving down with no friction present; B → C moving horizontally with friction operating; C → D moving upward on a frictionless surface. We will consider each section separately and apply the steps to each.

$A \rightarrow B$

STEP 1 a):

Determine whether external forces, other than the weight and the normal reaction act on the surface.



The two forces indicated are the only forces acting.

STEP 1 b):

Total work done = 0

STEP 2:

Identify the first situation of the system and the reference level.

The most suitable reference level is BC. The first situation of the system is A.

$$\therefore \quad (E_k)_A = 0 \qquad (E_p)_A = M.gxh_1 = M.\ g.\ 1.$$

STEP 3

Identify the final situation of the system.

For the movement of the mass from A to B, the final situation is obviously at B.

$$(E_k)_B = \tfrac{1}{2}\,M.v_B^2 \qquad (E_p)_B = 0$$

STEPS 4 AND 5:

$$(F\cos\theta - f)s = (E_k + E_p)_B - (E_k + E_p)_A.$$

Using the values above:

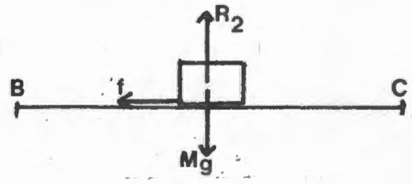
$$0 = \tfrac{1}{2}\,M.v_B^2 - M.g.\ 1.$$

$$\therefore \quad v_B = \sqrt{2\ g}\ \ m.s^{-1}.$$

$B \rightarrow C$

STEP 1 a):

Note that there is no force in the direction of C.



The mass will keep moving because of the kinetic energy acquired in moving from $A \rightarrow B$.

STEP 1 b):

Total Work Done = $(F\cos\theta - f)s = -f.2$.

STEP 2:

The reference level is BC while the first situation of the system in this case, is B.

$$\therefore\ (E_k)_B = \tfrac{1}{2} M.v_B^2 \qquad (E_p)_B = 0$$
$$= \tfrac{1}{2} M.(2g) \quad \text{(from previous section).}$$

STEP 3:

The final situation for movement from $B \rightarrow C$ is C.

$$\therefore\ (E_k)_C = \tfrac{1}{2} M.v_C^2 \qquad (E_p)_C = 0$$

STEPS 4 AND 5:

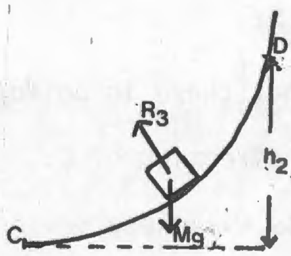
$$(F\cos\theta - f)s = (E_k + E_p)_C - (E_k + E_p)_B$$

$$\therefore\ -f.2 = \tfrac{1}{2} M.v^2{}_C - \tfrac{1}{2} M.2g$$
$$f = \mu R_2 = \mu M.g$$
$$\therefore\ -0{,}2 \times M.g.2 = \tfrac{1}{2} M.v_C^2 - \tfrac{1}{2} M.2g$$
$$\therefore\ v_C = \sqrt{12}\ m.s^{-1} \rightarrow$$

$C \rightarrow D$

STEP 1 a):

Since the surface is smooth, there is no friction.



STEP 1 b):

Total Work Done = 0.

STEP 2:

First situation is now C.

$$(E_k)_C = \tfrac{1}{2} M.v_C^2 = \tfrac{1}{2} M.12$$
$$(E_p)_C = 0$$

STEP 3:

Final situation is D

$$(E_k)_D = 0$$
$$(E_p)_D = M.gh_2$$

STEP 4 AND 5:

$$(F \cos\theta - f)s = (E_k + E_p)_D - (E_k + E_p)_C$$

$$\therefore \quad 0 = \tfrac{1}{2} M.12 - M.g.h_2.$$

$$\therefore \quad h_2 = \underline{0{,}6 \text{ m}} \rightarrow$$

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